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THE

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PHOTOGRAPHY OF THE SOLAR SPECTRUM FROM 6800 A TO 9600 A

By W. F. MEGGERS

Nearly thirty years have passed since Rowland photographed the solar spectrum from its limit in the ultra-violet at 3000 A into the visible red at 7300 A. Many attempts have been made to photograph the spectrum of the sun through the red and out into the adjacent infra-red, but thus far no results have been obtained which are comparable with those in the visible and ultra-violet regions.

The pioneer work in "Mapping the Least Refrangible End of the Solar Spectrum" was described in 1880 by Captain W. de W. Abney,[1] who prepared a silver-bromide emulsion which was sensitive to long waves. With this emulsion the prismatic spectrum of the sun was photographed to about 12000 A, and the diffraction spectrum to 10750 A. The dispersion of glass prisms was too small to show any detail in the spectrum, and less than 200 Fraunhofer lines were shown by the grating spectrum from 7600 A to 9825 A. In 1885 better photographs were obtained, and 590 Fraunhofer lines were recorded between the wave-length limits 7146 A and 9867 A.[2] The gratings were small and the photographic emulsion was

[1] *Philosophical Transactions*, **171**, 653, 1880.

[2] W. de W. Abney, *ibid.*, **177**, 457, 1886.

probably very coarse-grained; hence the definition was poor. The wave-lengths were determined only roughly, making exact coincidences with wave-lengths in spectra of known chemical elements impossible. W. Ritz[1] performed some experiments to improve Abney's emulsion, but the production of such red-sensitive emulsions appears to have been too difficult or uncertain to yield any further results in spectrum photography.

In 1891 George Higgs[2] prepared bisulphite compounds of alizarin blue and cerulin as photographic sensitizers, and succeeded in making an excellent map of the solar spectrum[3] out to 8300 A by the use of photographic plates stained with alizarin blue S. With a narrow slit and long exposures, results in the extreme red end of the spectrum were obtained which possessed most of the detail and definition usually so characteristic of the violet end. No measurements of wave-length were given except for the absorption bands of oxygen A, B, and α.[4]

In 1906 Millochau,[5] at Meudon Observatory, recorded some of the infra-red solar spectrum on photographic plates dyed with green malachite and "solarized" before use. Approximate wave-lengths of only 106 Fraunhofer lines between the limits 8025 A and 9325 A were given.

Up to the present time, therefore, no complete or accurate determinations of wave-lengths corresponding to Fraunhofer lines in the infra-red spectrum have existed, and there have been scarcely any reliable measurements in spectra of the chemical elements in this same region; identifications of absorption and emission lines have therefore been few and uncertain.

During the past year the Bureau of Standards[6] has had considerable success in photographing the red and adjacent infra-red regions of the spectra of chemical elements by means of ordinary photographic plates stained with dicyanin. The process of

[1] *Comptes Rendus*, **143**, 167, 1906.

[2] *Proceedings of the Royal Society*, **49**, 345, 1891.

[3] *Astrophysical Journal*, **7**, 86, 1898.

[4] *Proceedings of the Royal Society*, **54**, 200, 1894.

[5] *Comptes Rendus*, **144**, 725, 1907.

[6] *Bulletin of the Bureau of Standards*, **14**, 372, 1917.

sensitizing the plates was practically that recommended by Dr. Keivin Burns[1] in 1913. About 4 cc of a stock solution of one part dicyanin to one thousand parts of alcohol were added to 50 cc distilled water, 50 cc ethyl alcohol, and 5 cc of strong ammonia. Rapid photographic plates like Seed 27 or Graflex were immersed in such a bath from three to five minutes, rinsed in alcohol, and dried rapidly. Plates treated in this manner were found to be quite sensitive to wave-lengths between 6000 A and 9000 A. Such plates have been used with a six-inch concave grating, having 7500 lines per inch and 21 feet radius, to photograph the spectra of more than forty of the chemical elements. Exposures of thirty minutes recorded waves longer than 9000 A, and exposures of six to eight hours' duration have registered lines beyond 11000 A with arcs of the ferrous metals.

This success with artificial sources suggested an attempt to photograph the infra-red solar spectrum on plates stained with dicyanin. The work was undertaken at the Johns Hopkins University last April and May. For most of this work a five-inch plane grating with 20,000 lines per inch was used, with lenses of 18 feet (548.6 cm) focal length. This spectrograph gave a linear dispersion on the plate of 2 A per mm in the spectrum of the first order. A seven-inch grating with 15,000 lines per inch was also used. The apparatus was mounted on the fourth floor of the same laboratory, in the heart of Baltimore, where Rowland's work was done. In this laboratory disturbances are, no doubt, more frequent, vigorous, and troublesome now than they were in Rowland's time. Vibrations caused by passing street cars, railway trains, and heavy street traffic made it impossible to get satisfactory exposures of more than twenty minutes' duration. In spite of bad weather and hazy and smoky skies, these exposures sufficed to record the solar spectrum with good definition out to 9600 A. Under more favorable conditions this spectrum would have been photographed to still longer wave-lengths. The sensitiveness of dicyanin-stained plates to these long waves is demonstrated by the fact that the long wave limit of the visible spectrum (7600 A) was recorded by exposures of less than one minute. The region 8600 A required

[1] *Journal de physique* (5), **3**, 457, 1913.

only four or five minutes and 9600 A twenty minutes of exposure. Some of these photographs are reproduced in Plates I, II, and III, which show the solar spectrum from 6860 A to 9600 A.[1] Between the limits of wave-length 6800 A and 9600 A the original photographs show nearly 2000 Fraunhofer lines. Comparisons with Rowland's measurements in the region 6800 A to 7330 A show that some of the fine detail is lost by using coarse-grained emulsions for stained plates. By using fine-grained photographic plates and giving from five to ten times as much exposure, all of Rowland's faint lines could probably be registered by means of the dicyanin sensitizer.

Practically all of the 225 wave-lengths between 6800 A and 9600 A which have been photographed in the spectrum of the iron arc were easily identified with absorption lines in this region of the solar spectrum. These wave-lengths were therefore used as standards for the measurement of wave-lengths corresponding to the remainder of the absorption lines shown by these stained plates. Thus far nearly 400 of these solar wave-lengths have been identified with those of emission lines in the red and infra-red arc spectra of eighteen of the chemical elements. The spectra of eleven other elements have been measured in this spectral region, but none of their lines have been found among solar absorption lines, although some of these elements, notably the rare gases, have their strongest lines in the red and infra-red. Perhaps these rare gases should be looked for in the chromospheric spectrum of the sun. The table shows the number of Fraunhofer lines which have been measured in the region of long waves, and also indicates the number of lines which have thus far been identified with emission lines in the spectra of chemical elements.

In addition to the oxygen bands, many of the absorption lines in the long-wave region of the solar spectrum are, without doubt, of terrestrial origin. This is especially true of the lines near 8230 A and of the bands in the region 9300 A to 9600 A which Langley[2]

[1] It is difficult or impossible to reproduce the faintest lines and close lines which are near the limit of resolution, but at least 75 per cent of the total number of lines seen on the original photographs are shown in these Plates.

[2] *Annals of the Smithsonian Astrophysical Observatory*, 1, 1900.

designated ρ, σ, and τ, and believed to be due to water-vapor. There was no opportunity of photographing these when the sun was near the horizon because the walls of the building shielded the heliostat before nine o'clock in the morning and after four o'clock in the afternoon. Photographs taken at noon and three hours later do not show enough difference in line structure in these spectral regions to permit the separation of telluric from solar lines.

NUMBER OF LINES IN THE SOLAR SPECTRUM BETWEEN 6800 A AND 9600 A

Iron	200		Barium	1	
Nickel	63		Vanadium	1	
Titanium	27		Selenium	1	(?)
Cobalt	22		Silver	0	
Chromium	18		Lithium	0	
Silicon	10		Rubidium	0	
Manganese	6		Caesium	0	
Calcium	6		Strontium	0	
Tungsten	3	(?)	Gold	0	
Magnesium	2		Helium	0	
Sodium	2		Neon	0	
Potassium	2		Argon	0	
Copper	2		Krypton	0	
Molybdenum	2	(?)	Xenon	0	
Lanthanum	2	(?)	Unidentified	1600	

Dr. Schlesinger has recently given the Porter spectrograph[1] at Allegheny Observatory for the continuation of work on the infra-red solar spectrum. This spectrograph was designed for work on the rotation of the sun and is now being used to separate the solar from the telluric lines by means of the displacement suffered by solar lines in consequence of the solar rotation. At the same time an effort is being made to extend photography of the solar spectrum to waves longer than 10000 A.

Publication of the wave-lengths (from 6800 A to 9600 A) determined from the photographs of the solar spectrum made at the Johns Hopkins University will be postponed until further investigations, now in progress at the Bureau of Standards, in this

[1] *Publications of the Allegheny Observatory*, **3**, 99, 1914.

region of the spectra of the chemical elements have been completed. This will make possible the identification of more Fraunhofer lines. In the meantime the work at the Allegheny Observatory will give results for still longer waves, and will separate the solar from the telluric lines.

In conclusion it should be stated that the importance of dicyanin-stained plates for the purpose of photography in the red and infra-red regions of the spectrum cannot be overemphasized. Their convenience and efficiency bring an invisible long-wave spectral region as large as the entire visible spectrum within the reach of simple photography. Their sensitiveness to the long waves recommends their use for the extension of chromospheric and spot spectra as well as for the sun's photospheric spectrum.

Bureau of Standards
Washington, D.C.
November 9, 1917

A DETERMINATION OF THE SOLAR MOTION AND THE STREAM-MOTION FROM RADIAL VELOCITIES AND ABSOLUTE MAGNITUDES OF STARS OF LATE SPECTRAL TYPES[1]

By GUSTAF STRÖMBERG

In a recent memoir[2] by Adams and myself the results of an investigation concerning the relationship of average radial velocity to absolute magnitude have been published and we have shown that the intrinsically faint stars have a higher velocity than those that are intrinsically brighter. The stars were grouped in different shells according to distance from the sun, and no decided variation of velocity with distance could be found. Continuing this investigation, I have confirmed these results, and have further studied the solar motion and stream-motion as functions of absolute magnitude. This study, as well as the preceding, is based wholly on stars of the spectral types F, G, K, and M. For about 700 of these stars Adams has determined spectroscopically the absolute magnitudes and parallaxes.

When I commenced this work a number of the spectroscopic parallaxes were not definitely determined, and for this reason the absolute motions of the stars in space have not been computed, the radial velocities alone having been used. Hence the absolute magnitude has been employed only for the grouping of the stars. In order to increase the material, use has been made of about 600 determinations of radial velocity from the Lick and Mills Observatories. For these stars mean parallaxes have been computed from their proper motions and apparent magnitudes.

MEAN PARALLAXES

As mentioned in the paper cited, the recent determinations of stellar parallaxes by van Maanen, and of absolute magnitudes by Adams, indicate that stars of very small proper motion have parallaxes considerably larger than would be expected from their proper

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 144.

[2] *Mt. Wilson Contr.*, No. 131; *Astrophysical Journal*, **45**, 5, 1917.

motions, were the mean parallactic formula of Kapteyn employed. This is perhaps due to the fact that a list of apparently bright stars of late types, chosen on account of small proper motion, involves the selection, not only of distant stars of high intrinsic luminosity, but also of nearer stars whose velocity at right angles to the line of sight is nearly equal in direction and magnitude to the component of the sun's velocity in the same plane. Such a selection of stars of very small proper motion had actually been made in forming the list of stars to be observed for absolute luminosity and radial velocity at the Mount Wilson Observatory.

Kapteyn's formula for computing mean parallaxes from proper motion and apparent magnitude is[1]

$$\pi = A\mu^{b}\epsilon^{m}, \tag{1}$$

where μ is the proper motion and m the apparent magnitude, A, b, and ϵ being constants determined from measured parallaxes and from the parallactic motions of the stars. To render the material homogeneous, new constants were determined from 700 spectroscopically determined parallaxes. It was then found that the equation (1) gave a quite unsatisfactory representation of the smaller parallax. It is to be observed that hitherto the constants for the stars of later type have been based mainly on stars of large parallaxes, the formula for stars of small parallax being merely an extrapolation. Kapteyn's formula, however, is given a theoretical basis by Schwarzschild[2] on the assumption of a constant luminosity-curve and velocity-curve for all stars at all distances. Now if the velocity is a function of absolute magnitude, the velocity-curves and luminosity-curves are dependent on one another; and since among the distant stars we observe only those of high luminosity, both curves are dependent on the distance. There is further a strong tendency, especially among the K and M stars, to a separation of the absolute magnitudes into two groups, one of very high luminosity, the "giants," and one of very low luminosity, the "dwarfs."

[1] *Groningen Publications*, No. 8. Constants redetermined by van Rhijn, *Mt. Wilson Contr.*, No. 110; *Astrophysical Journal*, **43**, 36, 1916.

[2] *Astronomische Nachrichten*, **190**, 361, 1912.

We should also note that the small parallaxes determined spectroscopically can be measured with the same percentage of error as the large parallaxes, provided there are no large systematic errors. In using spectroscopic parallaxes we have the great advantage of securing logarithmic error-distribution within the different groups, as well as in their combination, while in the case of the directly measured parallaxes we have a normal distribution of error in the parallaxes themselves, assuming that we are dealing with a group of stars at the same distance. In Fig. 1 are plotted the values of $\log \pi$ as a function of $\log \mu$ for the spectral types F and G, K and M. The stars are treated separately according as they are near or distant from the sun's apices, and all have been reduced to the apparent magnitude 6.0 on the Harvard system. The points are the means of the logarithms of the parallaxes for different groups of stars of different proper motions. From the figure we see that the straight line

$$\log \pi = \log A + b \log \mu$$

gives a poor representation of the parallaxes of stars of small proper motion. Introducing the constants given by van Rhijn, which differ but slightly from those of Kapteyn, we have

$$\log \pi = -1.078 + 0.695 \log \mu - 0.0482 \,(m-6). \tag{2}$$

The equation

$$\log \pi = \log A + b \log (\mu + c) \tag{3}$$

gives a decidedly better representation of the observations. The constant b, which gives the direction of the asymptote for large proper motion, is very difficult of determination, the stars of large motion being too few in number for the purpose. Without affecting the accuracy appreciably, however, we may put $b = 1$, an error in b affecting only the parallaxes of stars of very large proper motion.

Table I gives the values of the constants in the equation

$$\log \pi = \log A' + \log (\mu + c) + (m-6) \log \epsilon \tag{4}$$

together with their mean errors. The mean error of $\log \pi$ is denoted by r, and the angular distance from the sun's apex by d.

From the comparison with the observations in Fig. 1 we see that there is a break in the continuity for the stars of types K and M between parallaxes 0".3 and 0".08 (log $\pi = -1.6$ and -1.1), corresponding to the division of the stars into giants and dwarfs.

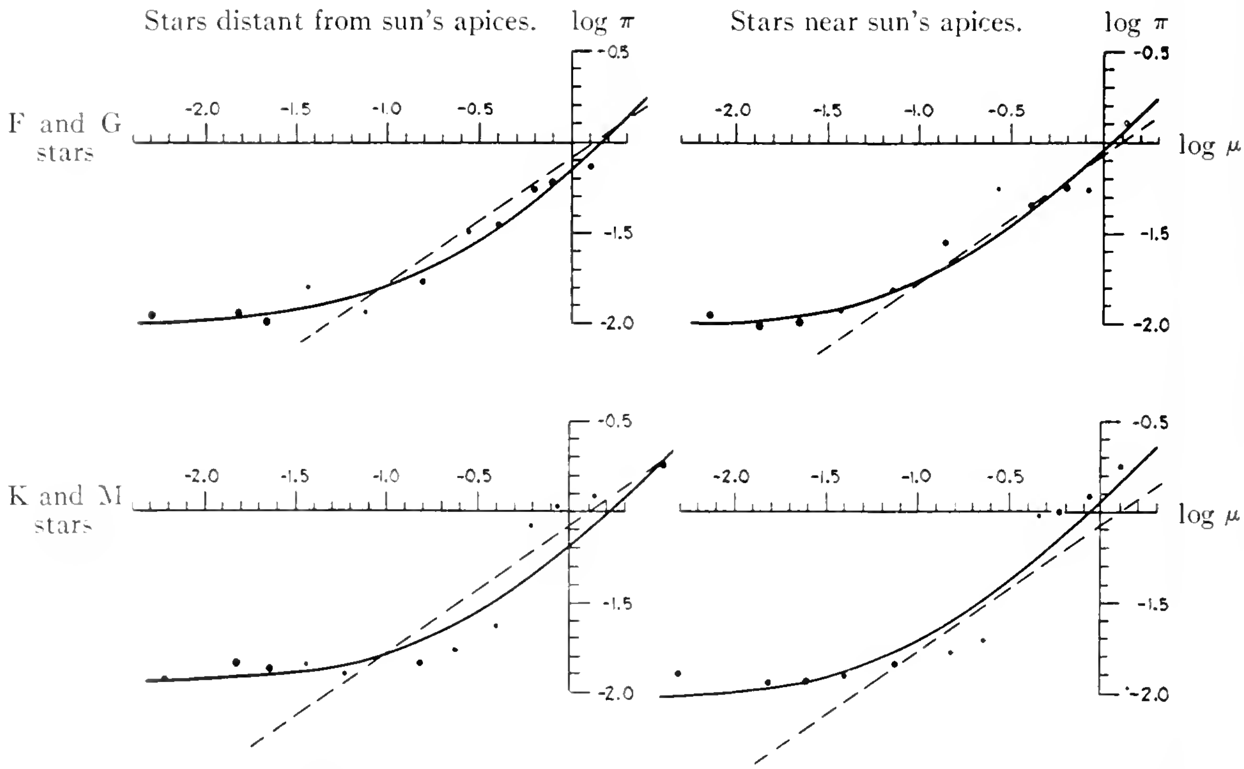

FIG. 1.—Relation between log π and log μ. The points represent means of log π, spectroscopically determined, for groups of stars, all parallaxes having been reduced to the sixth magnitude. The continuous curves represent

$$\log \pi = \log A + \log (\mu + c);$$

the broken lines represent

$$\log \pi = \log A + b \log \mu .$$

TABLE I

CONSTANTS FOR MEAN PARALLAX FORMULA

d	log A'	c	log ϵ	r	No.
	F and G Stars				
60°–120°	-1.208 ± 0.049	$+0''.157 \pm 0''.027$	-0.110 ± 0.015	± 0.073	244
0°–60°, 120°–180°	-1.081 ± 0.065	$+0.109 \pm 0.024$	-0.087 ± 0.040	± 0.093	202
	K and M Stars				
60°–120°	-1.253 ± 0.101	$+0''.200 \pm 0''.049$	-0.101 ± 0.020	± 0.15	117
0°–60°, 120°–180°	-0.965 ± 0.086	$+0.084 \pm 0.033$	-0.115 ± 0.018	± 0.15	135

Since there are scarcely any intermediate stars, I have not thought it necessary to compute separate constants for the stars of these two classes.

The difference in size of the points in Fig. 1 represents the difference in weights, which is based directly upon the number of stars in the different groups.

The values of log ϵ are probably not representative of all stars, since the faint stars are selected mainly on account of large proper motion and thus have parallaxes that are too large.

For values of μ smaller than about 0."03, the parallax is almost independent of proper motion, and we find

$$\log \pi = \log A' + \log c + (m-6) \log \epsilon . \tag{5}$$

SYSTEMATIC ERRORS OF THE PARALLAXES

In a paper by Kapteyn and Adams[1] it was suggested that in case the frequency function of the velocities were different from that given by Maxwell's law, its character would affect seriously the interpretation of the relationship found between proper motion and radial velocity. In order to eliminate this effect, Adams and I have used a formula for mean parallax which is independent of the nature of the frequency function of the velocity components, and based only on the assumption that for a group of stars scattered over the whole sky this function is the same in all directions. A brief account of the derivation of the formula and of the final equations used was included in our paper. A more detailed and complete statement is given here.

The star's velocity in space corrected for the sun's motion is projected on three axes, one coinciding with the radial velocity; of the other two, at right angles to the line of sight, one is directed toward the sun's apex and the other is at right angles thereto. The three components of the star's velocity in space, corrected for the sun's motion, are therefore

$$V' = V + V_0 \cos d ; \quad V_\nu = \frac{k\nu}{\pi} + V_0 \sin d ; \quad V_\tau = \frac{k\tau}{\pi} , \tag{6}$$

[1] *Mount Wilson Communications*, No. 1; *Proceedings of the National Academy of Sciences*, 1, 14, 1915.

in which V is the star's radial velocity, V_0 the sun's velocity, d the angle between the star and the sun's apex, ν and τ the components of the proper motion in the directions mentioned, and k a constant equal to 4.738 km/sec. Assuming the frequency function F to be the same in these three directions, we have

$$F(V') = F(V_\nu) = F(V_\tau).$$

In a group of stars the logarithms of the parallaxes are distributed more nearly in accordance with a normal frequency-curve than are the parallaxes themselves. In forming the mean parallax we therefore deal with the logarithms and use the geometrical instead of the arithmetical mean, and write accordingly,

$$\text{Geometrical Mean Parallax} = \underline{\pi} = \sqrt[n]{\Pi(\pi)}\,,$$

whence

$$\log \underline{\pi} = \frac{1}{n}\Sigma \log \pi = \overline{\log \pi},$$

in which n is the number of individual parallaxes. A further advantage of this method of procedure is that the ratios in equation (6) are thus transformed into differences from which we can form directly $\overline{\log \pi}$ or $\log \underline{\pi}$, whatever the range in the parallax.

We have, therefore,

$$F_1 (\log V') = F_1 (\log V_\tau) \text{ or } \overline{\log V'} = \overline{\log V_\tau}$$

where V' is to be taken regardless of sign. We then find

$$\overline{\log \pi} = \log k + \overline{\log \tau} - \overline{\log V'}$$

or

$$\underline{\pi} = \frac{k\underline{\tau}}{\underline{V'}}, \tag{7}$$

a bar above the symbol denoting the arithmetical mean, a bar below the symbol the geometrical mean.

For the ν component we have

$$F(V') = F(V_\nu) = F\left(\frac{k\nu}{\pi} + V_0 \sin d\right).$$

Hence,

$$F_1(V' - V_0 \sin d) = F_1\left(\frac{k\nu}{\pi}\right)$$

and

$$F_2[\log (V' - V_0 \sin d)] = F_2\left(\log \frac{k\nu}{\pi}\right)$$

or, finally,

$$\overline{\log \pi} = \log k + \overline{\log \nu} - \overline{\log (V' - V_0 \sin d)} \tag{8}$$

or

$$\pi = \frac{k\nu}{\underline{V' - V_0 \sin d}} .$$

Since $F(V')$ can be assumed to be equal to $F(-V')$ we also find

$$F(-V') = F\left(\frac{k\nu}{\pi} + V_0 \sin d\right)$$

or

$$F_1 [\log (-V' - V_0 \sin d)] = F_1[\log (V' + V_0 \sin d)] = F_1\left(\log \frac{k\nu}{\pi}\right).$$

Hence,

$$\overline{\log \pi} = \log k + \overline{\log \nu} - \overline{\log (V' + V_0 \sin d)} \tag{8a}$$

or

$$\pi = \frac{k\nu}{\underline{V' + V_0 \sin d}} .$$

Taking the mean of (8) and (8*a*) we find

$$\overline{\log \pi} = \log k + \overline{\log \nu} - \tfrac{1}{2}[\overline{\log (V' + V_0 \sin d)} + \overline{\log (V' - V_0 \sin d)}]. \tag{9}$$

For a large group of stars scattered over the whole sky there can be only a small effect of preferential motion, and the assumption of the equality of the frequency function in the three components probably is nearly fulfilled. Furthermore, among the stars of later type the *K*-term, which can be interpreted either as a systematic error in the determination of radial velocity or as a general dilatation or contraction of the stellar system, appears to be very small. In the paper by Adams and myself already cited, we found for parallaxes larger than 0.″017 a good agreement between the values computed from the formulae (7) and (8) and the parallaxes actually employed, the latter being in part spectroscopic determinations and in part computed from formula (5). For smaller parallaxes, however, equations (7) and (8) gave smaller values than those actually employed.

Formulae (7) and (9) have now been applied to a large number of parallaxes determined spectroscopically by Adams in order to find the systematic errors in his values. Stars of proper motion smaller than 0."020 annually have been excluded. Table II contains the result of the comparison. $\overline{M}_A$ is the mean absolute magnitude as determined by Adams, $\overline{m}$ the mean apparent magnitude, and $\pi_A = 10^{0.2(\overline{M}-\overline{m})-1}$ the geometrical mean of the spectroscopic parallaxes; π_τ and π_ν have been computed by equations (7) and (9). $\overline{V'}$ is the geometrical mean of the radial velocities and $\overline{M}_1$ the absolute magnitude computed from

TABLE II

SYSTEMATIC ERROR OF SPECTROSCOPIC PARALLAXES

M	No.	$\overline{m}$	$\overline{M}_A$	π_A	π_τ	π_ν	$\frac{1}{2}(\pi_\tau+\pi_\nu)$	$\overline{V'}$	$\overline{M}_1$	$O-C$ $\overline{M}_A-\overline{M}_1$
	A8–F7 Stars									
								km		
≦3.9	32	4.81	+2.54	0."035	0."034	0."043	0."038	10.3	+2.71	−0.17
≧4.0	33	5.89	+4.78	0.060	0.041	0.057	0.049	17.8	+4.34	+0.44
	F8–G7 Stars									
≦3.9	81	5.07	+1.49	0.019	0.024	0.020	0.022	11.0	+1.79	−0.30
≧4.0	104	6.61	+5.34	0.056	0.049	0.079	0.064	23.9	+5.64	−0.30
	G8–K4 Stars									
≦1.9	89	5.04	+1.00	0.0156	0.0161	0.0176	0.0168	10.3	+1.16	−0.16
≧2.0	91	6.87	+5.26	0.0477	0.0421	0.0460	0.0441	18.9	+5.09	+0.17

$$\overline{M}_1 = \overline{m} + 5 + 5 \log \pi, \tag{10}$$

where

$$\pi = \tfrac{1}{2}(\pi_\nu + \pi_\tau).$$

Formulae (7) and (9) for the mean parallax appear to possess some advantages over those commonly used when the range in proper motion is large. It is not necessary that the variation in π be very small, which is an essential condition both in Campbell's formula $\overline{\pi} = k\overline{\tau}/\overline{V'}$ and in the formula for computing mean parallaxes from the parallactic motion, namely,

$$\Sigma V_\nu = 0,$$

or

$$\pi = -\frac{\overline{kv \sin d}}{V_0 \overline{\sin^2 d}}.$$

In order to secure this result, the stars usually are divided into groups of the same proper motion and apparent magnitude, but we then often obtain groups which are too small to furnish values of π accurate enough for deriving absolute magnitudes. Further if the geometrical mean of π is used, we can compute directly from the mean apparent magnitude the mean absolute magnitude, whatever the range in π.

The largest difference in absolute magnitude in Table II is only 0.4, corresponding to an error in the parallax of $\frac{1}{5}$ of its value. We may therefore assume that the systematic errors in the spectroscopic parallaxes are comparatively small.

AVERAGE RADIAL VELOCITY AND ABSOLUTE MAGNITUDE

In the paper by Adams and myself it was stated that a very pronounced correlation exists between absolute magnitude and average radial velocity. In that investigation types F and G were combined as well as types K and M. I have now treated types F, G, and K separately and have found as before that no appreciable effect of distance can be detected by studying separately stars at nearly the same distance. In addition, I have computed the so-called K-term, i.e., the algebraic mean of the radial velocities corrected for the sun's motion. With reversed sign this term can be regarded as the systematic correction to the measured radial velocities, on the assumption that the algebraic sum of the radial velocities corrected for the sun's motion is equal to zero.

The results are in Table III. $\overline{M}$ is the mean absolute magnitude, $\overline{m}$ the mean apparent magnitude, π the geometrical mean of the parallaxes, $\underline{V}'$ the geometrical mean of the radial velocities corrected for the sun's motion, θ the average radial velocity regardless of sign, and K the algebraic mean of the radial velocities. The radial velocities are here corrected for the sun's motion, the constants

$$A = 270^\circ, \qquad D = +30^\circ, \qquad V = 20.0 \text{ km}$$

being assumed. The ratio of the number of stars within galactic latitudes $+30°$ and $-30°$ to the whole number of stars in the group is denoted by c, which is, accordingly, a measure of the galactic condensation. Were the stars uniformly distributed over the sky the value of c would be 0.5.

TABLE III

RADIAL VELOCITY AND ABSOLUTE MAGNITUDE

M	No.	$\overline{M}$	$\overline{m}$	π	V'	c	No.	θ	K
				F Stars					
					km			km	km
$\leqq 0.9$	56	0.24	4.16	0″.016	7.0	0.86	56	10.8	+0.8
1.0–1.9	73	1.54	4.83	.022	9.7	.55	73	14.4	+0.8
2.0–2.9	66	2.42	5.32	.026	8.6	.64	65	12.9	−1.5
$\geqq 3.0$	104	4.28	5.70	.052	15.3	.45	92	16.3	−1.0
	299	2.44	5.12	0.029	10.4	0.59	286	14.0	−0.3
				G Stars					
$\leqq 0.9$	152	0.31	4.91	.012	7.8	.66	149	11.3	+1.3
1.0–1.9	112	1.38	5.76	.013	10.0	.50	108	13.1	+0.2
2.0–3.9	58	2.74	5.60	.027	14.5	.50	51	17.2	−2.7
4.0–5.9	73	4.97	6.49	.050	15.3	.42	59	15.7	−2.3
$\geqq 6.0$	36	6.90	7.36	.081	20.2	.39	30	21.2	−4.9
	431	2.25	5.70	0.020	11.0	0.53	397	14.1	−0.5
				K Stars					
$\leqq 0.9$	124	0.52	4.18	.018	8.5	.72	124	13.5	+1.6
1.0–1.9	251	1.42	4.84	.021	11.6	.54	251	16.5	+1.8
2.0–4.9	108	2.59	5.08	.032	14.7	.53	107	21.0	+6.1
$\geqq 5.0$	78	7.10	7.45	.085	17.4	.46	78	27.6	−7.9
	561	2.23	5.11	0.027	12.0	0.56	560	18.3	+1.2

Both the geometrical means, V', and the arithmetical means, θ, of the radial velocities show a decided increase with decreasing brightness. There is, however, for all three types an apparent deviation from a linear relationship between radial velocity and absolute magnitude. If we plot the numbers, we find a break in the straight line at absolute magnitude 2 for F stars, at 3 or 4 for G stars, and between 4 and 6 for K stars. The same break occurs in the K-term, the absolutely bright stars having a positive K-term

and the absolutely faint a negative K-term. This break in the continuity is probably related to the division of stars into the giant and dwarf classes.

If we assume the mean velocity to be a function of the mass rather than of the absolute brightness, the more massive stars having smaller velocity than the less massive, we find that the ratio of luminosity to mass is larger for the giants than for the dwarfs. This is in harmony with Adams' results that the low-temperature lines are strengthened in the spectra of the dwarf stars.

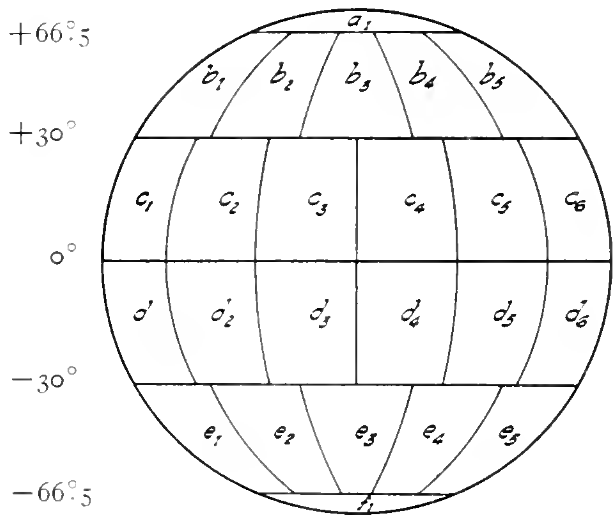

FIG. 2.—Distribution of forty-eight standard regions in the sky.

From Table III we find further that the very luminous stars show a much larger galactic concentration than the absolutely faint stars, the latter seeming rather to avoid the galaxy.

SOLAR MOTION AND ABSOLUTE MAGNITUDE

In order to find whether any decided relationship exists between the solar motion and the absolute magnitude, I have grouped the stars according to absolute magnitude, taking types F and G together. Only the radial velocities have been used in this investigation. Adopting a method of Charlier,[1] I divided the sky into 48 regions of equal area; but instead of using the equator as a plane of symmetry, I have grouped the stars in areas symmetrical to the galactic plane. This was done mainly to simplify the study of the stream-motion. Fig. 2 shows the location and designation of the areas.

Regions c_1 to c_{12} and d_1 to d_{12} are situated along the galactic equator between latitudes $+30°$ and $-30°$, the origin of longitude

[1] *Meddelanden från Lunds Observatorium*, Ser. II, No. 8, 1912.

being the intersection of the galactic and celestial equators. The regions b_1 to b_{10} and e_1 to e_{10} are situated between latitudes $\pm 31°$ and $\pm 66°$. Regions a and f are in latitudes higher than $\pm 67°$.

The centers of gravity of regions a_1 to c_{12} are given in Table IV. The centers for areas d, e, and f have the same longitudes, but galactic latitudes $-14°48$, $-45°1$, and $-80°$, respectively.

TABLE IV

CENTERS OF GRAVITY OF STANDARD REGIONS

Region	Gal. Long.	Gal. Lat.	Region	Gal. Long.	Gal. Lat.
a_1	90°	+80°	c_1	15°	+14°48
a_2	270	+80	c_2	45	+14.48
b_1	18	+45.1	c_3	75	+14.48
b_2	54	+45.1	c_4	105	+14.48
b_3	90	+45.1	c_5	135	+14.48
b_4	126	+45.1	c_6	165	+14.48
b_5	162	+45.1	c_7	195	+14.48
b_6	198	+45.1	c_8	225	+14.48
b_7	234	+45.1	c_9	255	+14.48
b_8	270	+45.1	c_{10}	285	+14.48
b_9	306	+45.1	c_{11}	315	+14.48
b_{10}	342	+45.1	c_{12}	345	+14.48

The pole of the galactic plane is assumed to have the co-ordinates[1] $\alpha=190°6$, $\delta=+27°2$ (1900).

The equations for finding equatorial from galactic co-ordinates are then

$$\left.\begin{aligned} \cos\alpha\cos\delta &= +0.1846x+0.4494y-0.8740z \\ \sin\alpha\cos\delta &= -0.9828x+0.0844y-0.1642z \\ \sin\delta &= \qquad\qquad\;\; +0.8893y+0.4573z \end{aligned}\right\} \tag{11}$$

where

$$x=\cos l\cos b, \qquad y=\sin l\cos b, \qquad z=\sin b. \tag{12}$$

The use of the galactic co-ordinates instead of equatorial is of no advantage in the computation of the solar apex, but is very convenient in the study of the stream-motion.

The equation of condition for the determination of the sun's motion from radial velocities is

$$\gamma_1 X_0+\gamma_2 Y_0+\gamma_3 Z_0+K=V, \tag{13}$$

[1] *Groningen Publications*, No. 18, 1908.

which is to be computed for each region separately, γ_1, γ_2, and γ_3 being the direction-cosines of the center of gravity of the regions, V the algebraic mean of the observed radial velocities, K the K-term, and X_0, Y_0, and Z_0 the components of the motion of the centroid of the group of stars relative to the sun. Further,

$$X_0 = -V_0 \cos B_0 \cos L_0,\quad Y_0 = -V_0 \cos B_0 \sin L_0,\quad Z_0 = -V_0 \sin B_0 \qquad (14)$$

where L_0 and B_0 are the galactic co-ordinates of the sun's apex and V_0 the sun's velocity.

The different regions have been given weights proportional to the number of stars in each. The results, with mean errors appended, are in Table V; θ is the average radial velocity.

TABLE V

SUN'S APEX AND VELOCITY

M	No.	M	m	A	D	V_0	K	θ
					F and G Stars			
						km	km	km
≦0.0	211	0.31	4.68	251°.6 ± 6°.6	+22°.7 ± 6°.1	19.4 ± 1.8	+ 0.2 ± 1.8	11.4
1.0–1.9	177	1.44	5.42	267.5 ± 11.0	+36.3 ± 12.4	16.6 ± 2.4	+ 0.0 ± 1.5	14.6
2.0–3.9	167	2.76	5.27	271.6 ± 9.0	+34.6 ± 10.0	22.0 ± 2.8	− 2.1 ± 1.7	16.2
≧4.0	170	5.29	6.41	279.7 ± 9.5	(+ 7.4 ± 9.9)	(26.5 ± 4.1)	− 1.9 ± 2.7	23.9
	725	2.32	5.40	268.1 ± 4.1	+25.2 ± 4.9	19.94 ± 1.38	− 0.69 ± 0.85	17.15
					K Stars			
≦0.9	122	0.54	4.22	282°.4 ± 9°.2	+32.8 ± 8.8	23.6 ± 2.6	+ 2.0 ± 1.6	13.6
1.0–1.9	245	1.41	4.86	268.6 ± 8.3	+36.5 ± 9.2	20.5 ± 2.4	+ 1.1 ± 1.4	16.6
2.0–4.9	99	2.58	5.12	282.1 ± 9.9	+16.1 ± 9.4	26.4 ± 4.6	+ 5.8 ± 2.4	18.1
≧5.0	79	7.07	7.41	(285.9 ± 28.9)	(−13.0 ± 34.4)	(15.3 ± 6.6)	(−11.1 ± 4.9	26.5
	545	2.25	5.13	277.5 ± 5.5	+32.2 ± 5.8	22.23 ± 1.73	+ 1.05 ± 1.00	18.40
					Giant M Stars			
Bright	135	1.5	4.98	264°.8 ± 7°.4	+26.3 ± 8.2	27.0 ± 3.3	+ 1.25 ± 1.86	16.9

There is an indication that the absolutely faint stars give a smaller declination of the sun's apex than the brighter stars, which is in harmony with the result that stars of large proper motion give a smaller declination of the apex than stars of small motion. This is usually explained on the two-drift theory: Drift I having the

greater velocity relative to the sun, the stars belonging to it have on an average greater proper motions than those of Drift II. The relation to absolutely faint stars is partly the result of a selection of stars of large proper motion. The results for the faint stars, however, are very uncertain, as these have been observed mainly in the northern sky only (Mount Wilson Observatory).

Putting $K=0$, the discrepancy in the declination of the sun's apex is much diminished. The results corresponding to this assumption are in Table V*a*.

TABLE V*a*

SUN'S APEX AND VELOCITY—K-TERM $=0$

$\overline{M}$	A	D	V_0	θ
		F and G Stars		
			km	km
0.31	251°4	+22.5	19.4	11.3
1.44	267.5	+36.3	16.6	14.6
2.76	272.1	+36.4	22.8	16.3
5.29	279.6	+10.9	27.1	23.9
2.32	268.3	+26.1	20.13	17.15
		K Stars		
0.54	279.6	+33.8	24.0	13.7
1.41	268.1	+37.2	20.4	16.6
2.58	284.5	+20.1	26.0	18.6
7.07	289.0	+26.5	22.1	26.2
2.25	277.0	+32.5	22.16	18.5
		Giant M Stars		
1.5	264.2	+26.1	26.8	16.9

From all later-type stars (excluding dwarf M stars) we find for the solar motion

$A_0 = 270°9 \pm 3°28$	$K=0$ (assumed)
$D_0 = +\ 29.2 \pm 3.45$	$A_0 = 270°7$
$V_0 = 21.48 \pm 1.02$ km/sec	$D_0 = +29.0$
$\theta = 17.71$ km/sec	$V_0 = 21.42$ km/sec
$K = +\ 0.36 \pm 0.60$ km/sec	$\theta = 17.71$ km/sec

No. of stars 1405

The values of θ are found a posteriori from the least-squares solution. Their increase with the decreasing brightness is well shown. There is no pronounced relationship between absolute magnitude and solar motion, except the decrease of declination for the intrinsically faint stars, which probably is partly an effect of different proportions of stars belonging to the two drifts. It is to be observed that these faint stars cannot be compared with apparently faint stars, which according to Dyson and others give a high declination for the sun's apex. The latter are mainly very distant stars, while the dwarfs are stars at very small distances.

PREFERENTIAL MOTIONS

The preferential motion of the stars used in this investigation has been determined from the radial velocities alone. Instead of using the dispersion—that is, the square root of the sum of the squares of radial velocities corrected for the solar motion divided by the number of stars—I have used the average radial velocity as a measure of the mobility in the line of sight. Squaring the velocities exaggerates the effect of a few high velocities, and even if we exclude velocities larger than a certain limit, there is often too great a number of high velocities near this limit. (Cf. Eddington, *Stellar Movements*, p. 147.)

The grouping of stars according to absolute brightness is very useful for the study of preferential motion, since the different groups are then more homogeneous as regards their mean velocities and only a very few stars of high velocity have to be excluded. Since the reductions for each star were computed before the solar motion was determined, I assumed for the latter $A=270°$, $D=+30°$, and $V_0=20.0$ km. These differ slightly from the values just found for the later-type stars, but the effect of the difference is quite inappreciable. No systematic correction has been applied to the measured radial velocities (K-term) since the values found in Table V have rather large errors, and are themselves probably too small to affect the result appreciably. The average velocity corrected for the sun's motion was computed for each region, the corrections for the sun's motion being determined separately for each star, and the

co-ordinates of the center of the region were used for computing the velocity surface.

I have made use neither of the ellipsoidal theory nor of the method for computing the two drifts, but have employed a more general method, which enables me to take into account the difference in mean velocity at opposite points of the sky. We know that the sun is somewhat north of the real galactic plane, and, further, that the B-type stars and the apparently faint stars in general are not uniformly distributed in galactic longitude. If in accordance with Turner's suggestion we assume the stream-motion to be a gravitational effect, we cannot expect a priori the velocity surface to be symmetrical with respect to the sun, but may anticipate it to be related in some way to the eccentric position of the sun in the stellar system. Since the mean distance of the late-type stars is of the same order of magnitude as the distance to the center of the stellar system, we may expect asymmetrical (odd) terms in the analytical expression of the velocity surface. Such asymmetrical terms would also exist if the velocity of the stream-motion were different in different regions of space, which has been suggested by Kapteyn (*Mount Wilson Annual Report*, 1916, p. 255).

We desire to express the average radial velocity as a continuous function of the longitude and latitude of the different regions. We start from the following rational integral function of the direction-cosines:[1]

$$
\begin{aligned}
r = a_0 &+ b_1x + b_2y + b_3z + c_1x^2 + c_2y^2 + c_3z^2 + c_4xy + c_5xz \\
&+ c_6yz + d_1x^3 + d_2y^3 + d_3z^3 + d_4x^2y + d_5xy^2 + d_6x^2z \\
&+ d_7xz^2 + d_8y^2z + d_9yz^2 + d_{10}xyz + \dots \qquad (15)
\end{aligned}
$$

in which r is the radius vector and equal to the average radial velocity θ, and x, y, and z are the direction-cosines of a point on the surface as defined by equation (12). We then have

$$x^2 + y^2 + z^2 = 1. \qquad (16)$$

The intersection of this surface with the XY-plane has the equation

$$r = a_0 + b_1x + b_2y + c_1x^2 + c_2y^2 + c_4xy + d_1x^3 + d_2y^3 + d_4x^2y + d_5xy^2 + \dots \qquad (17)$$

[1] Such a surface including terms of second order only was used by Eddington, *Monthly Notices*, 75, 521, 1915, as representing approximately an ellipsoidal velocity-distribution.

Introducing the longitudes we find

$$r = a_0 + \beta_1 \cos l + \beta_2 \sin l + \gamma_1 \cos 2l + \gamma_2 \sin 2l + \ldots \quad (18)$$

whence we see that (17) can be represented by harmonic terms of the same order as the degree of equation (15). This property holds for every intersection of the surface with a plane through the origin, for the form of equation (15) is unaltered if we change the direction of the axes of the system of co-ordinates. The arguments of the harmonic terms are thus multiples of longitudes reckoned in the intersection plane. Taking into account this property, we find that the number of independent coefficients in the equation of the surface is $(n+1)^2$ where n is the degree of the equation. In equation (15) some of the coefficients are thus superfluous, a fact depending on the circumstance that x, y, and z are connected by equation (16). These we shall eliminate as follows.

It is convenient to define the coefficients in the expression for the surface in such a way that the terms in x, y, and z represent deviations from the mean radius vector. We therefore introduce the condition $r = a_0$, r being the mean of all the radii vectores to the surface. The integral over the sphere for all other terms must be equal to zero. This is the case for each of the terms as they stand, except those involving x^2, y^2, and z^2. The condition that the integrals for the sum of these terms may also be zero is found as follows: Since x, y, and z are direction-cosines, the definite integrals of x^2, y^2, and z^2 over the sphere are equal. Denoting the value of these integrals by σ, the required condition is

$$(c_1 + c_2 + c_3)\,\sigma = 0,$$

whence

$$c_1 = -c_2 - c_3.$$

To eliminate the remaining superfluous coefficients we write the identities

$$\left.\begin{aligned} \alpha x^3 + \alpha x y^2 + \alpha x z^2 - \alpha x &= 0 \\ \beta x^2 y + \beta y^3 + \beta y z^2 - \beta y &= 0 \\ \gamma x^2 z + \gamma y^2 z + \gamma z^3 - \gamma z &= 0 \end{aligned}\right\} \quad (19)$$

where α, β, and γ are quite arbitrary. Assuming

$$d_7 - \alpha = d_8 - \gamma = d_9 - \beta = 0$$

and adding equations (15) and (19) and simplifying, we find

$$r=\bar{r}+B_1x+B_2y+B_3z+C_1x^2+C_2y^2-(C_1+C_2)z^2+C_3xy+C_4xz \\ +C_5yz+D_1x^3+D_2y^3+D_3z^3+D_4x^2y+D_5xy^2+D_6x^2z+D_7xyz, \tag{20}$$

which contains no superfluous coefficients.

Introducing now longitudes and latitudes defined by equation (12), we find the following trigonometrical expression for the surface:

$$\begin{aligned} r=\bar{r}&+\beta_1 \cos b \cos l+\beta_2 \cos b \sin l+\beta_3 \sin b+\gamma_1 \cos^2 b \cos 2l \\ &+\gamma_2 \cos^2 b \sin 2l+\gamma_3(2/3-\cos^2 b)+\gamma_4 \sin 2b \cos l \\ &+\gamma_5 \sin 2b \sin l+\delta_1, \cos^3 b \cos 3l+\delta_2 \cos^3 b \sin 3l \\ &+\delta_3 \cos^3 b \cos l +\delta_4 \cos^3 b \sin l+\delta_5 \sin^3 b \\ &+\delta_6 \cos^2 b \sin b \cos 2l+\delta_7 \cos^2 b \sin b \sin 2l. \end{aligned} \tag{21}$$

Equation (21) is the expression for the surface to terms of the third order inclusive. For application I have changed the form of (21) somewhat in order to separate the terms more easily and to secure nearly the same variation in the different trigonometrical combinations, so that the coefficients may indicate nearly the real effect of the different terms.

The form used is:

$$\begin{aligned} \theta=a_0&+a_1 \cos b \cos l+a_2 \cos b \sin l+a_3 \sin b+a_4 \cos^2 b \cos 2l \\ &+a_5 \cos^2 b \sin 2l+a_6 (2-3 \cos^2 b)+a_7 \sin 2b \cos l \\ &+a_8 \sin 2b \sin l+a_9 \cos^3 b \cos 3l+a_{10} \cos^3 b \sin 3l \\ &+a_{11} \cos 3b \cos l+a_{12} \cos 3b \sin l+a_{13} \sin 3b \\ &+a_{14}\, 8/3 \cos^2 b \sin b \cos 2l+a_{15}\, 8/3 \cos^2 b \sin b \sin 2l. \end{aligned} \tag{22}$$

The terms with coefficients a_1 to a_3 are of the first order, a_4 to a_8 of the second order, and a_9 to a_{15} of the third order: i.e., they have 1, 2, and 3 maxima and minima, respectively. Each term represents a deviation from the sphere $\theta=\bar{\theta}=a_0$ but does not affect the mean value of $\bar{\theta}=a_0$ for the whole sphere.

Turning to Tables III and V, we find that average velocity is more closely related to absolute luminosity than to spectral type. For this reason and in order to obtain groups large enough for studying the variation in average velocity for different regions in the sky, the stars were divided into three groups having different radial velocities, each including stars of types F, G, and K. The first group contains the stars intrinsically brightest with absolute mag-

nitudes equal to or less than 1.9 for F and G stars, and equal to or less than 0.9 for the K stars. The average radial velocity for this group varies from 11.4 to 14.6 km (see Table V). In the second group of stars, of absolute magnitudes 2.0–3.9 for F and G stars and 1.0–4.9 for the K stars, the average radial velocity ranges from 16.2 to 18.1 km. In the last group all stars fainter than these limits were taken together, with average velocity ranging from 23.9 to 26.5 km. This group also includes 11 dwarf M stars with an average radial velocity of about 30 km. These groups are more homogeneous with regard to velocity and probably even to mass than groups based solely on spectral types. Further data are given in Table VI.

TABLE VI

GROUPS FOR STUDY OF PREFERENTIAL MOTION

Group	No.	$\overline{M}$	$\overline{m}$	$\overline{\pi}$	θ
					km
I Very bright.......	509	0.76	4.83	0″.015	13.11
II Bright...........	513	2.08	5.09	0.025	17.13
III Faint............	260	6.05	6.82	0.070	25.88

The average velocity for the stars of these groups in each of the standard regions, corrected for the sun's motion, is given in Table VII. The problem is the representation of these data by the surface defined by equation (22).

TERMS OF EVEN DEGREE

Since the terms of odd degree are equal and opposite in sign for opposite points in the sky, we may eliminate them by using the means for opposite regions. We can thus derive independently the terms of even degree. The corresponding equation,[1] representing radial velocities, is

$$\theta = a_0 + a_4 \cos^2 b \cos 2l + a_5 \cos^2 b \sin 2l + a_6 (2 - 3 \cos^2 b) + a_7 \sin 2b \cos l + a_8 \sin 2b \sin l .$$

In combining opposite regions the average radial velocity was computed by giving weights proportional to the number of stars; but for the determination of the constants of the surface, the 24 means

[1] This expression was used by Hough and Halm for computing the solar motion on the assumption of two opposite streams (*Monthly Notices*, **70**, 94, 1909).

TABLE VII

VELOCITIES FOR STANDARD REGIONS

REGION	I θ	I No.	II θ	II No.	III θ	III No.	REGION	I θ	I No.	II θ	II No.	III θ	III No.
	km		km		km			km		km		km	
a_1 . . .	. . . . .	0	8.3	7	35.3	11	f_1	11.5	10	8.4	6	15.2	3
a_2	5.9	4	18.2	13	25.8	3	f_2	16.0	7	23.6	9	5.4	2
	5.9	4	14.7	20	33.3	14		13.3	17	17.5	15	11.3	5
b_1	11.8	12	18.4	17	31.4	14	e_1	12.2	14	11.2	9	39.7	4
b_2	8.2	5	17.6	10	16.2	8	e_2	12.2	6	15.3	12	27.9	9
b_3 . . .	13.0	12	20.3	6	(20.3)	1	e_3	13.0	16	15.7	11	25.2	13
b_4 . . .	11.6	12	16.9	9	32.2	13	e_4 . . .	13.9	9	21.3	7	21.0	9
b_5 . . .	14.8	17	16.2	10	26.4	13	e_5	15.8	4	23.6	8	23.7	3
b_6 . . .	10.1	6	16.3	10	(34.5)	2	e_6	14.0	4	8.8	15	(36.6)	1
b_7 . . .	11.6	5	21.9	8	8.6	5	e_7	9.4	7	19.6	14	25.2	3
b_8 . . .	5.9	5	13.6	5	4.1	7	e_8	8.6	5	18.1	16	(18.1)	1
b_9 . . .	8.0	4	17.7	11	31.0	8	e_9	11.1	8	21.4	8	(30.8)	5
b_{10} . . .	14.0	3	17.0	14	35.8	8	e_{10} . . .	12.5	9	17.6	9	(36.3)	1
	12.20	81	17.63	100	25.7	79		12.25	82	16.68	109	27.1	49
c_1 . . .	10.8	15	22.1	12	20.2	9	d_1 . . .	12.9	10	9.2	8	30.3	11
c_2 . . .	11.4	24	14.5	11	18.2	8	d_2 . . .	14.3	20	10.1	18	23.8	8
c_3 . . .	14.8	12	15.3	21	22.9	7	d_3 . . .	14.0	18	14.0	12	19.0	11
c_4 . . .	5.0	9	9.6	12	14.5	6	d_4 . . .	16.0	18	19.9	21	26.7	6
c_5 . . .	10.4	13	16.2	8	39.0	9	d_5	11.7	16	18.9	7	36.9	3
c_6 . . .	17.2	18	14.8	8	19.0	6	d_6	18.0	12	20.2	8	32.0	8
c_7	18.2	11	20.0	7	27.5	4	d_7	18.9	12	35.9	7	. . . . . .	0
c_8 . . .	14.4	8	15.4	9	(0.8)	1	d_8	11.2	14	14.3	10	(4.7)	1
c_9 . . .	9.2	12	9.7	10	. . . . . .	0	d_9	13.3	19	11.6	12	(0.7)	1
c_{10}	9.1	4	16.3	14	(68.0)	1	d_{10}	8.3	10	16.3	13	(25.7)	1
c_{11}	18.6	9	15.7	12	(6.0)	2	d_{11}	16.6	12	26.3	13	. . . . . .	0
c_{12} . . .	16.6	8	17.6	8	29.7	8	d_{12} . . .	13.7	15	27.8	8	43.0	2
	13.07	143	15.47	132	24.1	61		14.00	182	19.03	137	26.5	52

thus obtained were given equal weight. The following values of the coefficients and their mean errors were found from a least-squares solution.

	Coefficient	I	II	III
		km	km	km
	a_0	12.97 ± 0.42	17.29 ± 0.58	25.46 ± 1.33
$\cos^2 b \cos 2l$	a_4	$+ 2.54 \pm 0.75$	$+ 2.62 \pm 1.04$	$+ 5.13 \pm 2.37$
$\cos^2 b \sin 2l$	a_5	$- 0.51 \pm 0.75$	$- 1.81 \pm 1.04$	$- 5.68 \pm 2.37$
$2-3 \cos^2 b$ . . .	a_6	$- 0.58 \pm 0.46$	$- 0.42 \pm 0.63$	$+ 0.80 \pm 1.44$
$\sin 2b \cos l$ . . .	a_7	$- 0.09 \pm 0.76$	$+ 0.90 \pm 1.05$	$- 1.76 \pm 2.30$
$\sin 2b \sin l$ . . .	a_8	$- 0.86 \pm 0.75$	$+ 0.77 \pm 1.03$	$- 0.30 \pm 2.35$

The last three coefficients have rather large errors as compared with the coefficients themselves, and hence they might be excluded in computing the axes. Since all the terms, however, are of equal geometrical significance, we can secure a little more accuracy by taking all into account. The very small term $-0.09 \sin 2b \cos l$, however, has been omitted.

Introducing direction-cosines, we find

$$\begin{array}{ll}
\text{I} & \theta=12.97+3.12x^2-1.96y^2-1.16z^2-1.02xy-0.18xz-1.72yz \\
\text{II} & \theta=17.29+3.04x^2-2.20y^2-0.85z^3-3.62xy+1.80xz+1.54yz \\
\text{III} & \theta=25.46+4.33x^2-5.93y^2+1.60z^2-11.36xy-3.52xz-0.60yz
\end{array}$$

In forming the maxima and minima of these expressions for the determination of the position of the axes of the surfaces, we must take into account the condition (16). We therefore write equal to zero the partial derivatives of the function

$$\theta-\lambda\,(x^2+y^2+z^2-1),$$

in which λ is an arbitrary coefficient.

The direction-cosines x, y, and z of the axes of the surface are therefore given by the equations

$$\text{Group I}\ldots\left\{\begin{array}{llll}
(+3.12-\lambda)x & -0.51y & -0.09z & =0 \\
-0.51x & +(-1.96-\lambda)y & -0.86z & =0 \\
-0.09x & -0.86y & +(-1.16-\lambda)z & =0
\end{array}\right.$$

$$\text{Group II}\ldots\left\{\begin{array}{llll}
(+3.04-\lambda)x & -1.81y & +0.90z & =0 \\
-1.81x & +(-2.20-\lambda)y & +0.77z & =0 \\
+0.90x & +0.77y & +(-0.85-\lambda)z & =0
\end{array}\right.$$

$$\text{Group III}\ldots\left\{\begin{array}{llll}
(+4.33-\lambda)x & -11.36y & -3.52z & =0 \\
-11.36x & +(-5.93-\lambda)y & -0.60z & =0 \\
-3.52x & -0.60y & +(1.60-\lambda)z & =0
\end{array}\right.$$

The discriminating cubics of these equations are

$$\begin{array}{ll}
\text{I} & \lambda^3-8.469\,\lambda-5.025=0 \\
\text{II} & \lambda^3-12.081\,\lambda-5.940=0 \\
\text{III} & \lambda^3-170.04\,\lambda+223.64=0
\end{array}$$

The roots of the cubics are

	λ_1	λ_2	λ_3
I	+ 3.17	−0.62	− 2.55
II	+ 3.70	−0.50	− 3.20
III	+12.32	+1.33	−13.65

The axes of the surfaces thus computed are in Table VIII.

The maximum axis, which can be assumed to be identical with the axis of preferential motion in the ellipsoidal and the two-drift theory, lies in all cases near the galactic equator. The intermediate

TABLE VIII

VERTICES OF MOTION. TERMS OF EVEN DEGREE

Group	Axis	θ	l	b	α	δ
		km				
I	Maximum	16.14	174°.3	+ 0°.1	98°.0	+ 5°.1
	Minimum	10.42	84.3	+31.9	236.0	+83.2
	Third direction	12.35	264.4	+58.1	187.7	− 4.5
II	Maximum	20.98	164.0	− 8.0	85.9	+10.3
	Minimum	14.09	70.4	−23.6	348.4	+35.7
	Third direction	16.79	91.8	+64.9	189.4	+52.3
III	Maximum	32.38	148.5	+14.1	99.6	+34.1
	Minimum	17.32	56.2	+ 9.1	303.7	+53.4
	Third direction	25.58	294.2	+73.1	197.6	+11.7

axis lies in all cases nearest to the galactic pole.

For comparison I give here a list of other determinations of the principal vertex mainly from Eddington, *Stellar Movements*, p. 124:

		α	δ
Distant stars:	Kapteyn	91°	+13°
	Rudolph	96	+ 7
	Hough and Halm	90	+ 8
	Eddington	95	+ 3
	Schwarzschild	93	+ 6
	Eddington	94	+12
	*Hough and Halm	88	+27
	*Gyllenberg[1]	84	+ 5
	Raymond[2]	92	+ 4
	*Present Papers (I and II)	92	+ 8
Nearer stars:	Charlier	103	+19
	Dyson	88	+21
	Beljawsky	86	+24
	Raymond[2]	92	+35
	*Present Paper (III)	100	+34

(The asterisk indicates that the determination is based on radial velocities.)

[1] *Meddelanden från Lunds Observatorium*, Ser. II, No. 13, 1915.

[2] *Astronomical Journal*, 30, 191, 1917.

There is here some indication that the nearer stars yield a higher declination for the principal vertex. The same result is found in the present investigation, the declination of the vertex having values $+5^\circ.1$, $+10^\circ.3$, and $+34^\circ.1$; and it is even more marked in the galactic longitudes $174^\circ.3$, $164^\circ.0$, $148^\circ.5$.

ASYMMETRICAL TERMS

In order to find if there are systematic differences in the average radial velocities in opposite regions, I have computed the terms of the first and third orders in the expression for the surface given by equation (22). One-half of the difference of the average velocities for opposite regions represents their effect. The number of stars in some regions of the third groups being too small, I have discussed in this way only stars of the first and second groups.

The expression used for the asymmetrical terms is

$$\begin{aligned}\Delta\theta = {} & a_1 \cos b \cos l + a_2 \cos b \sin l + a_9 \cos^3 b \cos 3l + a_{10} \cos^3 b \sin 3l \\ & + a_{11} \cos 3b \cos l + a_{12} \cos 3b \sin l + a_{13} \sin 3b \\ & + a_{14}\ 8\ 3 \cos^2 b \sin b \cos 2l + a_{15}\ 8\ 3 \cos^2 b \sin b \sin 2l.\end{aligned}$$

The term $a_3 \sin b$ is omitted, the stars near the pole of the galaxy being too few to yield a good determination. In the second group I have replaced the term $a_{13} \sin 3b$ by the fifth-order term $a_{16} \sin 4b \cos b$, which proved to be very large.

Assigning equal weights to the different regions, a least-squares solution gave the following values for the coefficients:

		I	II
		km	km
$\cos b \cos l$	a_1	-1.21 ± 0.45	-0.47 ± 0.91
$\cos b \sin l$	a_2	$+0.84 \pm 0.45$	-0.35 ± 0.91
$\cos^3 b \cos 3l$	a_9	-1.94 ± 0.52	-1.87 ± 1.07
$\cos^3 b \sin 3l$	a_{10}	-2.15 ± 0.52	-2.47 ± 1.07
$\cos 3b \cos l$	a_{11}	$+0.74 \pm 0.53$	$+0.25 \pm 1.09$
$\cos 3b \sin l$	a_{12}	-1.34 ± 0.53	-0.29 ± 1.09
$\sin 3b$	a_{13}	-0.66 ± 0.38	....
$8\ 3 \cos^2 b \sin b \cos 2l$	a_{14}	$+0.98 \pm 0.47$	$+0.14 \pm 0.97$
$8\ 3 \cos^2 b \sin b \sin 2l$	a_{15}	$+0.32 \pm 0.47$	$+2.73 \pm 0.97$
$\sin 4b \cos b$	a_{16}	....	-2.35 ± 0.89

For the second group a new solution was made putting $a_1 = a_2 = a_{11} = a_{12} = a_{14} = 0$. The values were not changed, but the mean

errors of the four remaining terms were reduced to ± 0.94, 0.94, 0.85, and 0.78, respectively.

The complete expressions for the surfaces used in computing the maxima and minima are thus

$$\text{I} \quad \theta = 12.97 - 1.21 \cos b \cos l + 0.84 \cos b \sin l + 2.54 \cos^2 b \cos 2l - 0.51 \cos^2 b \sin 2l - 0.86 \sin 2b \sin l - 1.94 \cos^3 b \cos 3l - 2.15 \cos^3 b \sin 3l + 0.74 \cos 3b \cos l - 1.34 \cos 3b \sin l + 0.98 \times 8.3 \cos^2 b \sin b \cos 2l + 0.32 \times 8.3 \cos^2 b \sin b \sin 2l - 0.58 (2 - 3 \cos^2 b) - 0.66 \sin 3b.$$

$$\text{II} \quad \theta = 17.29 + 2.62 \cos^2 b \cos 2l - 1.81 \cos^2 b \sin 2l + 0.90 \sin 2b \cos l + 0.77 \sin 2b \sin l - 1.87 \cos^3 b \cos 3l - 2.47 \cos^3 b \sin 3l + 2.73 \times 8.3 \cos^2 b \sin b \sin 2l - 0.42 (2 - 3 \cos^2 b) - 2.35 \sin 4b \cos b.$$

In Figs. 3 and 4 are shown the intersections of these surfaces with the galactic equator. The radii vectores of the dots represent the average radial velocity for the squares along the galaxy, i.e.,

TABLE IX

INTERSECTIONS OF VELOCITY SURFACE WITH GALACTIC EQUATOR

Region	Gal. Long.	I No.	I $\theta_{obs.}$	I $\theta_{comp.}$	I O−C	II No.	II $\theta_{obs.}$	II $\theta_{comp.}$	II O−C
			km	km	km		km	km	km
c_1+d_1	15°	25	11.6	12.0	−0.4	20	16.9	16.0	+0.9
c_2+d_2	45	50	12.9	12.2	+0.7	29	17.3	15.5	+1.8
c_3+d_3	75	30	14.3	13.4	+0.9	33	14.8	17.6	−2.8
c_4+d_4	105	27	12.3	11.4	+0.9	33	16.1	16.8	−0.7
c_5+d_5	135	29	11.1	11.1	0.0	15	17.5	16.5	+1.0
c_6+d_6	165	30	17.5	16.2	+1.3	16	17.5	20.4	−2.9
c_7+d_7	195	23	18.6	19.0	−0.4	14	28.0	22.1	+5.9
c_8+d_8	225	22	12.3	13.9	−1.6	19	14.8	16.3	−1.5
c_9+d_9	255	31	11.7	8.8	+2.9	22	10.7	11.5	−0.8
$c_{10}+d_{10}$	285	14	8.5	11.8	−3.3	27	16.3	15.9	+0.4
$c_{11}+d_{11}$	315	21	17.4	17.0	+0.4	25	21.2	22.6	−1.4
$c_{12}+d_{12}$	345	23	14.8	15.8	−1.0	16	22.7	21.3	+1.4
		325				269			

c_1+d_1, c_2+d_2, etc., whose values are in Table IX. The dotted curves are the intersections with the galactic equator of the surfaces containing only terms of second order.

For both groups of stars we find a very marked three-lobed curve of nearly the same general appearance and position in the galactic equator. The three maxima are produced by the large third-order terms $a_9 \cos^3 b \cos 3l$ and $a_{10} \cos^3 b \sin 3l$. The sums of these terms for the two groups are

I $2.90 \cos^3 b \cos 3(l-76^\circ\!.0)$ II $3.10 \cos^3 b \cos 3(l-77^\circ\!.6)$

with mean errors in the coefficients of ± 0.74 and ± 1.33, respectively. They are of the same magnitude as the second-order terms which determine the stream-motion, namely,

I $2.59 \cos^2 b \cos 2(l-174^\circ\!.3)$ II $3.18 \cos^2 b \cos 2(l-162^\circ\!.7)$

the mean errors here being ± 1.06 and ± 1.47. The evidence for these third-order terms has therefore much the same definiteness as that for the existence of the stream-motion.

The maxima of the surfaces were obtained by assuming approximate values of the longitudes and latitudes of the axes and computing corrections to their co-ordinates with the aid of the formulae

$$\Delta b \frac{\partial^2\theta}{\partial b^2} + \Delta l \frac{\partial^2\theta}{\partial b \partial l} + \frac{\partial\theta}{\partial b} = 0$$

$$\Delta b \frac{\partial^2\theta}{\partial b \partial l} + \Delta l \frac{\partial^2\theta}{\partial l^2} + \frac{\partial\theta}{\partial l} = 0$$

the derivatives being calculated with approximate value of b and l. The two principal maxima of the radius vector for each surface are given in Table X.

TABLE X

CO-ORDINATES OF MAXIMUM RADIAL VELOCITIES

Group	Maximum θ	l	b	α	δ
	km				
I	19.60	190°	+4°	109°	−7°
	17.50	324	−2	264	−33
II	24.23	170	−18	84	−7
	26.33	322	−19	283	−41

An approximately symmetrical plane perpendicular to the galactic equator was found for the two surfaces in longitudes

I $L = 258^\circ\!.6 \pm 5^\circ\!.9$ II $L = 256^\circ\!.6 \pm 3^\circ\!.0$

Each of these directions nearly bisects the angle between the two maximal axes of the surfaces. In Figs. 3 and 4 the directions of the planes of symmetry are indicated by arrows.

VELOCITY SURFACE OF THE DWARF STARS

In order to find the longitudes of the maxima of the average radial velocities, a harmonic analysis was made taking together all stars between galactic latitudes $-66°$ and $+66°$. The values of θ for the different regions are found in Table XI.

TABLE XI

AVERAGE VELOCITIES—DWARF STARS

Region	Gal. Long.	No.	θ_{obs}	θ_{comp}	O−C
			km	km	km
$b_1+e_1+c_1+d_1$	16°5	38	29.3	29.6	−0.3
$b_2+e_2+c_2+d_2$	49.5	33	21.7	21.6	+0.1
$b_3+e_3+c_3+d_3+c_4+d_4$	90.0	44	21.9	23.2	−1.3
$b_4+e_4+c_5+d_5$	130.5	34	31.7	28.6	+3.1
$b_5+e_5+c_6+d_6$	163.5	30	26.2	29.7	−3.5
$b_6+e_6+c_7+d_7$	196.5	7	30.8	27.3	+3.5
$b_7+e_7+c_8+d_8$	229.5	10	12.4	15.3	−2.9
$b_8+e_8+c_9+d_9+c_{10}+d_{10}$	270.0	11	12.8	11.7	+1.1
$b_9+e_9+c_{11}+d_{11}$	310.5	15	27.6	27.6	0.0
$b_{10}+e_{10}+c_{12}+d_{12}$	343.5	19	34.0	33.9	+0.1

The computed average velocities were derived from the expression

$$\theta = 24.35 + 1.78 \cos l + 3.76 \sin l + 6.88 \cos 2l - 4.88 \sin 2l - 0.17 \cos 3l - 2.01 \sin 3l,$$

which is the result of the analysis.

The maximum values of θ are in longitudes 157° and 340°, and there is further a pronounced minimum in longitude 252°. The longitude of the symmetrical plane is 254°, in close agreement with that found for the more distant stars. The curve is given in Fig. 5.

TENTATIVE EXPLANATION OF THE PROPERTIES OF THE VELOCITY SURFACE

It is possible that the asymmetrical form of surface representing the average radial velocities $\theta = f(b, l)$ depends upon the eccentric position of the sun in the galactic system of stars. This suggestion

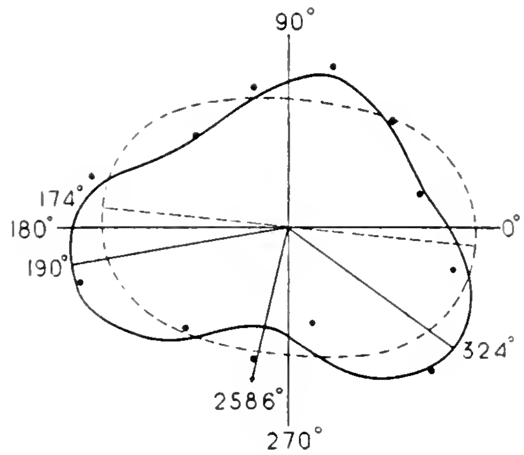

FIG. 3

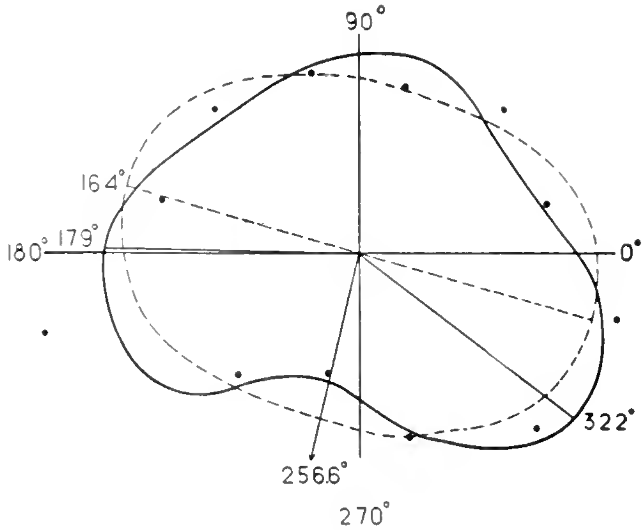

FIG. 4

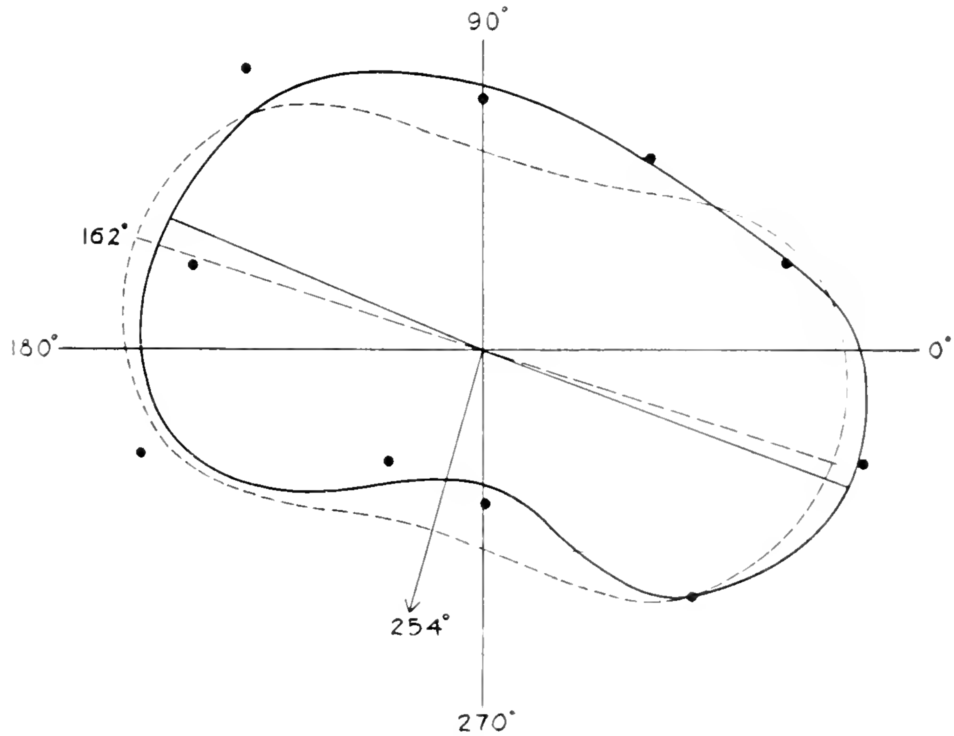

FIG. 5

FIGS. 3, 4, 5.—Intersections between the average radial-velocity surfaces and the galactic equator for the three groups of stars—Fig. 3 for the stars intrinsically brightest and most distant, Fig. 5 for those intrinsically faintest and nearest, and Fig. 4 for the stars intermediate in luminosity and distance. The radii vectores of the points represent the average radial velocity (1 cm=9 km/sec.) in regions adjacent to the galactic equator. The full curves are the intersections when terms to the third order inclusive are taken into account; the dotted curves, when terms of the second order only are used. The projections of the maximal axes of the surfaces are indicated by straight lines; the arrows indicate the longitudes of approximately symmetrical planes perpendicular to the galactic equator.

is supported by the fact, derived by a consideration of the terms depending on galactic latitude alone, that in all groups the average radial velocity is a maximum somewhat south of the galactic equator.

We thus find the following expressions for the variation in latitude alone:

$$\begin{array}{ll} \text{I} & \theta = 12.97 - 0.58\ (2-3\cos^2 b) - 0.66\sin 3b \\ \text{II} & \theta = 17.29 - 0.42\ (2-3\cos^2 b) - 2.35\sin 4b\cos b \\ \text{III} & \theta = 25.46 - 0.37\ (2-3\cos^2 b) + 4.95\sin b - 4.94\sin 3b \end{array}$$

the last being derived from a separate analysis of the average velocities in the six different zones. The curve corresponding to Group II is given in Fig. 6.

All these functions have maximum values of θ for negative values of the latitude. Thus,

$$\begin{array}{lll} \text{I} & \theta_{\text{max.}} = 13.9\ \text{km} & b = -19\overset{\circ}{.}1 \\ \text{II} & \theta_{\text{max.}} = 19.7\ \text{km} & b = -19.9 \\ \text{III} & \theta_{\text{max.}} = 28.4\ \text{km} & b = -22.8 \end{array}$$[1]

Since the sun is situated north of the real galactic plane at a distance of about 20 parsecs, the result may possibly indicate a maximum of space velocity in this plane.

The position of the center of the galactic system has recently been determined by Charlier[2] and by Walkey.[3] Charlier gives for the co-ordinates of the center of 800 B-type stars

$$\alpha = 115\overset{\circ}{.}5, \quad \delta = -55\overset{\circ}{.}6; \qquad l = 236^\circ, \quad b = -14^\circ$$

Distance = 18.21 siriometers = 88.3 parsecs.

Walkey finds for 30,736 stars of all types $l = 246^\circ$, and for the B and A stars separately

834 B stars, $l = 239^\circ$
10,337 A stars, $l = 250^\circ$

The last value agrees well with the longitude of the planes symmetrical to the velocity surfaces ($258\overset{\circ}{.}6$, $256\overset{\circ}{.}6$, 254°).[4]

[1] This, however, is not the principal maximum which lies at the galactic north pole.

[2] *Meddelanden från Lunds Observatorium*, Ser. II, No. 14, 1916.

[3] *Monthly Notices*, **74**, 649, 1914.

[4] A new determination by H. Nort of the star-densities as a function of galactic longitude has recently appeared. He finds from the Harvard Map of stars down to eleventh magnitude a largest concentration at longitude 275° (*Recherches Astronomiques de l'observatoire d'Utrecht*, **7**, 1917).

We may assume that the maximum of average radial velocity occurs when the angle between the line of sight and the direction of preferential motion in space is a minimum. The fact that the two maximal axes of the surface are inclined to each other might therefore be interpreted as a result of a preferential motion around the center in both directions. The plane of symmetry of the surface would then contain the center around which the stars are moving. The agreement between the longitudes of the symmetrical planes and the longitude of the center of the galaxy as determined by Charlier and Walkey supports this hypothesis. The increase in the values of the difference in the longitudes of the axes of maximum radial velocity (134°, 143°, and 183°), as we pass from the distant stars to the nearer, is also in harmony with such a suggestion. Further, if the stars are moving around the center of the stellar system, we may expect a minimum of orbital velocity near the center.[1] Such a minimum is perhaps indicated by the exceptionally small values of the radial velocity near longitude 257°.

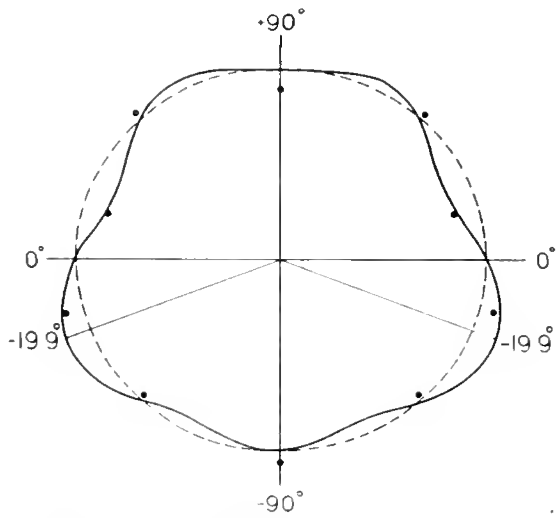

FIG. 6.—Intersection of a plane through the axis of the galaxy with the surface of revolution representing the average radial velocities for stars of Group II when terms depending on galactic latitude alone are included. Dotted curve corresponds to terms of even orders.

That the stream-motion possibly is a motion around the center of the galaxy is further indicated by the fact that the axis of preferential motion is in the galactic plane and nearly perpendicular to the direction toward the center of the stellar system.

The increase in average radial velocity with decreasing brightness is well shown in Figs. 3–5, which are drawn to the same scale. For almost every region when separately considered the radius

[1] Strömgren, "Über Bewegungsformen in Globular Clusters," *Astronomische Nachrichten*, **203**, 17, 1916.

vector increases with decreasing brightness in passing from Group I to Group III.

SUMMARY OF RESULTS

The radial velocities of stars of spectral types F, G, and K have been studied, the stars being grouped according to absolute luminosity with the aid of Adams' spectroscopic parallaxes. The relationship of proper motion to parallax has been determined, and the systematic errors in these parallaxes have been calculated by assuming the same frequency-distribution of the velocity-components in the line of sight as at right angles to it. The variation with absolute brightness of average radial velocity, of the K-term, of galactic concentration, of the solar motion, and of the stream-motion has been studied. The following are the main results.

1. In the formula of Kapteyn for computing mean parallax from proper motion and apparent magnitude it is necessary to add a term c to the proper motion in order to secure satisfactory agreement with the measured parallaxes of the very distant stars. The parallaxes of these stars appear to be almost independent of proper motion.

2. The systematic errors in Adams' spectroscopically determined parallaxes are small, the maximum error being one-fifth of the parallaxes.

3. For stars of types F, G, and K there seems to be a steady increase of average radial velocity with decreasing brightness. For all three types there seems to be a break in the linear relationship, corresponding to the division into giants and dwarfs. If we suppose the relation between velocity and luminosity to be an effect of mass, the giants probably have a higher ratio of luminosity to mass than the dwarfs.

4. The intrinsically bright stars have a higher galactic concentration than the fainter stars.

5. The K-term seems to be positive for the brightest stars and negative for the fainter stars.

6. No decided relationship between the constants of the solar motion and absolute brightness could be found. The intrinsically faint stars show, however, a somewhat lower declination for the

sun's apex than the brighter ones. Since the fainter stars have been observed mainly in the Northern Hemisphere, the declination of the apex is somewhat uncertain.

7. The preferential motions of three different groups of stars have been studied, the groups including stars of different luminosities and distances. A general expression for a surface representing a continuous variation of the radius vector (=average radial velocity) with direction has been used. This takes into account not only symmetrical terms affecting opposite points in the sky equally, but also asymmetrical terms with opposite effects at opposite points. The major axis of this surface agrees well with the axis of preferential motion usually adopted, if only terms of even order are used. Adding terms of odd order (asymmetrical terms), we find that the two axes of maximum radial velocity are not in a straight line, but are directed toward points near the galactic equator, and differing in galactic longitude by 134°, 143°, and 183°, respectively, for the three groups. The best approximation for a symmetrical plane perpendicular to the galaxy has for the three groups galactic longitudes 259°, 257°, and 254°, which agree well with the longitude of the center of the stellar system as determined by Charlier and by Walkey. This perhaps indicates that the stars studied are mainly moving around the center of the galactic system, with a preferential motion in the galactic plane.

Mount Wilson Solar Observatory
October 1917

PHOTO-VISUAL MAGNITUDES OF THE STARS IN THE PLEIADES

By HARRIET McWILLIAMS PARSONS

Although many photometric workers have investigated the stars in the Pleiades both visually and photographically, a determination of photo-visual magnitudes seemed advisable. This determination furnishes (1) the data for testing the photo-visual scale by comparing photo-visual with visual magnitudes, and (2) the data for finding directly the color-indices of the stars measured by comparing photo-visual with photographic magnitudes.

The term photo-visual is self-explanatory, i.e., visual magnitudes obtained photographically are called photo-visual. To obtain such magnitudes it is necessary to use a combination of plate and color-filter that will give a spectral intensity-curve like that of the human eye, having maximum sensitiveness at $\lambda = 5500$ approximately. For the determination of photo-visual magnitudes by Professor Parkhurst of the Yerkes Observatory, Cramer Isochromatic plates, both instantaneous and medium, are used, combined with carefully prepared and tested yellow color-filters known at Yerkes as β 10 (made by R. J. Wallace) and W12 (made by C. E. K. Mees). In order to determine the spectral intensity-curve for these combinations, exposures to northern skylight were taken with a Wallace spectrograph containing Wallace grating replica No. 53. The maximum sensitiveness of the Cramer Isochromatic plate with either filter occurs at $\lambda = 5500$, although W12 cuts off more sharply in the blue than does β 10. Eberhard has stated that it is not safe to rely upon an untested grating in a case like this, since the relative intensities in various parts of the spectrum may vary greatly with different gratings or grating replicas. An attempt was made, at the suggestion of Professor Parkhurst, to compare Wallace replica No. 53 with another Wallace replica used in the Ryerson Physical Laboratory at the University of Chicago. No noticeable difference in relative intensity was found. However, this subject

should be more thoroughly investigated, and it would be well to compare a number of gratings with a "standard prism," that is, one of which the constants are accurately determined. Judging from the results of the present investigation, this combination of color-filter and Cramer Isochromatic plates gives the desired photo-visual magnitudes.

The problem of a star's color is an important one in stellar photometry. The difference between the photographic and visual magnitudes is known as the color-index, which is a measure of the "redness" of the star. This color-index has been found to be closely related to the spectral type, i.e., the more advanced the spectral type the greater the color-index. For faint stars this relation gives a means of inferring the spectral type if the color-index is known. Also the fainter stars seem to be redder, according to present information, and it is seen that the color-indices are greater for the fainter stars.

Of the twenty plates used in this determination, fifteen were taken with the Zeiss doublet camera by Professor Parkhurst and the writer, and five with the Yerkes two-foot reflector. The doublet has lenses of "ultra-violet" glass and is of the Petzval type, with aperture of 14.5 cm and focal length of 81.4 cm. This instrument has a two-inch guiding telescope and gives sharp focal images. One of two parallel-wire gratings, known as R8 and R9, is placed over the objective for part of the exposure. In the case of the former the diameter of the wire is equal to the free space. Only two plates used here were taken with this grating. Since we wish to compare "free" images (those taken without the grating) with the central and first diffraction images obtained with the grating, it is necessary that atmospheric conditions should remain constant during the two exposures. If the sky is changing uniformly, it is still possible to make the comparison by using the mean of the first and third exposures, taken with the grating, compared with the second exposure, taken without the grating. The plates taken with the two-foot reflector were useful in determining faint magnitudes. Since the field of the reflector is small, only a part of the region of the Pleiades may be photographed on a plate. The images are not so sharp or so distinct as those obtained with the camera, because

guiding is more difficult. The grating used with this instrument is known as AI.

The engraving, Plate IV, was made from two separate prints, the junction being faintly visible east of Alcyone, between the stars 241 and 547. The two parts are separated by 2 mm too much space. The scale of the eastern part is 1 per cent greater than that of the western, which is 1 mm = 35.″5.

The plates were measured with the Hartmann "Mikrophotometer." The measurement consists in comparing the stellar images with an artificial scale formed by successive exposures on a given star, varying in a constant ratio. After the plate is measured, a correction is applied to reduce to the center of the plate. This correction is necessary because the size of an image varies slightly with its distance from the optical axis. On the camera plates the correction is small, while on the reflector plates it is much larger and more uncertain.

The next step in the reduction consists in changing the corrected scale-readings into relative magnitudes. This is accomplished by means of the known intervals of magnitude obtained by the use of the grating. With a given grating the difference in magnitude between the "free" and central image from grating is

$$5 \log_{10} \frac{a+d}{a} . \tag{1}$$

The interval in magnitude between the central and first diffraction image is given by

$$5\left(\log_{10} \frac{a\pi}{a+d} - \log \sin \frac{a\pi}{a+d} \right) , \tag{2}$$

where a = free space between wires, d = diameter of wires.[1] The method used in reducing is substantially that used by Miss Leavitt and explained in *Harvard Annals*, **71**, 147.

The last step consists in changing the relative magnitudes into magnitudes on the International Scale. Up to this point the

[1] The constants for reduction of the two gratings are:

	R8	R9
Central − Free	$= 1^{\mathrm{M}}.50$	$0^{\mathrm{M}}.68$
Spectral − Central	$= 0.98$	2.24

reduction has been independent of other determinations, depending only upon the constants of the grating. Stars of spectral type A between magnitudes 5.5 and 6.5 are called standard stars, since their photographic and visual magnitudes are assumed to be equal. Seven of these standards[1] were used in this work, the determinations of magnitude by Müller and Kempf[2] being taken as a basis, reduced to the International Scale by subtracting $0^{M}.30$.

In the accompanying tables are found the photo-visual magnitudes of 111 stars in the Pleiades, the visual magnitudes according to Müller and Kempf, the photographic magnitudes determined by Münch from the plates taken by Hertzsprung, the differences between the visual and photo-visual magnitudes, and the color-index. The second table gives the fainter stars which have Wolf's designation (underscored on the engravings), and thirteen stars with Roman numerals for identification upon the photographs. The third table is in the form of a summary, for classes of stars arranged according to magnitude, giving the average deviations from the mean for the different plates used (showing accordance of the photo-visual magnitudes), the means of the differences between visual and photo-visual magnitudes, and the means of the color-indices. Table IV classifies the stars according to color-index, giving the means of the color-indices and the means of the differences between visual and photo-visual magnitudes.

Tables III and IV give the two important results of the investigation. In Table III the column of mean differences between visual and photo-visual magnitudes shows whether our photo-visual scale differs systematically from that of Müller and Kempf, and whether the color-index changes with the brightness. In regard to the first point, a slight increase is noted as the magnitudes increase, which may be due to the fact that the determinations of magnitude for the fainter stars are more uncertain, since photographic images are often indistinct. The color-index is seen to increase with the fainter stars. Rosenberg[3] finds an increase in the redness of the stars in the Pleiades as they become fainter, especially

[1] B.D.+23° 505, +22 563, +24 546, +24 553, +23 563, +23 536, +24 556.

[2] *Astronomische Nachrichten*, **150**, 193, 1899.

[3] *Ibid.*, **186**, 71, 1910.

TABLE I

B.D. No.	Photo-Visual Mag.	Müller and Kempf Visual Mag.	Münch Photog. Mag.	M & K−Ps	Color-Index
+23° 541	2.ᴹ94	2.ᴹ89	2.ᴹ88	−0.ᴹ05	−0.ᴹ06
23 557	3.73	3.62	3.60	−0.11	−0.13
23 507	3.76	3.66	3.59	−0.10	−0.17
23 516	3.93	3.91	3.80	−0.02	−0.13
23 522	4.28	4.18	4.08	−0.10	−0.20
+24 547	4.34	4.27	4.17	−0.07	−0.17
23 558	5.15	5.08	4.96	−0.07	−0.19
23 505	5.53	5.52	5.35	−0.01	−0.18
22 563	5.59	5.54	5.36	−0.05	−0.23
24 546	5.73	5.68	5.52	−0.05	−0.21
+24 553	5.83	5.87	5.68	+0.02	−0.17
23 563	6.23	6.21	6.07	−0.02	−0.16
23 536	6.24	6.38	6.35	+0.12	+0.09
23 556*	6.56	6.42	6.86	−0.14	+0.30
24 556	6.47	6.45	6.33	−0.02	−0.14
+23 561	6.66	6.71	6.54	+0.05	−0.12
23 569	6.82	6.80	6.72	−0.02	−0.10
24 562	6.88	6.85	6.73	−0.03	−0.15
23 540	6.93	6.88	6.82	+0.05	−0.01
23 553	6.99	6.93	7.03	−0.06	+0.04
+23 535*	7.07	6.94	7.50	−0.13	+0.43
23 570	6.98	6.98	7.00	0.00	+0.02
23 537	7.04	7.01	6.96	−0.03	−0.08
23 512	7.18	7.23	7.22	+0.05	+0.04
23 538	7.19	7.23	7.17	+0.04	−0.02
+23 523	7.37	7.33	7.38	−0.04	+0.01
24 578	7.46	7.48	7.32	+0.02	−0.14
23 560	7.54	7.52	7.54	−0.02	0.00
23 567	7.44	7.54	7.55	+0.10	+0.11
23 519*	7.69	7.54	9.06	−0.15	+1.37
+24 566	7.68	7.69	7.82	+0.01	+0.14
23 542*	7.82	7.73	9.30	−0.09	+1.48
23 530	7.81	7.75	7.82	−0.06	+0.01
23 517	7.87	7.82	7.95	−0.05	+0.08
23 520	7.94	7.99	8.21	+0.05	+0.27
+23 562	8.01	8.01	8.05	0.00	+0.04
23 510	8.06	8.03	8.15	−0.03	+0.09
23 524	8.13	8.10	8.41	−0.03	+0.28
23 559	8.14	8.13	8.31	−0.01	+0.17
23 504	8.22	8.14	8.29	−0.08	+0.07
+23 549	8.31	8.23	8.52	−0.08	+0.21
24 563	8.33	8.24	8.61	−0.09	+0.28
23 546	8.27	8.32	8.43	+0.05	+0.16
23 528	8.46	8.35	8.51	−0.11	+0.05
24 550*	8.69	8.55	8.78	−0.14	+0.09

TABLE I—*Continued*

B.D. No.	Photo-Visual Mag.	Müller and Kempf Visual Mag.	Münch Photog. Mag.	M & K−Ps	Color-Index
+23°508	$8^{M}.51$	$8^{M}.66$	$8^{M}.78$	$+0^{M}.15$	$+0^{M}.27$
23 544*	8.88	8.70	10.15	−0.18	+1.27
23 531	8.68	8.72	9.05	+0.04	+0.37
24 567*	8.91	8.75	8.76	−0.16	−0.15
23 503	9.12	8.99	9.34	−0.13	+0.22
+23 509	8.90	9.04	9.20	+0.14	+0.30
22 570	....	9.10	....	....	....
23 554	9.12	9.11	9.34	−0.01	+0.22
23 565*	9.14	9.14	9.19	0.00	+0.05
23 530*	9.37	9.27	9.75	−0.10	+0.38
+23 548	9.32	9.28	9.61	−0.04	+0.29
24 577	9.20	9.33	9.51	+0.13	+0.31
23 526	9.30	9.35	9.55	+0.05	+0.25
23 564*	9.46	9.37	....	−0.09	....
23 513	9.47	9.40	9.73	−0.07	+0.26
+23 529	9.47	9.46	9.71	−0.01	+0.24
22 568	....	9.53	....	....	....
23 506	9.10	9.58	....	+0.48	....
24 552*	9.40	9.70	10.10	+0.30	+0.70
24 545	9.71	9.73	10.18	+0.02	+0.47
+23 545	9.53	9.76	10.21	+0.23	+0.68
23 568	9.56	9.78	10.19	+0.22	+0.63
24 574	9.52	9.84	....	+0.32	....
23 566	9.49	9.85	....	+0.36	....
23 546	....	9.90	....	....	....
+23 547	9.58	9.90	10.26	+0.32	+0.68
23 552	9.82	9.91	10.39	+0.09	+0.57
23 551	9.82	9.99	10.50	+0.17	+0.68
22 560	10.10	10.01	....	−0.09	....
23 525	10.43	10.05	10.61	−0.38	+0.18
+23 511	10.25	10.06	10.48	−0.19	+0.23
24 564	....	10.12	....	....	....
24 579	....	10.13	....	....	....
23 550	9.83	10.22	10.72	+0.39	+0.89
24 573	....	10.23	10.04	....	....
+23 500	....	10.24	....	....	....
23 555	9.48	10.31	....	+0.83	....
23 502	....	10.41	....	....	....
23 527	....	10.42	....	....	....
23 533*	10.70	10.42	....	−0.28	....
+23 518	10.45	10.43	....	−0.02	....
23 532	10.53	10.44	....	−0.09	....
23 521	....	10.46	....	....	....
24 541	....	10.52	....	....	....
23 543	10.23	10.54	....	+0.32	....
+23 501	....	10.78	....	....	....
23 515	....	10.78	....	....	....
24 542	....	10.75	....	....	....
23 514	....	11.00	....	....	....
+24 544	....	11.51	....	....	....

TABLE II*

Wolf's No.	Ps No.	Photo-Visual Mag.	Wolf's No.	Ps No.	Photo-Visual Mag.
318		$9^{\mathrm{M}}98$		VII	$10^{\mathrm{M}}87$
	I	10.01		VIII	10.91
	II	10.09		IX	10.92
247		10.18	364		10.95
328		10.19	356		11.02
	III	10.29	341		11.02
265		10.30		X	11.03
	IV	10.31	348		11.07
335		10.33		XI	11.07
	V	10.36	289		11.09
301		10.40	244		11.10
333		10.44	267		11.10
	VI	10.64	295		11.14
239		10.74	357		11.17
241		10.82		XII	11.27
				XIII	11.29

In these tables Ps (Parsons) refers to the values of the present paper.

TABLE III

Mag. (M & K)	Average Deviation from Mean	Mean Difference M & K−Ps		No. of Stars	Mean Color-Index	No. of Stars
		Arithmetical	Algebraic			
$3^{\mathrm{M}}0$– $5^{\mathrm{M}}5$...	$\pm 0^{\mathrm{M}}13$	$-0^{\mathrm{M}}07$	$-0^{\mathrm{M}}07$	7	$-0^{\mathrm{M}}15$	7
5.5– 6.5...	±0.06	−0.05	−0.02	8	−0.09	8
6.5– 7.5...	±0.10	±0.04	−0.01	12	−0.01	12
7.5– 8.5...	±0.13	±0.06	−0.03	17	+0.26	17
8.5– 9.5...	±0.19	±0.09	−0.03	16	+0.28	15
9.5–10.5...	±0.18	±0.27	+0.15	18	+0.57	10
10.5–11.5...	....	+0.32	+0.32	1	....	......

TABLE III*a*

Mag. (M & K)	Average Deviation from Mean	Mean Difference M & K−Ps		No. of Stars	Color-Index
		Arithmetical	Algebraic		
$3^{\mathrm{M}}0$– $5^{\mathrm{M}}5$...........	$\pm 0^{\mathrm{M}}13$	$-0^{\mathrm{M}}07$	$-0^{\mathrm{M}}07$	7	$-0^{\mathrm{M}}15$
5.5– 6.5...........	±0.06	±0.04	−0.01	7	−0.12
6.5– 7.5...........	±0.16	±0.04	−0.01	11	−0.04
7.5– 8.5...........	±0.13	±0.05	−0.04	14	+0.13
8.5– 9.5...........	±0.19	±0.09	−0.03	12	+0.21
9.5–10.5...........	±0.18	±0.27	+0.14	17	+0.57
10.5–11.5...........	....	+0.32	+0.32	1	....

between the seventh and ninth magnitudes, which change in color he considers a real function of the "Glühstand," and not due to the absorption in space. Hertzsprung[1] states that the color-index from sixth to ninth magnitude increases to $0^{M}.3$, which fact may be noted in Table III. Table IV shows whether the combination of plate and color-filter differs in color-perception from the Müller and Kempf eye. From the results any differences might seem to be accidental.

TABLE IV

Color-Index	No. of Stars	Mean Color-Index	Mean Difference M & K−Ps	
			Arithmetical	Algebraic
$-0^{M}.25$– $0^{M}.00$	22	$-0^{M}.13$	$\pm 0^{M}.05$	$-0^{M}.03$
0.00–+0.20	18	+0.08	±0.07	−0.04
+0.20–+0.50	19	+0.29	±0.08	−0.03
+0.50–+1.00	7	+0.69	+0.25	+0.25
+1.00–+1.50	3	+1.37	−0.14	−0.14

Trümpler[2] states that ten stars in the Pleiades by their proper motions are shown to have no physical connection with the rest of the group. Omitting these ten stars (having asterisks in Table I), we have Table III*a*, similar to III. On the whole, the results would seem to indicate that the scale employed is practically the same as that of Müller and Kempf, and that the combination of plate and color-filter has color-perception similar to that of the human eye.

Whatever of merit may be found in these determinations of magnitude and the results deduced is due to the suggestion and guidance of Professor J. A. Parkhurst, of the Yerkes Observatory, who took several of the plates used and under whose direction the other plates were taken, measures made, computation carried out, and the subject discussed in its present form. To Professor Parkhurst I wish to express my gratitude and appreciation.

Vassar College Observatory
January 1917

[1] *Astronomische Nachrichten*, **200**, 137, 1915. [2] *Ibid.*, **200**, 222, 1915.

NOTE ON THE CEPHEID VARIABLE SU CASSIOPEIAE[1]

By WALTER S. ADAMS and HARLOW SHAPLEY

A variation in the radial velocity of Boss 637, SU Cassiopeiae,[2] was found by Adams in the course of the regular spectroscopic work with the 60-inch reflector. The observations and measures were made without knowledge of the variations in light, as the star is not listed as variable in the *Preliminary General Catalogue*. The spectrograms on hand permit an inquiry into spectroscopic parallax, variation of velocity, and changes in spectral type, and the last may be compared with the analogous variations previously found from objective-prism plates. The material is discussed in the present note, which also includes some remarks on the variation in light.

The variability in brightness was detected in 1906 by Müller and Kempf[3] from the discordances in measures for the *Potsdam Photometric Durchmusterung*. A special photometric study indicated a variation between magnitudes 5.93 and 6.26, with a period defined by the formula,

$$\text{Max.} = \text{J.D. } 2417287.30 + 1.9498 \cdot E, \quad \text{G.M.T.}$$

The visual light-curve was found to differ from that of typical Cepheids in its small amplitude and symmetrical form.

In 1906–1908 the star was observed photographically by J. A. Parkhurst,[4] who also found the variation peculiarly small and nearly symmetrical with respect to maximum light. The elements by Müller and Kempf being unsatisfactory, the period $1^{d}.9490$ was substituted for the value above. From the photographic magnitudes at maximum and minimum, 6.52 and 6.99, respectively, Parkhurst derived a variation in color-index from 0.59 to 0.73. An objective-prism plate at phase $1^{d}.28$ gave a spectrum of F3, in

[1] *Contribution from the Mount Wilson Solar Observatory*, No. 145.

[2] $\alpha = 2^{h}43^{m}2^{s}.9$, $\delta = +68°28'27''$ (1900.0).

[3] *Astronomische Nachrichten*, **173**, 307, 1907.

[4] *Astrophysical Journal*, **28**, 279, 1908.

fair agreement with the color-index. Because of the star's faintness, the attempt to find a variation in radial velocity with the Bruce spectrograph and the 40-inch refractor was considered inconclusive, although the observed range was from -1 km to -16 km.

An examination of the light-curve by Shapley[1] has shown that the variation cannot be interpreted as the result of the rotation of a simple ellipsoidal body. The conclusion that the star is a Cepheid, notwithstanding the form and amplitude of its light-curve, is verified by the spectroscopic work described below.

Variations in spectral type from A8 to F5 were recorded in 1916 on plates made with the 10-inch photographic telescope at Mount Wilson.[2] The type of spectrum and the brightness of the spectral images (compared with those of neighboring stars) indicated that neither the formula by Müller and Kempf nor its modification by Parkhurst then represented the variation in light; but the epoch given by Müller and Kempf with an intermediate value of the period, $1^{d}.94935$, was found sufficiently accurate for the work on spectral variation.

A new investigation of the period has now been made, employing (1) the observations made at the Potsdam and Yerkes observatories; (2) a redetermination of the magnitudes on the objective-prism plates of 1915 and 1916, and (3) a series of visual estimates on four nights in August 1917. The result has been checked with the observations of spectral type mentioned in the preceding paragraph, as well as those derived from the slit spectrograms, and with the observations of radial velocity given below. The variations in light, velocity, and spectrum are all relatively small and gradual, making high accuracy for the elements impossible. The following formula, however, gives satisfactory phases for all variations:

$$\text{Max.} = \text{J.D. } 2417287.10 + 1.9495 \cdot E, \quad \text{G.M.T.}$$

There is some evidence that the period is not constant.

The spectroscopic observations with the 60-inch telescope are given in Table I. The phases have been computed from the

[1] *Astronomische Nachrichten*, **194**, 357, 1913.

[2] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916.

preceding maximum of the light-curve. Describing the velocity-variations in terms of the orbital elements of a spectroscopic binary, the following results are obtained from a graphical solution:

$$\text{Epoch of Max. Vel. of Approach} = \text{J.D. } 2420197.65, \text{ G.M.T.}$$
$$\text{Period} = 1^{\mathrm{d}}9495$$
$$e = 0.0$$
$$K = 11.0 \text{ km/sec.}$$
$$V_0 = -7.0 \text{ km/sec.}$$
$$a \sin i = 295{,}000 \text{ km}$$
$$\frac{m^3 \sin^3 i}{(m+m_1)^2} = 0.0003 .$$

The epoch of maximum negative velocity precedes the maximum of light by $0^{\mathrm{d}}05$. The elements are entirely typical of Cepheid

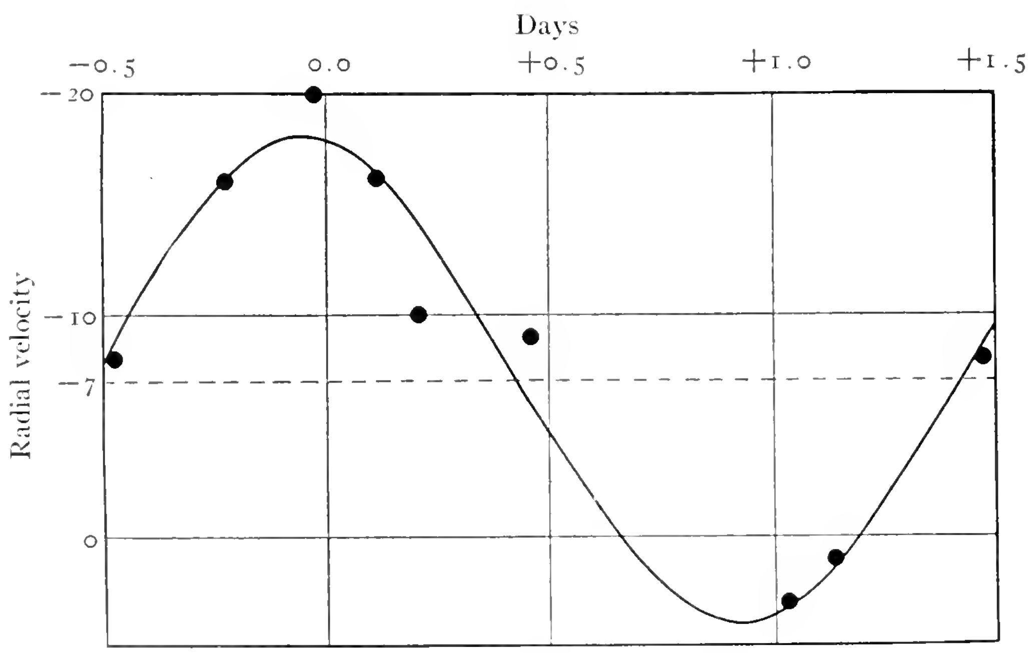

FIG. 1.—Velocity-curve of SU Cassiopeiae

variables that have small ranges of velocity and nearly symmetrical velocity-curves. In Fig. 1 the observations are plotted along the computed velocity-curves. The average deviation is ± 1.5 km.

The spectral types determined by Adams from the spectrograms made with the 60-inch reflector are given in the last column of

Table I. The types derived by Shapley from the plates made with the 10-inch refractor are in Table II,[1] with phases based on

TABLE I

OBSERVATIONS WITH 60-INCH REFLECTOR

Plate	Date	J.D. and G.M.T.	Epoch	Phase	Velocity	Spectrum
					km	
γ 2897....	1913 Nov. 16	2420088.745	1437	0ᵈ21	−10	F2
3072....	1914 Feb. 1	0165.692	1476	1.13	+ 1	F5
3171....	Mar. 5	0197.675	1492	1.93	−20	F0
3808....	Nov. 24	0461.915	1628	1.04	+ 3	F6
5116....	1916 Oct. 7	1144.943	1978	1.73	−16	F0
5239....	Nov. 9	1177.821	1995	1.47	− 8:	F3
5311....	Dec. 7	1205.706	2010	0.12	−16	F2
6200....	1917 Sept. 6	1478.986	2150	0.47	− 9	F4

the new elements. The variation of spectrum with magnitude is illustrated in Fig. 2.

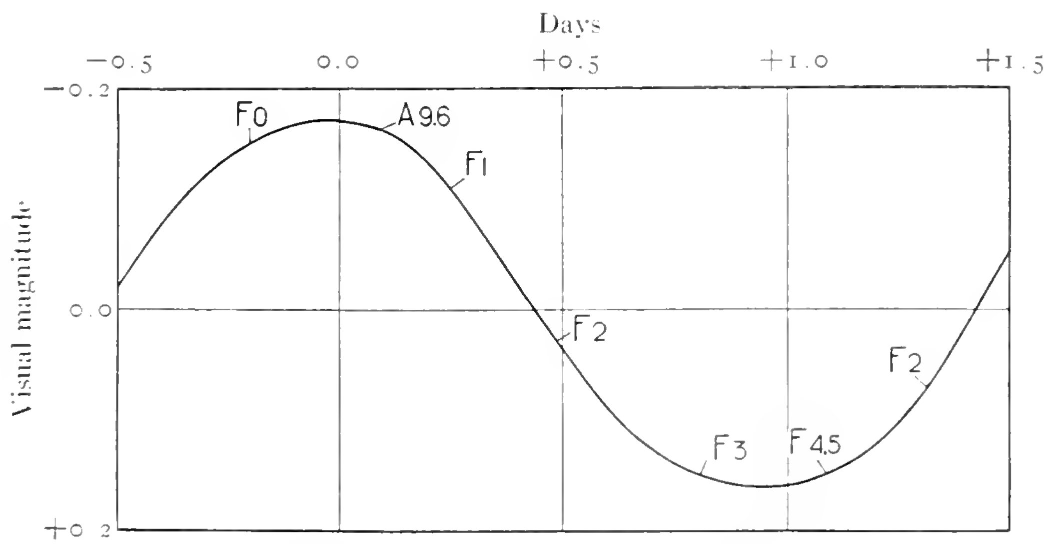

FIG. 2.—Variation of the spectral type of SU Cassiopeiae

The proper motion of SU Cassiopeiae is mainly parallactic, being 0″.016 in position-angle 108°. The parallax, according to a

[1] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916. Types marked as uncertain in the earlier paper are now omitted, and a typographical error is corrected. A new classification of all plates yielded but one correction; for the plate on Julian Day 2420841 the spectrum is F3 instead of F1.

recent determination by van Maanen,[1] is +0.″010 ±0.003; the spectroscopic parallax is in exact agreement if the visual magnitude

TABLE II

OBSERVATIONS WITH 10-INCH REFRACTOR

J.D. and G.M.T.	Epoch	Phase	Spectrum	J.D. and G.M.T.	Epoch	Phase	Spectrum
2420841.836....	1823	0ᵈ80	F3	2420900.727...	1853	1ᵈ21	F2
0894.775....	1850	1.11	F3	0900.731...	"	1.21	F1
0894.782 ...	"	1.11	F4	0901.685...	1854	0.21	A9
0896.711....	1851	1.09	F5	0901.694...	"	0.22	F0
0899.622....	1853	0.10	A9	0901.702...	"	0.23	F2
0899.631....	"	0.11	A8	0901.714 ...	"	0.24	F1
0899.820....	"	0.30	F2	0901.725...	"	0.25	F0
0899.832....	"	0.31	F0	0901.737...	"	0.27	F0
0900.722....	"	1.20	F2				

is taken to be 6.23, as given in the *Preliminary General Catalogue*. With the corresponding value of the distance, the total velocity in space with respect to the sun is 10 kilometers a second.

MOUNT WILSON SOLAR OBSERVATORY
November 1917

[1] *Mt. Wilson Contr.*, No. 136, 1917.

Minor Contributions and Notes

STARS IN THE GROUP OF THE PLEIADES BUT NOT BELONGING TO THE PHYSICAL SYSTEM

In another place in this number is printed Miss Parsons' paper entitled "Photo-visual Magnitudes of Stars in the Pleiades," in which she calls attention to Trümpler's list[1] of twelve stars projected on the group but having proper motions so different from the rest as to suggest detachment from the system. Most of these stars possess the added peculiarity of a larger color-index than the rest of the group. The accompanying table, which collects the data from the two papers cited, will therefore be of interest.

TABLE I

B.D.	Magnitudes		Color-Index		Relative Proper Motion
	Photo-visual Parsons	Photographic Muench	Parsons	Trümpler	
+23°510	7ᴹ69	9ᴹ06	+1ᴹ37	+1ᴹ30	2″51
530	9.37	9.75	+0.38	+0.40	5.25
533	10.70	(11.3)	(+0.6)	...	(5.63)
535	7.07	7.50	+0.43	+0.53	5.48
542	7.82	9.30	+1.48	+1.23	3.39
+23°544	8.88	10.15	+1.27	+1.17	3.86
556	6.56	6.86	+0.30	+0.40	2.40
564	9.46	(10.6)	(+1.1)	(+2±)	5.18
565	9.14	9.19	+0.05	−0.15	4.17
+24°550	8.69	8.78	+0.09	+0.13	2.12
552	(9.40)	10.10	(+0.70)	+0.25	2.11
567	8.91	8.76	−0.15	−0.13	3.70
Mean	....	....	+0.76	+0.75	....

In the table the magnitudes and color-indices are given from Miss Parsons' paper in columns two, three, and four, with the following additions:

1. B.D. +23°533: The photographic magnitude of this star is not given explicitly by Muench, but comparisons with neighboring

[1] *Astronomische Nachrichten*, 200, 222, 1915.

stars on his photographic print show the value to be about $11^{M}.3$, with a resulting color-index of +0.6.

2. B.D. +23°564: By the same process as used for the foregoing star the magnitude 10.6 and the color-index +1.1 were obtained.

3. B.D. +24°552: This is the only star for which the color-index found by Miss Parsons differs materially from Trümpler's value. Investigation of the original data showed that it had been measured by Miss Parsons on only one plate and that the image was somewhat elongated, so that the resulting magnitude could not be reliable and the star should be omitted from the list.

4. Omitting the above-mentioned data, inclosed in parentheses, the mean of the color-indices of the remaining nine stars is almost identical in the two lists, differing by only 1 per cent.

From the foregoing data and the list in Miss Parsons' paper we therefore reach the following remarkable conclusions:

1. Down to magnitude 8.7 all stars having color-indices as large as 0.30 are shown by their large relative proper motions (compared with Alcyone) to belong outside the system.

2. All stars (with two or perhaps three exceptions) having large relative proper motions have also large color-indices and are therefore in a different stage of evolution from the members of the system.

Thus we have some additional data tending to prove the homogeneity of the physical system of the Pleiades.

J. A. Parkhurst

Yerkes Observatory
January 1918

CHANGES IN THE SPECTRUM OF THE WOLF-RAYET STAR GAMMA ARGUS

Spectrographic investigations were recently begun of bright-line hydrogen stars, the Wolf-Rayet stars, and the brighter stars of Classes B and A. Among the first of the Wolf-Rayet stars to be observed was the premier star of this class, γ Argus. These observations show unmistakable changes in its spectrum. On account of the importance of the star a preliminary note on the subject is thought desirable.

The observations so far made have been obtained with a 20° prism of 5 inches aperture attached to the astrographic equatorial as an objective-prism. This combination gives spectra in good focus for practically the entire photographic region, the dispersion at $H\gamma$ being 35 A per millimeter. This dispersion is sufficient to show clearly the widening of lines in spectra of early types and much detail. The resolution is good, the H and $H\epsilon$ lines in Sirius, for example, being clearly separated. The negatives were made on Seed 30 plates.

Several exposures were secured on August 15, 16, and 17 and a fourth plate with three different exposures on November 13, 1917. In this interval of three months there has been a well-defined change in the structure of $H\beta$ and a less marked change in the region of λ 450. At both epochs there is weak, narrow $H\beta$ absorption, with bright borders of unequal intensity on the two sides. In August the bright border on the red side is the stronger. In November the violet border is the stronger. In both cases the differences are very pronounced, and their reality is confirmed by several images. I have spoken of the $H\beta$ as an absorption-effect. It is doubtful, however, if the position of that line is in reality any weaker than the continuous spectrum outside of the bright borders, although it shows as a weak absorption line between the two brightenings.

The changes in the region of λ 450 are not so pronounced, although they appear to be certain. They are in the nature of a change of width of the bands.

A comparison of these recent observations with the Harvard reproduction of the spectrum of this star[1] and with Campbell's observations in 1893–1894[2] shows that large changes have almost certainly occurred in the interval of twenty years. Indeed, the intensities of the bands in the recent observations bear little resemblance to the Harvard reproduction. On account, however, of the well-known peculiarities which are introduced in photographic copies, it is not possible at this time to discuss the observed differences.

[1] *Harvard Annals*, **56**, No. 6, Plate II, Fig. 4.

[2] *Astronomy and Astrophysics*, **12**, 555, 1893; **13**, 457, 1894.

Campbell's statement, "At F the spectrum appears to be strictly continuous," with regard to his 1893 observation, and the statement in his later article—"Two negatives show what seems to be a very faint band at Hβ, with a fine, partially dark line through its center at 4861; but the contrasts are very slight indeed"—seem to furnish a secure basis for the conclusion that the structure recently observed in that region has become more pronounced. His observations were also photographic, and Hβ is now much stronger than the line at λ 479, which he also recorded.

The recent observations also show a broad, very faint brightening in the general region of the chief nebular line. Although very faint, this strengthening seems to be certain. A general strengthening is *suspected* in the region of the second nebular line.

A more detailed discussion of the spectrum of this star as well as of others will be published later.

Two conclusions are suggested by these observations: that there has been a secular change in the spectrum and that there have been changes in the structure of the Hβ line analogous to those observed in the bright-line hydrogen stars. This latter conclusion appears to be especially significant as tending still further to link together the stars of Classes B and O.

I have had the efficient assistance of Second Astronomer Winter in taking the photographs.

C. D. PERRINE

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
November 14, 1917

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THE
ASTROPHYSICAL JOURNAL
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GENERALIZATION OF THE PROBLEM OF THE ROTATION OF PRISMS PRODUCING CONSTANT DEVIATION BY TWO REFRACTIONS AND ONE INTERNAL REFLECTION

By HORACE SCUDDER UHLER

While I was reading the instructive paper on "The Rotation of Prisms of Constant Deviation" recently published by W. E. Forsythe[1] a number of questions arose in my mind and revived my old interest in the subject. Among these may be mentioned: (*a*) Is the axis of rotation given by Forsythe unique, or is it only one position of the generatrix of an extended cylindrical locus? (*b*) Does the existence of the axis in question depend upon the circumstance that the angle between the faces of incidence and emergence was assumed to be 90°? (*c*) If, in general, an axis of rotation, endowed with the properties specified by Forsythe, exists for a prism having any reasonable angle between the refracting planes, what is the location of this axis with respect both to the prism and to the optic axes of the collimator and telescope? In attempting to answer these and other questions I was led to a fuller appreciation of the fact that the papers by Pellin and Broca,[2]

[1] *Astrophysical Journal*, **45**, 278, 1917.

[2] *Journal de Physique*, **8**, 314, 1899.

by myself,[1] and by Forsythe[2] deal with special cases of very general properties of constant-deviation prisms. Since the results of my analytical investigation of the generalized problem are presumably of theoretical interest to the student of geometrical optics, and as they may also be found of practical value, it seems appropriate to present them in this place.

To prepare the way for the solution of the problem of the axis of rotation it is desirable to make a few introductory remarks concerning the broad point of view which will be taken, and also to generalize the fundamental equations pertaining to the type of constant-deviation prism of which the Pellin and Broca form is a very special case. A principal section of a prism of this kind is shown, by the outline *ABCD*, in Fig. 1.

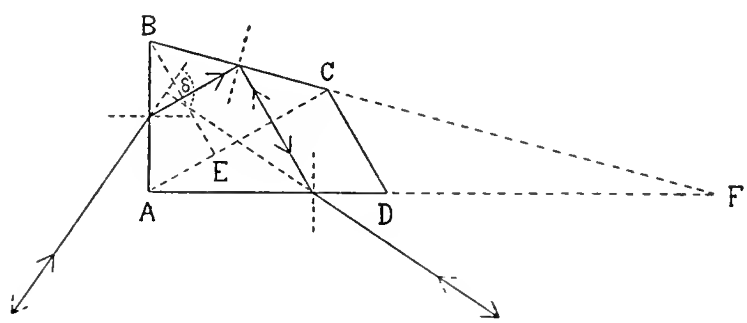

FIG. 1

In this prism $\angle DAB=90°$, $\angle ABC=75°$, $\angle BCD=135°$, and $\angle CDA=60°$. The path of a ray at minimum deviation, so called, is indicated by the broken line with the arrow-heads. The total deviation of the ray, δ, equals 90°. The particular numerical values of the angles suggest at once the usual synthesis of the rhomboidal prism from the "halves" of an ordinary equilateral prism (*ABE* and *DAC*), and the 90° total reflecting prism *BCE*. Since, in my opinion, this conception affords no help in gaining a full appreciation of the general case, but is rather a hindrance to progress, it will be entirely abandoned in the following pages. For economy of time, space, and material, the greater part of the optically ineffective extension *CFD*, of the figure *ABCD*, is omitted by the manufacturers. For the present purposes, however, it will

[1] *Physical Review*, **29**, 37, 1909. [2] *Astrophysical Journal*, **45**, 278, 1917.

be found advantageous to consider the general constant-deviation prism as having a triangular principal section. Although the path of the light is reversible, it will be convenient to designate the planes $\overline{AB}$, $\overline{BC}$, and $\overline{AD}$ (see also later diagrams) as the incidence face, the total reflection face, and the emergence face, respectively. The angles ABC and DAB will be symbolized by σ and ω, in the order named. These angles may have any values consistent with the proper operation of the prism. In my earlier paper[1] a partial generalization was introduced by allowing σ (formerly ξ) to have any feasible value, but ω was kept at $90°$ throughout. This amounted to studying all prisms (used as specified) giving the constant deviation $90°$ and equivalent to ordinary isosceles prisms having the variable refracting angle $2\tau = 2\sigma - 90°$. For well-known reasons the isosceles prisms designed for producing spectra are almost invariably given the equilateral form, so that the advance made by changing σ from $75°$ to a variable value was probably less important than the gain which will arise from changing ω from $90°$ to any reasonable value. As an immediate consequence of the variable character of ω it will be possible to design prisms giving any prescribed constant deviation while retaining all the important properties of the equivalent equilateral direct-transmission prism. The present paper, as stated above, restricts neither σ nor ω, and hence it covers all families of prisms that produce constant deviation by two refractions and one internal reflection at three different planes.

Attention will now be directed to the fundamental optical and mathematical properties of the general constant-deviation prism. In Fig. 2 let $\overline{AB}$ and $\overline{AD}$ denote respectively portions of the incidence and emergence faces. $\angle DAB = \omega$. The angles of incidence and emergence will be symbolized by ι_1 and ι_2, and they will be reckoned positive when generated by counter-clockwise rotation from their respective normals. The (total) deviation δ will be defined as the angle through which the emergent ray must be turned anti-clockwise to bring it into (vectorial) parallelism with the incident ray. Diagrams (*a*) and (*b*) show the cases where $\iota_1 > \iota_2$ and $\iota_1 < \iota_2$, respectively. In Fig. 2 (*a*) the emergence face, the

[1] *Physical Review*, **29**, 37, 1909.

emergent ray, and the normal at the point of emergence may be considered as constituting a rigid figure which is to be revolved counter-clockwise around the pivot A until the emergence face coincides with the incidence face. This rotation will contribute the amount ω to the value of δ. To complete the value of δ the emergent ray $\overline{OE}$ (in its new position) must be turned anti-clockwise

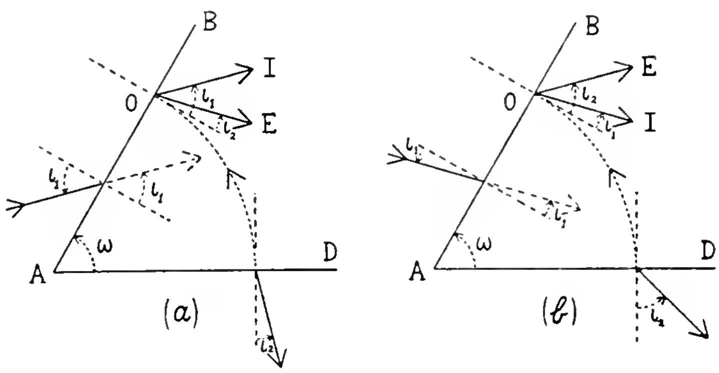

FIG. 2

through the angle EOI to the direction $\overline{OI}$ of the incident ray. The last rotation is evidently equal to $\iota_1-\iota_2$, hence $\delta=\omega+(\iota_1-\iota_2)$. In Fig. 2 ($b$) the first rotation is again equal to ω, but the second component rotation must be performed *clockwise* by an amount $\iota_2-\iota_1$. Therefore the total (anti-clockwise) deviation $\delta=\omega-(\iota_2-\iota_1)$ or $\delta=\omega+\iota_1-\iota_2$. Consequently, for any case

$$\delta=\omega+\iota_1-\iota_2. \tag{1}$$

This method of proof has been given because of its complete generality. It emphasizes the fact that when ι_1, ι_2, and ω are given, δ is uniquely determined. Since equation (1) represents a purely geometrical relation, and nothing more, it is clear that the value of the total deviation does not involve, in any manner whatsoever, the history of the ray between the occasions of incidence and emergence. Hence the range of applicability of this equation is not restricted to prismatic forms, so that the lines $\overline{AB}$ and $\overline{AD}$ of Fig. 2 need not pertain to the faces of a prism. Only when the values of ι_1 and ι_2 are not known does it become necessary to introduce quantities which depend upon the arbitrarily prescribed path of the ray between the points of incidence and emergence.

The immediate consequences of the hypotheses that, between the points of incidence and emergence, the ray shall remain in one optically homogeneous medium, and shall experience internal reflection at a plane surface, will now be considered. For the sake of generality, no reference will be made to any particular diagram. The angles of refraction associated with ι_1 and ι_2 will be denoted respectively by ρ_1 and ρ_2.

Since the segment of the ray between the points of incidence and internal reflection is straight (homogeneous, isotropic medium), a triangle will always be formed by this segment together with the sides of the angle σ. (This triangle will be finite under all circumstances except the practically impossible case where the incident ray strikes the edge common to the faces of incidence and of total reflection.) The angles of this triangle are $90^\circ - \rho_1$, σ, and $90^\circ - \phi_1$, where ϕ_1 denotes the angle between the ray in question and the normal to the total reflection face. Hence $180^\circ = (90^\circ - \rho_1) + \sigma + (90^\circ - \phi_1)$, or

$$\rho_1 = \sigma - \phi_1. \tag{2}$$

In like manner the segment of the ray between the points of internal reflection and emergence will always form a triangle when taken in conjunction with the total reflection and emergence planes. The angles of this triangle are $90^\circ + \rho_2$, $180^\circ - (\sigma + \omega)$, and $90^\circ - \phi_2$, where ϕ_2 symbolizes the angle between the ray under consideration and the normal to the face of total reflection. Hence $180^\circ = (90^\circ + \rho_2) + (180^\circ - \sigma - \omega) + (90^\circ - \phi_2)$, or

$$\rho_2 = \sigma + \phi_2 + \omega - 180^\circ. \tag{3}$$

At internal reflection $\phi_1 = \phi_2$, according to the law of reflection, therefore addition of equations (2) and (3) leads to

$$\rho_1 + \rho_2 = 2\sigma + \omega - 180^\circ. \tag{4}$$

Like formula (1), the last equation is a purely geometrical relation involving primarily the equality of the angles ϕ_1 and ϕ_2. If now the hypothesis be made that the incidence and emergence faces of the prism are in contact with the same medium, then the condition $\iota_1 = \iota_2$ (or $\rho_1 = \rho_2$) leads to the equation $\rho_1 = \rho_2$ (or $\iota_1 = \iota_2$). This conclusion does not depend upon the form of the law of refraction:

in particular, the trigonometrical part of Snell's law is not involved. Also the index of refraction of the material of the prism relative to the surrounding medium may be less than, or equal to, unity as well as greater than this value. When $\iota_1 = \iota_2$ and $\rho_1 = \rho_2$, equation (1) gives $\delta = \omega$, equation (4) reduces to $\rho = \sigma + \frac{1}{2}\omega - 90^\circ$, and hence equation (2) becomes $\phi = 90^\circ - \frac{1}{2}\omega$. The last two equations may be thrown into more symmetrical forms by introducing the angle (σ') between the emergence and total reflection faces. Since $\sigma + \sigma' + \omega = 180^\circ$ it follows that $\rho = \frac{1}{2}(\sigma - \sigma')$ and $\phi = \frac{1}{2}(\sigma + \sigma')$.

Emphasis should be laid on the fact that, when the direction of the incident beam is kept invariable while the prism is rotated around any axis perpendicular to a principal plane, each transmitted color (wave-length) will experience the constant deviation ω at the instant when $\iota_1 = \iota_2$ and $\rho_1 = \rho_2$. ι_1 and ι_2 are explicit functions of n [$\sin \iota_1 = -n\cos(\sigma + \frac{1}{2}\omega)$], or implicit functions of the wave-length, whereas ρ_1 and ρ_2 have the constant value $\sigma + \frac{1}{2}\omega - 90^\circ$. So far as the prism itself is concerned, the question whether the axis of the emergent beam does or does not experience pure translation normal to its direction is not germane to the properties of the prism just discussed. The additional condition that the axis of the emergent beam shall maintain a fixed position during the angular displacement of the prism will be given full consideration in later paragraphs.

As is well known, when light passes through an isosceles prism under the advantageous condition of minimum deviation, $\rho_1' = -\rho_2' = \tau$ and $\iota_1' = -\iota_2'$, where 2τ denotes the refracting angle of the prism. By identifying ρ_1' or ρ_2' with ρ an analytical definition of equivalence between the constant-deviation prism and the ordinary isosceles prism may be obtained. The equation $\rho = \frac{1}{2}(\sigma - \sigma')$ shows that ρ will be positive or negative according as σ is greater or less than σ'. Therefore ρ must be identified with ρ_1' or τ when σ exceeds σ' in value, and with ρ_2' or $-\tau$ when σ is inferior to σ'. Consequently the formula of equivalence is

$$\sigma - \sigma' = \pm 2\tau. \tag{5}$$

As $\sigma + \sigma' + \omega = 180^\circ$, equation (5) may also be written

$$2\sigma + \omega = 180^\circ \pm 2\tau. \tag{5'}$$

Let σ_1 and σ_2 denote the values of σ pertaining to two (supposedly) different prisms having the same value of ω and equivalent to the same direct-transmission prism characterized by τ. Then $2\sigma_1+\omega=180^\circ+2\tau$ and $2\sigma_2+\omega=180^\circ-2\tau$; hence, by addition, $\sigma_1+\sigma_2+\omega=180^\circ$. Now, for one and the same constant-deviation prism (the section of which is simply a plane triangle) having the angles σ_1, σ_1', and ω the relation $\sigma_1+\sigma_1'+\omega=180^\circ$ must hold. Comparing the last equation with the one immediately preceding, it is seen that $\sigma_1'=\sigma_2$ and so the angles σ_1 and σ_2 belong to one single triangle. Consequently the plus signs alone in formulae (5) and (5′) will lead to all constant-deviation prisms of the general type under consideration. The double sign merely means that the path of the ray through the prism has been reversed. By viewing Figs. 1 and 4 through the paper, and fixing the attention on the dotted arrow-heads, the circumstances prevailing when ρ is negative will be perceived at a glance.

The condition of equivalence just laid down is not merely a formal analytical definition. It is easy to show that the difference between the total lengths of the two rays, which lie entirely within a constant-deviation prism and which form the extreme lateral boundaries of the widest beam that can be transmitted by the prism (in the manner prescribed), is equal to $-2b\cos(\sigma+\frac{1}{2}\omega)$ or $2b\sin\tau$, where b symbolizes the actual or the effective [see Fig. 4 (*c*)] width of the incidence face. But $2b\sin\tau$ is precisely the expression for the width of the base of a principal section of the equivalent isosceles direct-transmission prism, the equal sides of which have each the length b. Hence, when made of the same material, the two prisms will have the same spectroscopic resolving power (Rayleigh). This result, as well as the equivalence of the magnitude of the angular dispersion, can also be obtained at once from a consideration of the nature of (internal) reflection at a plane. The chief points of difference between the optical effects produced by the two kinds of prism are: (*a*) greater absorption for the constant-deviation type; (*b*) when $\iota_1=\iota_2$ and $\rho_1=\rho_2$, the deviation effected by the isosceles prism is a function of the wave-length, whereas that produced by the internal reflection type is constant; (*c*) as the angle of incidence for one wave-length is varied, an algebraic

minimum of deviation may always be obtained with the isosceles form, while no stationary value can arise with the constant-deviation prism; and (*d*) the order of the spectral colors is reversed in the two cases.

With regard to the size of the angles of the constant-deviation prism the following remarks should be made. Since, when $\iota_1=\iota_2$, $\phi=90^\circ-\frac{1}{2}\omega$, it follows that ω must not be greater than $180^\circ-2\sin^{-1}\left(\frac{1}{n_r}\right)$ in order that the internal reflection may be *total* over the entire range of the spectrum within which the prism is to be used. n_r symbolizes the least value of the index of refraction pertaining to this range. (By silvering the total reflection face this limit may be raised.) When ρ_1 and ρ_2 are positive or zero, equation (4) $[\sigma+\frac{1}{2}\omega=90^\circ+\frac{1}{2}(\rho_1+\rho_2)]$ shows that $\sigma+\frac{1}{2}\omega$ cannot be acute. Similarly, $\sigma'+\frac{1}{2}\omega$ cannot be obtuse.

Before the question of the position of the axis of rotation of the constant-deviation prism is taken up, attention will be called to some interesting properties associated with equation (5′). For a given value of τ this equation may be represented by a straight line having the intercepts $90^\circ+\tau$ and $2(90^\circ+\tau)$ on the axes of σ and ω, respectively. Since there are an infinite number of points on this line in the positive quadrant, it follows that the number of constant-deviation prisms equivalent to a single isosceles prism is likewise infinite. (For obvious reasons certain pairs of values of σ and ω may not be admissible.) In particular, suppose $\tau=0$ so that, at constant deviation, $\rho_1=\rho_2=0$. Then, by Snell's law, $\iota_1=\iota_2=0$ irrespective of the value of n. Accordingly the ray enters and leaves the prism normally, and no opportunity of rotating the prism (while maintaining constant deviation) is afforded. This is the familiar condition of deviation without dispersion. When $\phi=\sigma=\sigma'=30^\circ$, $\delta=\omega=120^\circ$, but total reflection would not obtain; when $\phi=\sigma=\sigma'=45^\circ$, $\delta=\omega=90^\circ$; and when $\phi=\sigma=\sigma'=60^\circ$, $\delta=\omega=60^\circ$. The three numerical examples are illustrated in Fig. 3, outlines (*a*), (*b*), and (*c*).

When $n=1.5$ with $\tau=30^\circ$, the critical value for ϕ equals $41^\circ48'37''$. Then $\iota_1=\iota_2=48^\circ35'25''$, $\sigma=71^\circ48'37''$, and $\delta=\omega=96^\circ22'46''$. This extreme case is illustrated by Fig. 4 (*a*). When

$\tau = 30°$ with $\sigma = 75°$, $\phi = 45°$ and $\delta = \omega = 90°$. The last set of data corresponds to the original Pellin and Broca prism; see Fig. 1. When $\tau = 30°$ with $\sigma = 90°$, $\delta = \phi = \omega = 60°$. This is shown by outline (*b*) of Fig. 4. When $\tau = 30°$ with $\sigma = 95°$, $\phi = 65°$ and $\delta = \omega = 50°$; see (*c*) of Fig. 4. These diagrams illustrate the manner in which the optically ineffective portion of the prism changes as the angle ω decreases, τ being kept throughout at the constant value $30°$. In general, for a given family of prisms (τ constant), the region containing the angle σ' is ineffective when ω is greater than $90° - \tau$.

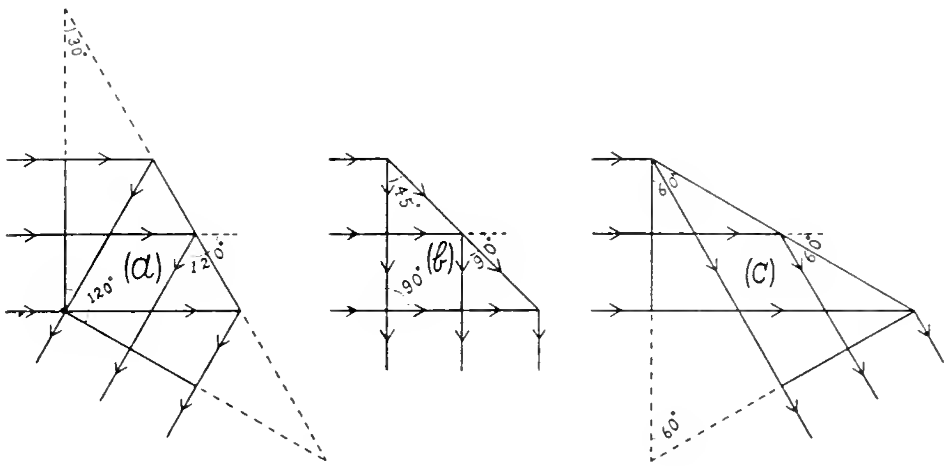

FIG. 3

When the segment of the complete ray between the points of incidence and total reflection is parallel to the emergence face, no portion of the prism is ineffective. Then $\omega = 90° - \tau$, $\phi = 45° + \frac{1}{2}\tau$, and $\sigma = 45° + \frac{3}{2}\tau$. Under this condition ω is always acute and ϕ is large enough to insure total reflection for all kinds of optical glass. When ω is inferior to $90° - \tau$, the ineffective portion of the prism includes the angle ω. In this case the width of the transmitted beam is limited by the length of the total reflection face, whereas in the case where ω exceeds $90° - \tau$ it is controlled by the breadth of the incidence face. In order that internal reflection may occur with a finite length of the total reflection face, the first internal ray must not be parallel to this face. Accordingly the following conditions must be fulfilled: $\sigma < 90° + \tau$ and $\omega > 0°$.

The transitional case,[1] $\omega = 90^\circ - \tau$, seems to me to afford certain advantages over the original Pellin and Broca prism when $\tau = 30^\circ$; see Fig. 4 (*b*). Some of these are: (i) No superfluous glass is involved. (ii) The prism having $\omega = 60^\circ$ causes the telescope to be more nearly in line with the collimator than is the case when $\omega = 90^\circ$. The former construction gives better mechanical balance to the spectroscope than the latter. Moreover, it is sometimes desirable for the observer to be farther away from a high potential discharge than is possible when the deviation is 90°. (iii) By following the

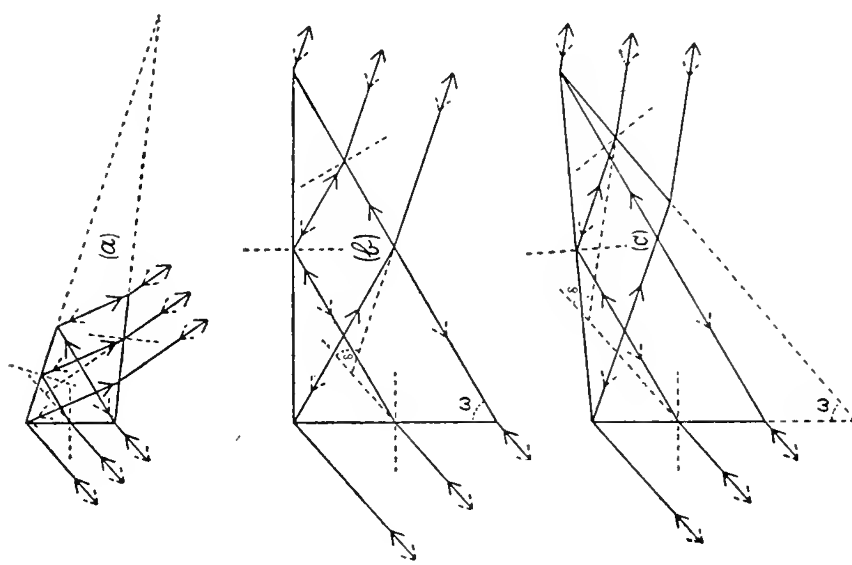

FIG. 4

paths of rays which have experienced multiple reflection, it will be seen that when $\omega = 60^\circ$ the conditions are more favorable for minimizing the intensity of scattered light and diverting secondary spectra from the telescope than when $\omega = 90^\circ$ and the corner (σ') is ground matt. (iv) As actually made, prisms are often not geometrically perfect near the vertical edges. When $\omega = 60^\circ$, the prism can be made larger than is theoretically necessary for the accommodation of the incident beam of light—so as to employ only the best portions of the optical surfaces—without placing a greater demand on the stock of glass than when $\omega = 90^\circ$; for, with

[1] I have tested this experimentally with a prism made and kindly loaned by Professor Charles S. Hastings. The results were highly satisfactory and in complete accord with the theory.

identical incidence faces, the ratio of the volume of the original Pellin and Broca prism to that of the $\omega=60°$ prism is at least equal to $\frac{1}{12}(7+5\sqrt{3}) \doteq 1.305$. (v) The $30°-60°-90°$ prism can be used as a direct-transmission equilateral prism. (vi) Two $30°-60°-90°$ prisms, of the same kind of glass, can be cemented together with their faces of intermediate area in optical contact so as to form one larger equilateral prism of relatively large resolving power. The only hypothetical disadvantage associated with $\omega=60°$ to which my attention has been called involves the question of economy in manufacture. It depends on the fact that the faces of the $60°$ prism are somewhat larger than the homologous faces of the $90°$ type. More precisely, for prisms of identical incidence faces, the ratio of the area of the emergence face for $\omega=60°$ to that of the least value of the like face for $\omega=90°$ equals $3-\sqrt{3} \doteq 1.268$, and for the total reflection faces the ratio equals $\sqrt{2} \doteq 1.414$, at most.

Before taking up the next problem it seems desirable to state that no attempt has been made in the preceding paragraphs to follow out in detail the paths of rays which pass through the constant-deviation prism when ι_1 and ρ_1 are respectively not equal to ι_2 and ρ_2. In other words, the entire field of view in the telescope on both sides of the optic axis (line of "minimum" or constant deviation) has not been considered. The investigation of this matter would involve Snell's law and a knowledge of the values of the indices of refraction of the material of the prism for the extreme wave-lengths that enter the ocular for each position of the prism.

A perfectly general proof of the following (supposedly new[1]) theorem will now be given. *When a ray of light passes through a triangular prism in a principal plane in such a manner as to suffer one internal reflection and two refractions, and having the angle of emergence equal to the angle of incidence (minimum deviation), there exists one, and only one, axis of rotation which will cause the emergent ray to remain in a fixed position while the prism is turned around this axis and the incident ray is maintained immovable. The axis is the intersection of the face at which total reflection occurs with*

[1] W. E. Forsythe proved a part of this theorem for the special case where $\omega=90°$; *loc. cit.*

the plane bisecting the interior angle between the incidence and emergence faces.

In Fig. 5 $X'O'Y'$ represents a principal section of the prism. In this diagram $(\tau=30°)$ $\phi=52°$, $\sigma=82°$, and $\delta=\omega=76°$. The length $\overline{O'Y'}$ will be denoted by b. The broken line $CITED$ indicates the path of the complete ray when the angle of emergence equals the angle of incidence ι. Under this condition, as demonstrated above, the total deviation of the ray will have the constant value ω; hence, since the incident ray CI has an invariable position,

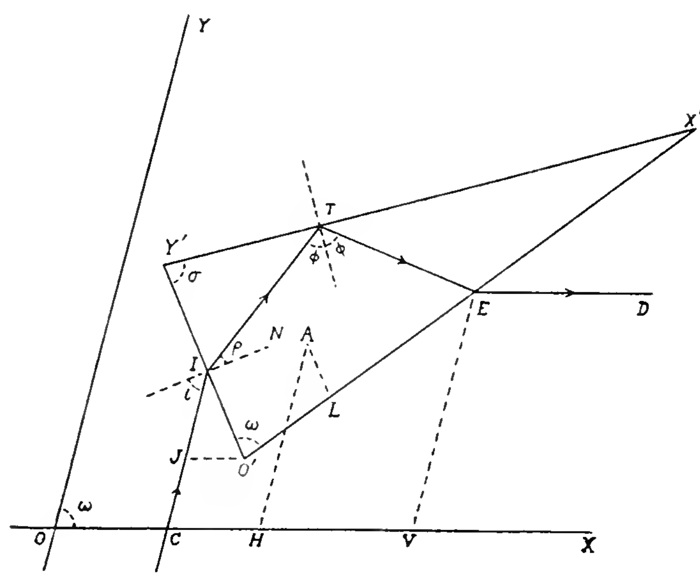

FIG. 5

a set of fixed oblique co-ordinate axes may be taken such that $\overline{OY}$ and $\overline{OX}$ are parallel to the incident and emergent segments ($\overline{CI}$ and $\overline{ED}$) of the ray, respectively. In other words, $\overline{OX}$ and $\overline{OY}$ are parallel respectively to the optic axes of the telescope and collimator. $\angle XOY=\omega$. The point A indicates the axis of rotation which is fixed with respect both to the immovable co-ordinate frame XOY and to the rotating frame $X'O'Y'$. $\overline{OC}=x_c$, $\overline{OH}=x_o$, $\overline{HA}=y_o$, $\overline{VE}=y_t$, $\overline{O'L}=x'_o$, and $\overline{LA}=y'_o$. When the prism is rotated around the axis through A (x_c, x_o, y_o, x'_o, y'_o, being kept constant) the point of emergence E will move along the face $\overline{O'X'}$ and, in general, the line $\overline{ED}$ will move normal to $\overline{OX}$, so that y_t will vary as ι changes. The problem is to determine, if possible, one set (or perhaps a locus of sets) of values of x_o, y_o, x'_o, and y'_o such that

y_t will remain constant independently of the changing values of ι (varying colors). Accordingly an analytical expression giving y_t as an explicit function of b, x_c, x_o, y_o, x'_o, y'_o, σ, ω, and ι will be derived. The condition that y_t shall be independent of ι is expressed mathematically by the vanishing of the algebraic coefficients of the terms containing circular functions of ι as factors.

The work can be conveniently systematized and appreciably simplified by commencing with the co-ordinate system $X'O'Y'$. The abscissa $\overline{O'E}$ of the point of emergence may be obtained by the aid of the equations of the lines $\overline{IT}$, $\overline{X'Y'}$, and $\overline{ET}$, as will now be shown. Let $\overline{O'I} \equiv a$. The angle made by the line $\overline{IT}$ with the axis $\overline{O'X'}$ equals $\omega-(90°-\rho)$ or $\sigma+\frac{3}{2}\omega-180° \equiv \theta$, since $\angle NIT = \rho = \sigma+\frac{1}{2}\omega-90°$, as proved in an earlier paragraph. Hence the point-slope formula $y-y_1=m(x-x_1)$, where $m \equiv \sin\theta / \sin(\omega-\theta)$, leads directly to the equation of $\overline{IT}$, which may be written as

$$x' \sin(\sigma+\tfrac{3}{2}\omega)+(y'-a)\sin(\sigma+\tfrac{1}{2}\omega)=0. \tag{6}$$

The line $\overline{X'Y'}$ has the slope-angle θ equal to $\sigma+\omega$ and it passes through the point $(0, b)$, hence its equation is

$$x' \sin(\sigma+\omega)+(y'-b)\sin\sigma=0. \tag{7}$$

The co-ordinates (x'_2, y'_2) of the point of total reflection T are obtained by solving the simultaneous equations (6) and (7). By taking advantage of the identity

$$\sin(\sigma+\omega)\sin(\sigma+\tfrac{1}{2}\omega)-\sin\sigma\sin(\sigma+\tfrac{3}{2}\omega)=\sin\tfrac{1}{2}\omega\sin\omega$$

it will be found that

$$\left.\begin{aligned} x'_2 &= \frac{(b-a)\sin\sigma\sin(\sigma+\frac{1}{2}\omega)}{\sin\frac{1}{2}\omega\sin\omega} \\ y'_2 &= \frac{a\sin(\sigma+\frac{1}{2}\omega)\sin(\sigma+\omega)-b\sin\sigma\sin(\sigma+\frac{3}{2}\omega)}{\sin\frac{1}{2}\omega\sin\omega} \end{aligned}\right\} \tag{8}$$

Next, to obtain the equation of the line $\overline{ET}$. Since, at minimum deviation, $\angle X'ET = 90°+\rho = \sigma+\frac{1}{2}\omega$, the equation of $\overline{ET}$ may be written

$$(x'-x'_2)\sin(\sigma+\tfrac{1}{2}\omega)+(y'-y'_2)\sin(\sigma-\tfrac{1}{2}\omega)=0.$$

Consequently, the abscissa $x'_3(y'_3=0)$ of the point E is given by

$$x'_3=\frac{x'_2 \sin(\sigma+\tfrac{1}{2}\omega)+y'_2 \sin(\sigma-\tfrac{1}{2}\omega)}{\sin(\sigma+\tfrac{1}{2}\omega)}. \qquad (9)$$

Substitution in (9) of the expressions (8) for x'_2 and y'_2 leads, after suitable reductions, to

$$x'_3=\frac{2b \sin\sigma \cos\tfrac{1}{2}\omega - a\sin(\sigma+\tfrac{1}{2}\omega)}{\sin(\sigma+\tfrac{1}{2}\omega)}. \qquad (10)$$

These reductions may be advantageously effected by making use of the following identities:

$$\sin^2(\sigma+\tfrac{1}{2}\omega)-\sin(\sigma+\tfrac{3}{2}\omega)\sin(\sigma-\tfrac{1}{2}\omega)=\sin^2\omega,$$

$$\sin\sigma\sin(\sigma+\tfrac{1}{2}\omega)-\sin(\sigma-\tfrac{1}{2}\omega)\sin(\sigma+\omega)=\sin\tfrac{1}{2}\omega\sin\omega.$$

It is now necessary to change from the co-ordinate frame $X'O'Y'$ to XOY. When the origin O is moved to $O'(x_1, y_1)$, and the new axes are turned through an angle θ, the old ordinate y of any point is connected with the new co-ordinates x' and y' according to the following equation of transformation:

$$y=y_1+[x'\sin\theta+y'\sin(\omega+\theta)]\csc\omega.$$

In the present case $\theta=90^\circ-\iota$, so that, for the point $E(x'_3, 0)$,

$$y_e=y_1+x'_3\cos\iota\csc\omega,$$

from which, by (10)

$$y_e=y_1+\frac{[2b\sin\sigma\cos\tfrac{1}{2}\omega-a\sin(\sigma+\tfrac{1}{2}\omega)]\cos\iota}{\sin\omega\sin(\sigma+\tfrac{1}{2}\omega)}.$$

The triangle $O'IJ$ shows that

$$a\cos\iota=(x_1-x_c)\sin\omega,$$

therefore

$$y_e=x_c-x_1+y_1+\frac{b\sin\sigma\cos\iota}{\sin\tfrac{1}{2}\omega\sin(\sigma+\tfrac{1}{2}\omega)}. \qquad (11)$$

The co-ordinates of $O'(x_1, y_1)$ will now be derived from the equations of the lines $\overline{O'X'}$ and $\overline{O'Y'}$. The angle which the normal drawn from O to $\overline{O'X'}$ makes with $\overline{OX}$ equals $360^\circ-\iota$. The per-

pendicular distance from $A(x_0, y_0)$ to $O'X'$ equals $-y'_0 \sin \omega$, so that, from the standard normal form

$$x \cos \alpha + y \cos (\omega - \alpha) - p = 0$$

the equation of $\overline{O'X'}$ is found to be

$$(x - x_0) \cos \iota + (y - y_0) \cos (\omega + \iota) - y'_0 \sin \omega = 0. \qquad (12)$$

For the line $\overline{O'Y'}$ $\alpha = \omega - \iota$, and the perpendicular distance from A to $\overline{O'Y'}$ is $x'_0 \sin \omega$, hence the equation of $\overline{O'Y'}$ becomes

$$(x - x_0) \cos (\omega - \iota) + (y - y_0) \cos \iota + x'_0 \sin \omega = 0. \qquad (13)$$

The identity $\cos^2 \iota - \cos (\omega + \iota) \cos (\omega - \iota) = \sin^2 \omega$ facilitates the reduction of the solution of equations (12) and (13) to

$$\left. \begin{aligned} x_1 &= x_0 + \frac{x'_0 \cos (\omega + \iota) + y'_0 \cos \iota}{\sin \omega} \\ y_1 &= y_0 - \frac{x'_0 \cos \iota + y'_0 \cos (\omega - \iota)}{\sin \omega} \end{aligned} \right\} \qquad (14)$$

Substitution of these expressions for x_1 and y_1 in equation (11) leads to the following final fundamental function:

$$y_t = x_c - x_0 + y_0 + (x'_0 - y'_0) \sin \iota - \left[x'_0 + y'_0 - \frac{b \sin \sigma}{\cos \frac{1}{2}\omega \sin (\sigma + \frac{1}{2}\omega)} \right] \cot \tfrac{1}{2}\omega \cos \iota. \qquad (15)$$

The position of the emergent ray may be made independent of the angle of incidence by equating to zero the coefficients of $\sin \iota$ and $\cos \iota$. Accordingly

$$\left. \begin{aligned} x'_0 - y'_0 &= 0 \\ x_0 + y'_0 &= \frac{b \sin \sigma}{\cos \frac{1}{2}\omega \sin (\sigma + \frac{1}{2}\omega)} \end{aligned} \right\} \qquad (16)$$

Since equations (16) are independent linear functions of x'_0 and y'_0, and as the processes involved in the derivation of formula (15) are perfectly general, it follows that one, and only one, axis of rotation exists. The first of these equations shows that (x'_0, y'_0) satisfies the relation $x' - y' = 0$, which represents a plane bisecting the angle between the incidence and emergence faces. To complete the

proof of the theorem under consideration it remains to show that the point (x'_0, y'_0) lies on the line $\overline{X'Y'}$. By combining equation (7) with $x'-y'=0$ it will be found that

$$x'=y'=\frac{b \sin \sigma}{\sin (\sigma+\omega)+\sin \sigma}=\frac{b \sin \sigma}{2 \cos \frac{1}{2}\omega \sin (\sigma+\frac{1}{2}\omega)}.$$

But this is precisely the solution of equations (16), consequently the theorem has been demonstrated in its entirety.

Not only is it necessary to know the position of the axis of rotation with respect to the faces of the prism, but the location of this line with reference to the optic axes of the collimator and telescope must also be determined. When equations (16) are satisfied, formula (15) reduces to

$$y_t=x_c-x_0+y_0.$$

Now the incident and emergent rays must coincide respectively with the optic axes of the collimator and telescope, so that, if these axes are to be taken as lines of reference, the co-ordinate axes $\overline{OY}$ and $\overline{OX}$ must be translated until they coincide with $\overline{CI}$ and $\overline{ED}$, in the order named. These two conditions are expressed mathematically by $x_c=y_t=0$. Then $x_0-y_0=0$; hence, *the axis of rotation must lie in a plane which bisects the angle* ω *between the optic axes of the collimator and telescope.*

The actual value of x_0 and y_0 will depend upon the arbitrary choice of the point on the incidence face at which a ray corresponding to a given index of refraction shall meet this plane. Accordingly let ι_0 be the value of the angle of incidence for the ray that strikes the incidence face at a point which divides b in the ratio $1:r$. That is, $\overline{O'I}:\overline{IY'}=a:(b-a)=1:r$. $a=b/(1+r)$, $x_c=0$, and $\iota=\iota_0$ so that the relation $a \cos \iota=(x_1-x_c) \sin \omega$, found earlier, reduces to

$$x_1=\frac{b \cos \iota_0}{(1+r) \sin \omega}.$$

Substituting this value of x_1, together with the common value of x'_0 and y'_0 given by (16), in the first of formulae (14), it will be found that

$$x_0=y_0=\frac{b}{\sin \omega}\left[\frac{\cos \iota_0}{1+r}-\frac{\sin \sigma \cos (\iota_0+\frac{1}{2}\omega)}{\sin (\sigma+\frac{1}{2}\omega)}\right]. \qquad (17)$$

As a numerical example, let $n = 1.60$, $r = \frac{4}{3}$, $\sigma = 82^\circ$, $\tau = 30^\circ$, and $\omega = 76^\circ$. Then $\iota_0 = 53^\circ 7' 48''$ and $x_0 = y_0 = 0.288b$. A sectional view of the prism in this position is shown in Fig. 6.

The value of x_0 from chosen values of r and ι_0 ($r = 1$, and ι_0 corresponding to 5893 A, say) having been determined, equation (17) may be used to find the value of r associated with some other angle of incidence ι_0' (for the C line or the F line, perhaps), as x_0 is now a known quantity. Let w and r_0' denote respectively the half-width of the incident beam and the value of r calculated from b, x_0, ι_0', σ, and ω. Then it is easy to show that the inequalities

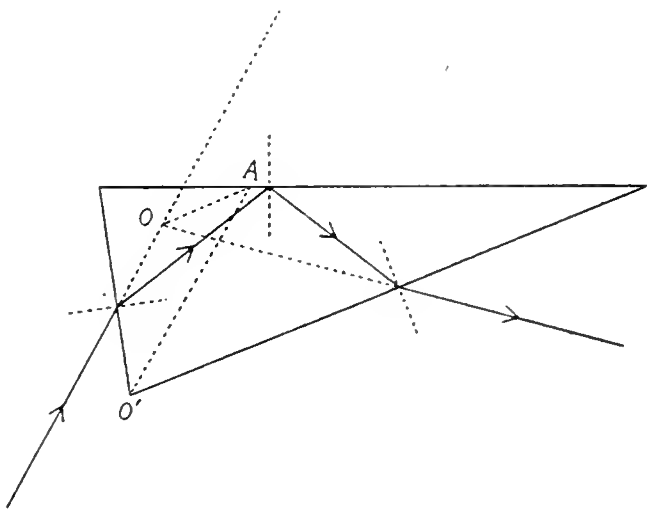

FIG. 6

$w < (b \cos \iota_0')/(1 + r_0')$ and $w < (br_0' \cos \iota_0')/(1 + r_0')$ must be fulfilled in order that the edge rays may strike the incidence face b. It has been tacitly assumed that r_0' comes out positive, for if it were negative b would be divided *externally* and the axial or chief ray itself would not hit the material segment b of the incidence plane. In any event, special care must be taken to avoid errors in such cases as are typified by Fig. 4 (c).

In conclusion I desire to call attention to the fact that the comments made by W. E. Forsythe, on page 278 of his paper, seem at least to imply that my earlier article did not cover precisely the same problem as the one which he so neatly solved. If this interpretation be correct, then his statements admit of appropriate modification for two reasons: (i) on page 42 of my earlier paper a feasible method for moving the prism which covers all cases was

given, and (ii) formula (18) on page 49 (*loc. cit.*)—which is a special case ($\omega=90°$) of equation (15) given above—leads at once to his result. It is perfectly true, however, that I did not explicitly deduce the co-ordinates of his axis from formula (18). My second solution was approximate to a sufficiently high degree of accuracy, whereas his solution is mathematically exact.

SUMMARY

1. The important properties possessed by so-called quadrilateral prisms giving a deviation of 90° have been generalized for any feasible constant deviation δ. The most fundamental single fact brought out by this part of the analysis is that, at effective minimum deviation, the deviation is always equal to the interior angle between the incidence and emergence faces of the prism ($\delta=\omega$).

2. The position of the axis of rotation given by W. E. Forsythe[1] for $\omega=90°$ has been shown to hold for any triangular prism producing a constant deviation δ.

3. It has been demonstrated that the axis of rotation, which must be perpendicular to the plane determined by the optic axes of the collimator and telescope, must lie in the plane which bisects the angle ω between these optic axes.

4. A formula connecting the angle of incidence with the ratio in which the point of incidence of the axial ray divides the incidence face has been derived. As corollaries of this equation, criteria are given for determining whether the extreme rays of a beam will or will not strike the incidence face of the prism.

SLOANE PHYSICAL LABORATORY, YALE UNIVERSITY
November 29, 1917

[1] *Loc. cit.*

THE VISIBILITY OF RADIATION IN THE BLUE END OF THE VISIBLE SPECTRUM[1]

By L. W. HARTMAN[2]

Through the courtesy of its director it was the privilege of the writer in June and July, 1916, to work at the Nela Research Laboratory of the National Lamp Works of General Electric Company, at Nela Park, Cleveland, Ohio. Inasmuch as it did not seem advisable to study the problem originally planned by the writer, Director Hyde suggested that a study be made of the visibility of radiation in the blue end of the spectrum. The results herewith presented were obtained as a consequence of such an investigation.

Among the most thorough as well as the most important determinations of visibility made thus far may be mentioned those of König,[3] Langley,[4] Thürmel,[5] Ives,[6] Bender,[7] Nutting,[8] Hyde and Forsythe,[9] and Coblentz and Emerson.[10] The visibility of radiation for the average eye varies with the wave-length, having the greatest value in the region of 555 $\mu\mu$ and decreasing rapidly to low minimum values at either end of the visible spectrum. The visibility for a given wave-length V_λ may be expressed by the relation

$$V_\lambda = \frac{I_\lambda}{E_\lambda},$$

where I_λ represents for the given wave-length interval the luminous intensity of the light-source measured in light units, and E_λ is the

[1] Communicated from Nela Research Laboratory, National Lamp Works of General Electric Company, Nela Park, Cleveland, Ohio.

[2] University of Nevada.

[3] *Ges. Abhandlungen.*

[4] *American Journal of Science*, **36**, 359, 1888.

[5] *Annalen der Physik*, **33**, 1139, 1910.

[6] *Philosophical Magazine* (6), **24**, 853, 1912.

[7] *Annalen der Physik* (4), **45**, 105, 1914.

[8] *Philosophical Magazine* (6), **29**, 301, 1915.

[9] *Astrophysical Journal*, **42**, 285, 1915.

[10] *Bulletin of the Bureau of Standards*, **14**, 167, 1917.

radiation from the source for the same interval of wave-length measured in energy units. The latter quantity can be computed from Wien's equation and the color-temperature of the source, after which corrections for dispersion, stray light, and absorption of the optical system must be made. When values of I_λ have been determined, V_λ can be computed.

Two methods have been used hitherto to obtain the value of I_λ in this ratio. The first, as in the ordinary photometer based on equality of brightness, involves a direct comparison of the illumination produced by light of successive wave-lengths in the visible spectrum with that produced by a second source considered as a standard; and the second involves the use of the flicker photometer, in which the disappearance of flicker is the criterion of equality. In either case the light from the comparison source may be kept constant in color, or the step-by-step method may be employed, in which case the color of the comparison source is changed at those points where the color-differences exceed a predetermined amount.

The direct-comparison method was used by König, Langley, and Hyde and Forsythe, while Ives, Bender, Thürmel, Nutting, and Coblentz and Emerson have used the flicker method. Although much work has been done on the subject, there is doubt that these two methods give the same results for very great color-differences; indeed, it has been shown that in certain cases they do not.[1] The measurements of Coblentz and Emerson[2] have extended from 400 $\mu\mu$ to 750 $\mu\mu$, while those of Hyde and Forsythe,[3] who carried their observations into the red end of the spectrum farther than anyone else, extended from 620 $\mu\mu$ to 770 $\mu\mu$.

The great difficulty in the way of determining the visibility of radiation far out in the red or the blue end of the spectrum is the small amount of light available. When it is realized that the visibility of radiation for the average eye in going from the position of maximum sensibility to about 400 $\mu\mu$ varies by a very large factor, perhaps 40,000 to 60,000, as may be seen by considering the result of others in conjunction with those given below, it will be evident

[1] Luckiesh, *Electrical World*, **67**, 621, 1913; *Physical Review* (2), **4**, 1, 1914.

[2] *Bulletin of the Bureau of Standards*, **14**, 167, 1917.

[3] *Astrophysical Journal*, **42**, 285, 1915.

that a source that would be very luminous, considered as a whole, would be quite weak if only a small interval of wave-length were taken in the deep blue.

The method utilized here was one that previously had been used in another investigation[1] in this laboratory. It consisted of an adaptation of the arrangement of the parts of the Holborn-Kurlbaum optical pyrometer. One advantage of this method is that it permits the use of greater brightness so that measurements may be made in the extreme blue region of the spectrum. The arrangement of the parts of the apparatus is shown in Fig. 1. By means of a projection lens at *B*, the image of the source of light, *A*, a broad, vertical, incandescent filament of a gas-filled tungsten

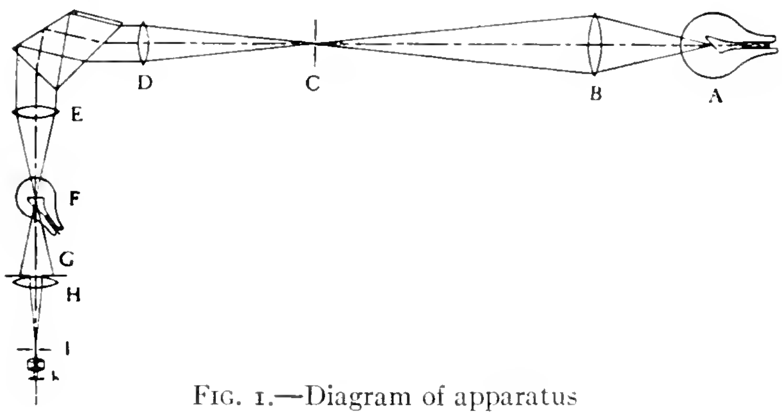

FIG. 1.—Diagram of apparatus

lamp, was thrown upon the collimator slit, *C*, of a Hilger constant-deviation spectrometer. After the light from this source had passed through the prism the spectrum was formed in the focal plane of the telescope of the spectrometer, where the horizontal filament of a small tungsten lamp, *F*, serving as a comparison lamp, was mounted. The lens, *H*, projected an image of the incandescent filament of this lamp, together with the spectrum of the source, *A*, on a narrow adjustable vertical slit, *I*, placed in the focal plane of the eyepiece, *K*. In making the observations with the different observers the width of the collimator slit was maintained at a constant width of 1.00 mm, and the slit in the eyepiece was kept at a constant width of 0.2 mm. Upon turning the drum attached to the prism-stand of the spectrometer, any desired portion of the

[1] *Ibid.*

spectrum of the source A could be brought into the field of view under the eyepiece slit of the telescope, and could be compared in brightness with the incandescent filament of the pyrometer lamp, which was supplied with constant current and therefore was maintained at a constant temperature. Throughout the whole series of observations the brightness of the source of light (A) was constant, as the current through it was kept at 13.54 amperes, giving a color-temperature[1] of 2695° K. This source possessed two distinct advantages: it was broad, the filament being 1.8 mm wide, and as the projection lens gave a magnification of 1.7 the image more than covered the collimator slit, the cone of rays still being large enough to fill the collimator lens, and the image was exceedingly bright. In order to eliminate as far as possible the effect of stray light and at the same time to reduce the color-difference between the incandescent filament and the particular portion of the spectrum under observation, a piece of blue glass of known spectral transmission was mounted in the eyepiece of the telescope. The image formed by the eyepiece was approximately 1.00 mm in diameter.

The value of the current in the filament of the comparison lamp (F) was set so as to match the brightness of a region far out in the blue end of the spectrum of the source (A); and then the current in the filament, and therefore the brightness of the filament, was maintained constant; and the different regions of the spectrum of the source (A) were compared in brightness with it. This was accomplished by introducing, immediately in front of the collimator slit, rotating sectored disks which reduced by a definite, known amount the apparent brightness of the source, and then finding the position of the wave-length drum of the spectrometer, which gave apparent equality between the two. The wave-length drum of the spectrometer had been calibrated by reference to known spectral lines, and the constancy of the calibration was frequently checked. Before making the final observations, trial tests were made with three different widths of the collimator slit, viz., 0.5 mm, 1.00 mm, and 1.50 mm, and with three different currents, and therefore, upon plotting the luminosity-curves, it was found that if they were arbitrarily made to agree at some one wave-length they then agreed

[1] Hyde, Cady, and Forsythe, *Physical Review* (2), **10**, 395, 1917.

within the limits of errors throughout the overlapping wave-lengths. Since the variations in brightness for the average eye for these curves were of the order of 40 to 1 (a ratio of 53 to 1 for the individual whose curves are given later in Fig. 4), it is believed that the possibility of an error due to the Purkinje effect has been eliminated. In addition it may be noted that a small field was used. As noted heretofore, throughout this series of observations the widths of the slits of the collimator and eyepiece were maintained constant, namely, at 1.00 mm and 0.20 mm, respectively.

The energy-curve of the light-source (A) was determined by comparison with a black body. Using the temperature thus obtained, 2695° K, the distribution of energy was calculated from Wien's equation, where the value of C_2 was taken equal to 14,350 micron degrees. These values were then corrected for dispersion and selective absorption of the prism-and-lens system, and for the absorption of the pyrometer lamp, and the luminosity was corrected for width of slits, for scattered light, and for the absorption of the blue glass in the eyepiece of the telescope. The corrections for the selective absorption of the prism-and-lens system of the instrument were based on determinations made by Dr. Forsythe for this same apparatus and used in the paper mentioned above.[1] In these determinations the transmission of the various parts of the optical system for the different wave-lengths was obtained from measurements of brightness with a spectral pyrometer. In case of the prism the dispersion effect was overcome by using an extended source that was fairly uniform.

The following method was used to determine the brightness of the scattered light: Inasmuch as the field of view of the spectrum in the eyepiece was limited in height by a diaphragm placed in front of the collimator slit (C), the filament of the comparison lamp (F) was moved so as to be about a millimeter above or below the field of the spectrum of the source (A). In this position the current in the comparison lamp was adjusted so that its brightness matched the brightness of the scattered light alone, as seen through the blue glass. The comparison lamp was then replaced in its proper position at the center of the field of the spectrum, as seen in the

[1] Hyde and Forsythe, *Astrophysical Journal*, **42**, 285, 1915.

eyepiece, and the light from the source (A) was reduced, by means of rotating sectors placed immediately in front of the collimator slit

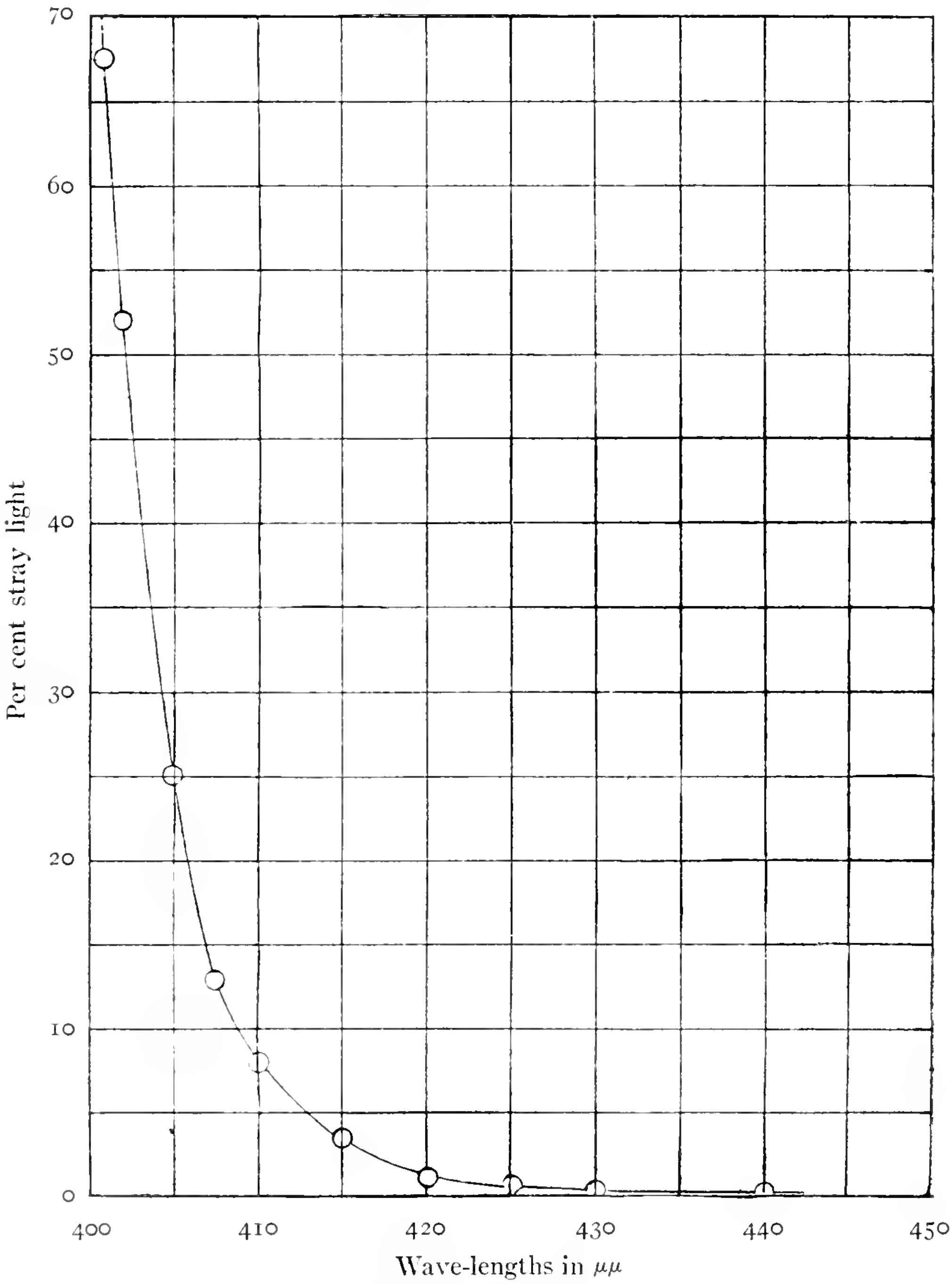

FIG. 2.—Curve showing percentage variation of stray light through blue glass, with wave-length of transmitted light.

(C), to a value such that the brightness of the comparison lamp, with still the same current flowing through it, matched the bright-

ness of the direct radiation plus the scattered light. By varying the current through the filament of the comparison lamp the comparison could be made for any desired wave-length. Knowing the fraction of the total light transmitted through the rotating sector, the ratio of the scattered light to the total radiation could be easily

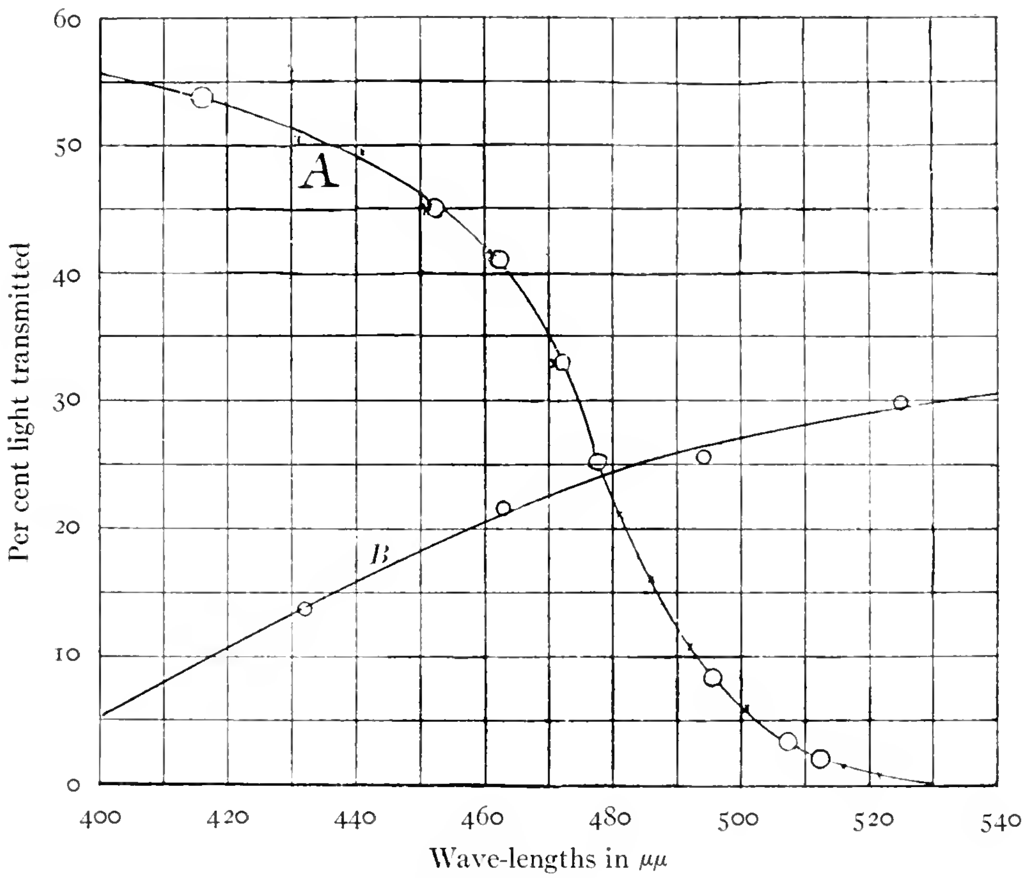

FIG. 3.—Transmission curves: *A*, Blue glass; *B*, Prism and lenses

computed. The results of these determinations are shown graphically in Fig. 2. That this method gives a satisfactory determination of the percentage transmission of the scattered light has been shown elsewhere.[1] The transmission of the blue glass in the eyepiece of the telescope was also determined by Dr. Forsythe, and the results are shown graphically in Fig. 3*A*. The transmission-curves of the other parts of the optical system of the apparatus are shown in Fig. 3*B*.

[1] Hyde and Forsythe, *Astrophysical Journal*, **42**, 289, 1915.

In this series of visibility observations, measurements were made by twenty different individuals, all of whom were trained laboratory observers. The final results are tabulated in Table I. In this table the results have been reduced to a common value at $\lambda = 450\ \mu\mu$. The settings of the spectrometer by each individual were made with at least three different values of current in the pyrometer lamp, and with from six to eight different rotating sectors, the calibration of which had been determined with great accuracy. Frequent check readings were also made by the individual observers. After the readings by an individual observer had been recorded, curves were plotted with wave-length for abscissae, and logarithms of luminosities for ordinates. In these plots the luminosities for the three different degrees of brightness of the pyrometer lamp were made equal in the portions of the spectrum where they overlapped. To do this a number of readings of the differences in the ordinates of the two curves of greater luminosities were taken, and an average was calculated by which the adjustments were made. Thus a smooth curve was obtained, extending from the shortest wave-length at which observations were made to the longest wave-length at which observations were made, and the values for each observer given in Table I are based on the readings from such a smooth curve. A set of such curves, together with the resultant curve, is shown in Fig. 4. In this set of curves, logarithms of luminosities are plotted as ordinates and wave-lengths as abscissae. The three different values of current used throughout in the pyrometer lamp gave values of brightness corresponding to color-match temperatures of 1312° K, 1424° K, and 1568° K, respectively, which in turn gave through the blue-glass filter brightnesses of 0.000033, 0.00019, and 0.0013 candles per square centimeter, respectively.

The relative values of luminosity for radiation for the different observers for a given wave-length in the blue end of the spectrum compared with the average luminosity of a black body at 1315° K through the blue-glass filter used are shown in Table II. In obtaining these data both the source and the comparison lamp were kept constant as in the regular determinations, the former being at 2695° K and the latter at 1312° K, as settings were made by each observer. Data were thus obtained as above, from which curves

TABLE I

VISIBILITY DATA ON TWENTY SUBJECTS IN THE BLUE END OF THE SPECTRUM

Wave-Length	L.W.H.	W.E.F.	E.O.H.	P.W.C.	W.W.	L.T.T.	F.E.C.	J.K.	C.F.S.	A.G.W.	C.F.L.	I.H.V.	C.F.K.	J.H.M.	W.E.B.	R.G.B.	H.C.M.	G.P.L.	H.M.J.	E.P.H.
410	1.15	2.15	1.40	2.50	1.40	2.15	1.90	2.95	2.60	1.65	0.95	0.85	1.05	1.50	2.65	1.55	1.55	1.40	1.40	2.15
420	9.0	13.0	10.0	13.5	9.6	13.5	15.0	15.5	14.5	10.0	10.5	8.9	7.3	9.0	11.9	11.0	11.1	10.0	9.8	15.5
430	29.0	32.0	39.5	30.5	33.5	35.0	37.5	37.0	37.5	36.5	28.0	23.0	27.0	30.0	30.0	28.0	35.0	32.0	33.5	38.0
440	60.	58.	58.	63.	62.	62.	71.	64.	64.	71.	56.	54.	57.	61.	59.	56.	66.	61.	63.	66.
450	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
460	155	160	170	150	160	170	125	155	160	145	165	160	155	150	155	160	145	150	150	125
470	220	265	285	220	245	265	155	250	270	235	265	255	240	220	250	250	245	215	265	190
480	315	425	480	340	395	420	205	355	485	315	450	410	385	350	400	415	385	255	485	255
490	470	725	770	530	550	670	295	510	850	705	755	600	581	545	695	770	570	535	895	380
500	650	1135	1075	755	590	1015	415	690	920	1250	1080	760	750	835	1110	1280	780	780	1640	590

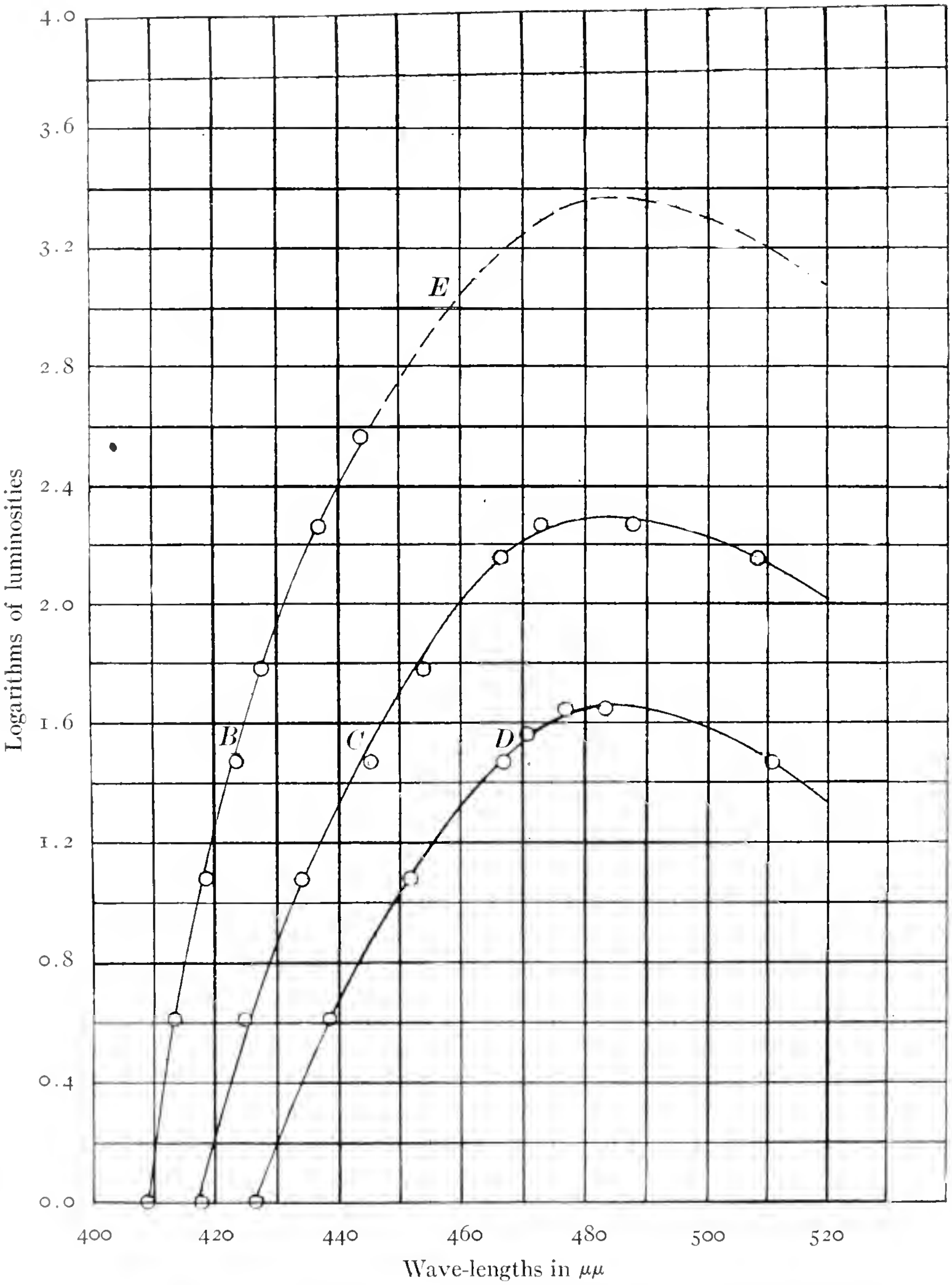

Fig. 4.—Curves showing the relation between logarithms of luminosities and wave-lengths with three different values of current through the comparison lamp. In curve *B* the current through the lamp (*F*) was 0.250 amperes; in curve *C*, 0.285 amperes; and in curve *D*, 0.325 amperes. The average differences between the ordinates of curves *B* and *C* and curves *C* and *D* for the wave-length interval where overlapping occurs were determined and the luminosities of *C* and *D* were made equal in this region to the luminosity of *B* for the given wave-length intervals. The curve *E*, represented by the broken line, gives the extended portion of the curve *B*, based on this adjustment of the luminosity values of curves *C* and *D*. The circles on curve *B* represent the observations made with the lowest current value in the lamp (*F*).

were plotted. From these curves one can determine the relative values of the luminosity for radiation for the different observers for light of the desired wave-length as already mentioned.

TABLE II

RELATIVE VALUE OF THE LUMINOSITY OF RADIATION FOR TWENTY DIFFERENT OBSERVERS FOR LIGHT AT $\lambda=430\ \mu\mu$ COMPARED WITH THE AVERAGE LUMINOSITY OF A BLACK BODY AT 1312° K THROUGH THE BLUE-GLASS FILTER USED

H. M. J.	58	E. O. H.	109
W. E. B.	69	G. P. L.	111
R. G. B.	69	I. H. V.	115
J. H. M.	70	P. W. C.	115
C. F. K.	85	E. P. H.	120
A. G. W.	87	W. W.	120
H. C. M.	100	C. F. S.	126
W. E. F.	101	J. K.	145
L. T. T.	102	L. W. H.	162
C. F. L.	107	F. E. C.	166

The average values of visibility for radiation of the twenty observers, together with the values obtained from the data of Nutting and of Coblentz and Emerson, are given in Table III, in

TABLE III

Wave-Lengths	Mean Visibility of Twenty Subjects	Mean Values Given by Nutting*	Mean Values Given by Coblentz and Emerson
$\mu\mu$			
410	1.7	9.5	24
420	11.4	17.1	42
430	32.6	30.3	59
440	61.6	58.0	71
450	100	100	100
460	153	168	137
470	240	266	202
480	376	392	305
490	620	566	474
500	905	828	770

*These values, kindly furnished by Dr. Nutting, differ slightly from his published data, owing to a redetermination of the distribution of energy in the spectrum of the acetylene flame used.

which all the observations have been reduced to 100 for $\lambda = 450\ \mu\mu$. It will be noted that the values obtained by the method outlined in this paper are lower for the extreme blue than the values given in the two comparison columns. Values for 400 $\mu\mu$ were obtained

by extrapolation from 405 $\mu\mu$ or 407 $\mu\mu$, but it was decided to include in the table only those values obtained from data based on actual observations in the region of the extreme short wave-lengths.

One can test the accuracy of the values of visibility of radiation for the wave-lengths indicated in Table III by computing for some definite temperature-interval the effective wave-length[1] of the blue glass mounted in the eyepiece of the telescope and comparing this with the value determined experimentally. Using the relation

$$\lambda_e = \frac{C_2 \log (e)}{\log (\phi_2/\phi_1)} \cdot (1/T_1 - 1/T_2) ,$$

the computed value of the effective wave-length for the temperature interval 1781° K to 2475° K, for two thicknesses of the glass used, was found to be 466.8 $\mu\mu$, while the value of the effective wave-length for the same two thicknesses of blue glass, determined experimentally by Dr. Forsythe, was found to be 467 $\mu\mu$. In the equation written above, λ_e is the effective wave-length of the blue glass, and C_2 equals 14,350 micron degrees,

$$\phi = \int_0^\infty E_\lambda \, V_\lambda \, t_\lambda ,$$

E_λ being the energy for the given wave-length interval computed from Wien's equation, V_λ the visibility value given in Table III, t_λ the transmission for the given wave-length of the blue glass used in this investigation, and T_1 and T_2 the two temperatures on the Kelvin scale for which ϕ_1 and ϕ_2, the corresponding luminous fluxes through the blue glass, were obtained. This seems to be a valuable check on the accuracy of the results obtained in the present work.

SUMMARY

1. With the aid of a suitable blue-glass screen used in connection with the direct-comparison method of determining the visibility of radiation in the blue end of the spectrum, measurements have been made by twenty different observers, and values of visibility between the limits 410 $\mu\mu$ and 500 $\mu\mu$ have been obtained.

[1] Hyde, Cady, and Forsythe, *Astrophysical Journal*, **42**, 294, 1915.

Corrections for slit-widths, for scattered light, for the absorption of the optical system of the apparatus, have been made. Inasmuch as a very bright source of light was employed, and as a bright but small retinal image was used, and as consistent results with no apparent shift of the visibility-curves were obtained with different widths of the collimator slit and with different brightnesses of the comparison source, it is believed that any error due to the Purkinje effect has been eliminated.

2. Using the values of visibility of radiation obtained by this method for the interval of wave-length mentioned above, and for the interval of temperature 1781° K to 2475° K, the effective wave-length for two thicknesses of the blue-glass filter has been found to be 466.8 $\mu\mu$. The experimentally determined value of the effective wave-length is 467 $\mu\mu$.

In conclusion the writer wishes to express his thanks to the various observers for their time, patience, and unfailing courtesy in making the observations, and especially to Dr. W. E. Forsythe and Dr. A. G. Worthing for many helpful suggestions, for the determination of numerous constants involved in this investigation, and for the free use of many constants and computations used in this paper. To the director, Dr. E. P. Hyde, and to Mr. F. E. Cady the writer wishes to acknowledge his indebtedness for the generous treatment accorded him in the loan of apparatus and the full privileges of the Research Laboratory.

Nela Research Laboratory
National Lamp Works of General Electric Company
Nela Park, Cleveland, Ohio
February 1918

NOTE ON THE SPECTRUM OF THE ISOTOPES OF LEAD

By LESTER ARONBERG

The spectra of isotopes were first investigated by Russell and Rossi[1] and Exner and Haschek,[2] who examined the spectra of thorium and ionium preparation. But no difference in the lines of the two spectra was found.

Aston[3] has examined the spectrum of the two isotopes of neon obtained by fractional diffusion, but found them to be identical.

More recently Soddy and Hyman,[4] Richard and Lembert,[5] Honingschmid and St. Horovitz,[6] and Merton[7] have compared the spectrum of lead of radioactive origin with that of ordinary lead. Soddy and Hyman, who worked with lead from thorite, found that the line λ 4760.1 was stronger in ordinary lead than in the thorite lead, but that the spectra in every other respect seemed to be identical. Richard and Lembert and Honingschmid and St. Horovitz also found that the spectra were identical.

Merton, who used a spectrograph of a higher dispersion than those used by the former investigators (about 10 A per mm), measured the principal lead lines between λ 3500 and λ 4100 in order to detect any difference in wave-length, but he found that no differences greater than 0.03 A (which was within the limit of experimental error) occur in the lines of the two different spectra. According to Professor Hicks's theory, the atomic-weight term enters exactly into the separation of doublets and triplets in series spectra. Assuming that lead has a doublet series spectrum with a separation of 50 A at λ 4000, it was calculated by J. W. Nicholson that if the radio-lead used by Merton was 0.5 unit less

[1] *Proceedings of the Royal Society of London*, **87**, 478, 1912.

[2] *Sitzungsberichte der k. Akad. Wiss., Wien* (II*a*), **121**, 175, 1912.

[3] British Association Meeting 1913, p. 403.

[4] *Chemical Society Transactions*, **105**, 1402, 1914.

[5] *Journal of American Chemical Society*, **36**, 1329, 1914.

[6] *Sitzungsberichte der k. Akad. Wiss., Wien* (II*a*), **123**, December 1914.

[7] *Proceedings of the Royal Society of London*, **91**, 198, 1914.

than ordinary lead, a shift of the order of 0.15 A should result. But, as stated above, no change in wave-length of this order occurs.

Merton also made a special examination of the line λ 4058 by means of a Fabry and Perot étalon, and found that there is no difference in wave-length as great as 0.003 A (which was within the limit of experimental error) for the line λ 4058 in the spectrum of ordinary lead and of lead from pitchblende.

The present investigation was undertaken at the suggestion of Professor W. D. Harkins, who wished to determine whether the electronic periods are wholly dependent upon the net charge of the nucleus of the atom. It was thought possible that the mass of the nucleus might have a small additional effect. The lead which was used was very kindly supplied by Professor T. W. Richards, who had determined the atomic weight as 206.318. This lead was obtained from Australian carnotite.

In order to obtain a bright source and at the same time to have the spectral lines very sharp and narrow (which is very necessary for work in the higher orders), a slightly modified form of the oxy-cathode arc *in vacuo*, employed by Wali-Mohammad,[1] was used, as shown in Fig. 1. This source had the additional advantage that the radio-lead, of which only about 3 grams were available, was not wasted, for the deposit on the walls of the tube could be dissolved and the solution saved.

Before proceeding with the grating spectrum a comparison spectrum of the two leads was taken on a Hilger quartz spectrograph, which has a dispersion of about 25 A per mm. The two leads were placed in two different vacuum arc lamps. No change of intensity or in number of lines, however, could be observed. A photograph of the two leads—the ordinary lead at a pressure of 0.04 mm and the radio-lead at a pressure of 0.07 mm—is given in Plate V, Fig. 2 (see plate facing p. 102). The hydrogen and nitrogen lines which appear stronger in one than in the other are due to the 0.03 mm pressure between the two arc lamps.

It seems interesting, however, to note here that at first a change of intensity in the lines λ 2833 and 2823 was observed; viz., in the ordinary lead the line 2833 had an intensity of 10 (arbitrary scale)

[1] *Astrophysical Journal*, **39**, 189, 1914.

and 2823 had an intensity of 5, while in the radio-lead the line 2833 was of intensity 5 and the line 2823 of intensity 10. But, after many repeated trials to find out whether it was a true change or not, it turned out that the pressure in the lamp containing the radio-lead was increased after the arc started from 0.05 mm to 0.25 mm. And it is that small difference in pressure which caused the change in intensity. This was verified by changing the pressure in the arc containing the ordinary lead to 0.25 mm, when the same change of intensity was observed, no matter whether the residual gas was hydrogen or air or whether it was ordinary lead or radio-lead.

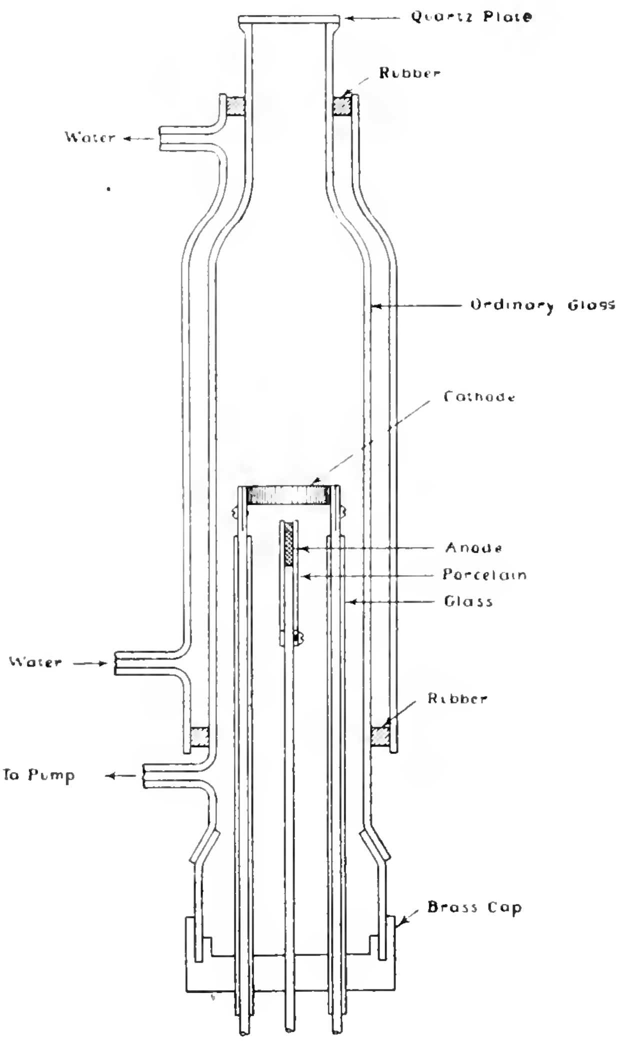

FIG. 1

The sixth order of a Michelson ten-inch plane grating in a Littrow mounting of 30 feet focus was tried next. But, unfortunately, the only line that could be photographed in the higher orders within reasonable exposure times was the line λ 4058, the strongest line of the lead spectrum.

The structure of λ 4058 was studied by Janicki[1] and Wali-Mohammad,[2] using an echelon and the same kind of a light-source as used in this experiment. They found it to possess two satellites, +0.032 and −0.041. In the present investigation it was found that both the ordinary lead and the radio-lead possess only one

[1] *Annalen der Physik*, **29**, 833, 1909. [2] *Astrophysical Journal*, **39**, 189, 1914.

component at -0.0480 and -0.0501 for the radio- and ordinary lead, respectively, the difference being within the experimental error of measuring on account of the faintness of the satellite.

In order to detect any small changes in wave-length, the two arc lamps were connected in parallel to the mercury pump so as to keep the pressure in both identical, about 0.04 mm of mercury, and exposed at the same time to avoid any mechanical shifts. The light from one went through a right-angled prism attached to the slit, while the light from the other lamp passed straight through the slit, thus forming a spectrum the middle of which belongs to one lamp, while the lines above and below belong to the other one.

The voltage, about 40 v., was kept the same in both arcs within one volt (except in a few exposures where one lamp was two or three volts below the other). The current of about 1.1 amp. was kept the same within 0.05 amp. Seven exposures were taken in such a position that the light from the lamp containing the radio-lead passed straight through, while that of the other went through the right-angled prism.

The plates were measured on a Gaertner comparator. The instrument reads directly to thousandths of a millimeter and may be estimated to ten-thousandths. Table I gives the differences in millimeters and angstroms of λ 4058 in the two kinds of lead, the radio-lead having the larger wave-length.

TABLE I

No. of Exposure	Difference in mm	Difference in A
1	0.013	+0.0046
2	.013	+ .0046
3	.019	+ .0068
4	.010	+ .0036
5	.016	+ .0057
6	.012	+ .0043
7	0.010	+0.0036

Then the positions of the lamps were interchanged and six more exposures taken. The measurement gave the values shown in Table II.

The lamps were placed again in their original position and three more exposures were taken, which gave the results shown in Table III.

The average of the foregoing differences in wave-length is 0.012 mm, corresponding to an increase in λ of 0.0043 A for the radio-lead, for the dispersion at λ 4058 in the sixth order $= 3.59$ A per cm.

TABLE II

No. of Exposure	Difference in mm	Difference in A
8................	0.010	+0.0036
9................	.010	+ .0036
10................	.011	+ .0039
11................	.013	+ .0046
12................	.016	+ .0057
13................	0.010	+0.0036

The satellite was measured on the first four exposures and found to have the same magnitude of change in wave-length. But no accurate measurements could be made on account of the faintness of the component, when the main line was properly exposed.

TABLE III

No. of Exposure	Difference in mm	Difference in A
14................	0.013	+0.0046
15................	.010	+ .0036
16................	0.013	+0.0046

The pressure in the arc must not exceed 0.1 mm of mercury, for if the pressure was higher than 0.1 mm the line broadened and masked the effect.

In order to be certain that the change in wave-length was not due to slight changes in voltage or amperage, the voltage of the ordinary lead was lowered intentionally by 3 volts in Nos. 6 and 7; for by lowering the heating current through the cathode the voltage is changed owing to a smaller electronic emission, while the amperage of the arc can be still kept the same. Nos. 12 and 13 represent the case where the voltage of the radio-lead was lowered by 3 volts. No. 15 was taken when the amperage of the radio-lead was lowered by 0.2 amp., and the voltage was kept the same by increasing the current through the cathode. From the values of the abovementioned exposures it is seen that slight changes in voltage or amperage have no effect on the shift of the line.

Finally, to establish the reality of the change in wave-length more firmly in a new manner, the radio-lead was removed and replaced by ordinary lead at the same distance from the cathode. The lamps were placed in the same position as that used for the last three exposures. But no difference greater than the experimental error of measuring was found. Four exposures were taken and gave the following values:

No. of Exposure	Difference in mm
1	+.001
2	−.002
3	−.003
4	+.0015

Nos. 3 and 4 were taken by lowering the voltage by 3 volts and the amperage by 0.2 amp., respectively, of the lamp containing the fresh ordinary lead.

The effect of the change of mass of the nucleus on series spectra of helium and hydrogen was studied by Evans[1] and by Paschen.[2] They found it to agree, within the experimental error, with the general theoretical formula deduced by Bohr, viz.:

$$V=\frac{2\pi^2e^2E^2Mm}{h^3(M+m)}\left\{\frac{1}{n_1^2}-\frac{1}{n_2^2}\right\},$$

where e and m are the charge and mass of the electron, E and M the charge and mass of the nucleus, and h is Planck's constant.

Assuming the ordinary lead to have an atomic weight of 207 and the radio-lead 206, E, the nuclear charge, being the same, it is calculated from Bohr's formula that λ 4058 of the radio-lead should be increased by 0.00005 angstrom, which agrees in sign but not in magnitude with the observed value.

The fact that there was an actual shift of the line in the ordinary lead and not a mere broadening of the line would suggest the conclusion that ordinary lead is not a mixture of thorium and radium lead, but is a pure isotope by itself.

In conclusion I wish to thank Professor H. G. Gale for his constant advice and suggestions in the investigation of this problem.

Ryerson Physical Laboratory, University of Chicago
December 1917

[1] *Philosophical Magazine*, **29**, 284, 1915. [2] *Annalen der Physik*, **50**, 207, 1916.

THE STRUCTURE OF THE BISMUTH LINE λ 4722

By LESTER ARONBERG

The structure of the bismuth line λ 4722 has been studied by von Baeyer,[1] von Baeyer and Gehrcke,[2] Takamine,[3] Lunelund,[4] and Wali-Mohammad.[5] The former three investigators have used the method of crossed spectra to obtain a very high resolving power, while the latter two have used an echelon grating of very great resolving power.

The results of Lunelund and Wali-Mohammad differ very greatly from those of the former three investigators as to the position of the satellites of the line. Professor H. G. Gale suggested clearing up the question by using a Michelson ten-inch plane grating in a Littrow mounting of 30 feet focus in the sixth order to examine the structure of the line.

TABLE I

Gehrcke and v. Baeyer	v. Baeyer	Takamine	Lunelund	Wali-Mohammad	Grating
+0.316	+0.318	+0.320	−0.031(=+0.314)	−0.029(=+0.316)	+0.318
+ .289	+ .283	+ .284	− .062(=+0.283)	− .061(=+0.284)	+ .284
+ .242	+ .242	+ .238	− .105(=+0.240)	− .103(=+0.242)	+ .240
+ .104	+ .100	+ .102	+ .103	+ .102	+ .102
+ .057	+ .056	+ .056	+ .059	+ .057	+ .056
0.000	0.000	0.000	.000	.000	0.000
....	....	....	.144?	−0.144?	....
....	....	....	0.166?	....	....

As a source of light an oxy-cathode arc was employed, in form slightly modified from that used by Wali-Mohammad (*loc. cit.*). Plate V, Fig. 1, is a photograph of the line taken with a current of 1.5 amp. A number of plates were taken and measured, the results as compared with those of the other investigators being given in Table I.

[1] *Verhandlungen der deutsch. phys. Gesells.*, **9**, 84, 1907.

[2] *Annalen der Physik*, **20**, 285, 1906.

[3] *Tokyo Sugaka-Buturigakkwai Kizi*, 2 ser. VIII, February 1915.

[4] *Annalen der Physik*, **34**, 505, 1911.

[5] *Astrophysical Journal*, **39**, 189, 1914.

From this table and from the accompanying photograph it is seen conclusively that the results of Lunelund and Wali-Mohammad are incorrect as to the actual position of the satellites of the line above.

In order to find the true wave-length of the strongest line of the group, as well as of each of its components, a trace of Zn was mixed with the bismuth metal used as the anode. Thus the Zn line λ 4722.164, which is very sharp and single, was photographed at the same time with the Bi line. In a second attempt an equal amount of Zn was mixed with the bismuth, and then the Zn line came out self-reversed, since the current of 1.2 amp. necessary to bring out the Bi line reverses the Zn line. Thus good measurements could be obtained. The true wave-length of each of the components in international units, starting with the strongest one, is as follows:

4722.379, 4722.435, 4722.481, 4722.619, 4722.663, 4722.697.

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ON PARALLAXES AND MOTION OF THE BRIGHTER GALACTIC HELIUM STARS BETWEEN GALACTIC LONGITUDES 150° AND 216°[1]

By J. C. KAPTEYN[2]

1. INTRODUCTION

In *Mount Wilson Contribution* No. 82 an attempt was made to derive the individual parallaxes and motions of practically all the known helium stars between galactic latitudes $\pm 30°$ and galactic longitudes 216°–360°. The present paper aims at a similar determination for stars within the same limits of latitude and between longitudes 150°–216°.[3]

As in the earlier publication, the limits in latitude were thus chosen in order that the overwhelming majority of the B stars should be included in such a way that a tolerable idea of their distribution in space could be obtained by projection on a single plane, namely, that of the galaxy. The limit at $l = 216°$ forms the boundary between the stars of *Mount Wilson Contribution* No. 82 and those now under consideration; that at $l = 150°$ has been chosen rather arbitrarily. There is at this point some indication of a natural limit in the arrangement of the bright helium stars of small proper motion ($\mu = 0''016$) shown by Plate IX in *Mount Wilson Contribution* No. 82. Whether later studies confirm this indication or not does not matter much, because it is my intention later to extend the study of the distribution of the helium stars to the remaining galactic longitudes, at least as far as possible. Down to longitude 110° provisional investigations already indicate that, here too, useful results may be expected.

The region now to be considered offers far greater difficulties than that treated in *Mount Wilson Contribution* No. 82. These are a consequence of the proper motions which, in general, are

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 147.

[2] Research Associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

[3] A small supplementary group of 8 stars between $b = -30°$ to $-42°$, $l = 156°$ to 178°, was added for a particular purpose (see Sec. 8).

exceptionally small and, superficially, show scarcely any recognizable regularity in their directions. Under the circumstances it may seem a bold undertaking to subject these stars to a treatment which in general is applicable only to motions approximately parallel and equal. One circumstance, however, proves that below the apparent irregularity there must be hidden a real similarity: the radial velocities of the stars throughout the region vary relatively little. Hence (*a*) the stars must lie near the antivertex[1] and (*b*) the range of real velocity must be small.

Circumstance (*a*) explains why the proper motions are so small and makes large differences in the position angles of the motions appear as a necessity; (*b*) makes it all but certain that the seeming irregularities in the proper motions are not greater than might be expected among stars moving along approximately parallel lines with approximately equal velocities. As a matter of fact, investigation shows that the proper motions, instead of contradicting the conclusions as to the approximate equality of motion derived from the radial velocities, prove that the spectroscopic velocities give a very inadequate idea as to the degree of this equality. The seeming irregularity of the motions is due to the fact that the systematic part (stream-motion) is to a great extent obliterated by foreshortening, while the irregular part (peculiar motion) shows as strongly as anywhere else.

As the present investigation furnishes the parallaxes of practically all the B stars within the region considered, a map showing the positions of the stars in space might have been given, as was done for longitudes 216°–360° in *Mount Wilson Contribution* No. 82. I have not done so, in the hope that further investigation will furnish data for extending this map to include all of the B stars between $-30°$ and $+30°$ of galactic latitude.

2. THE MATERIALS

In the main I have used (as in *Mount Wilson Contribution* No. 82) all the B stars in Boss's catalogue, 168 in all. The spectra

[1] The name "antivertex," in analogy with the name "antiapex," is used for the point around which the motions *converge*, whereas the name "vertex" is reserved for the divergent.

and the magnitudes have been taken from *Harvard Annals*, **50**, the proper motions from Boss. Besides these bright stars I had the good fortune to be able to include all the faint B and A stars within the limits

$$\text{R.A. } 5^h0^m \text{ to } 6^h0^m \quad \text{Dec. } -10° \text{ to } +20° \tag{1}$$

for which Miss Cannon has recently determined the spectra. This list contains 412 B and 760 A stars, many of them fainter than magnitude 9.0, some even fainter than 9.5.

It is a pleasure to me to extend my warmest thanks both to Professor Pickering and to Miss Cannon for their generous help and courtesy. In using their results I have had a foretaste of the vast possibilities that will be afforded by the completion of the undertaking known as the *Revised Draper Catalogue.*

3. THE NEBULA-GROUP

A glance at the lower map of Plate IX in *Mount Wilson Contribution* No. 82 shows that the bright B stars between 5^h20^m and 5^h40^m, on both sides of the equator, are particularly crowded. This crowding, however, is shown with perfect distinctness only by the Pickering-Cannon stars. Defining the position of the group by hour- and parallel-circles we have the limits at

$$\text{R.A. } 5^h13^m \text{ and } 5^h40^m \quad \text{Dec. } -9° \text{ and } +5° \text{ (1900).} \tag{2}$$

With a more complicated boundary-line we might diminish the area by about 20 per cent.

Within this boundary the B0–B5 stars are about 12 times and the B8–B9 stars about 5.7 times more numerous than in the surrounding regions. This alone proves, incontestably in my opinion, that we have to do with a local group which probably does not extend in depth much farther than it does laterally.

We have still further proof. The bright B stars, the stars of our list in Boss's catalogue within the limits (2), are almost exclusively of spectral types B0–B3, whereas outside the group they include a considerable number of B5–B9 stars. Omitting one star marked B whose subclass is unknown, I find the following numbers of stars:

	Oe5–B3	B5	B8–B9	
Within limits (2)	23	1	1	(P)
Outside limits (2)	80	21	41	

Add to this the facts that, as will presently appear, the radial velocities of the Oe5–B3 stars are very nearly equal, and consequently that the component u[1] of the peculiar motion is exceptionally small; that the component τ of the proper motion, as already explained, is not greater than the observational errors—all of which proves that the real motions of these stars are very nearly parallel and equal, much more so than is the case with the stars outside the limits (2); and there can be no doubt that we have to deal with a local group, somewhat of the nature of the Pleiades or the Hyades. As this local group surrounds the famous Orion Nebula, which very probably, though we cannot yet say certainly, forms part of the group, I will call it the Nebula-group.

4. SYSTEMATIC ERROR IN BOSS'S μ_α AND μ_δ

I will begin by studying the stars outside the Nebula-group. But such a study must of necessity be preceded by an investigation of the systematic errors in the proper motions of Boss's catalogue. In the case of proper motions so exceptionally small (a total motion as great as 0".04 is found for only 6 per cent of our stars), such errors are of very considerable importance. It will be our aim not only to find plausible values for them, but to arrange all our calculations in such a way that the residual uncertainty has a minimum effect.

The determination of these errors will be based on the hypothesis that the direction of motion of the stars of small proper motion is not systematically different from that of the stars whose motions are somewhat more considerable. If the systematic motion of the B stars is wholly a parallactic displacement, as is generally assumed, the hypothesis will be justified. The same will be true if all the stars considered have the same stream-motion; only in the case of different stream-motions will it fail. To diminish the probability of such a state of affairs, and to increase the number of objects on which we can base our conclusions, stars in neighboring parts of the sky and of different spectra were included in the investigation.

The method itself I have explained in several places, for example, *Mount Wilson Contribution* No. 82, p. 48. Let p_1 represent the

[1] For the meaning of our notation see *Mount Wilson Contr.* No. 82, p. 77.

average position angle of the motions of stars of small proper motion, p_2 the same quantity for the stars of large proper motion, and suppose the two reduced to the same point of the sky. If then p_1 and p_2 differ, the difference will be attributed to systematic errors in μ_α and μ_δ. The equation of condition given in *Mount Wilson Contribution* No. 82 supposes that the corrections $\delta\mu_\alpha$ and $\delta\mu_\delta$ are small as compared with the motions μ_α and μ_δ themselves. In the present case, where the proper motions are as a rule so exceedingly small, the validity of such a supposition seems doubtful, and it will be preferable to use the equation in its rigorous form:

$$\frac{\mu_2 \cos p_2 - \mu_1 \cos p_1}{\mu_1\mu_2}\delta\mu_\alpha - \frac{\mu_2 \sin p_2 - \mu_1 \sin p_1}{\mu_1\mu_2}\delta\mu_\delta = \sin (p_2 - p_1) \qquad (3)$$

I began with the four regions (A) including most of the stars contained in the present paper, with the exception of the Nebula-group and its nearest surroundings.

$$\left.\begin{array}{llllllll}
\text{Group } 1 & l^* & 150^\circ & \text{to} & 165^\circ & b & -12^\circ \text{ to} & +\ 4^\circ \\
\text{"} \quad 2 & \text{"} & 200 & \text{"} & 216 & \text{"} & -10 \quad \text{"} & +\ 3 \\
\text{"} \quad 3 & \alpha & 3^h30^m & \text{"} & 5^h12^m & \delta & -17 \quad \text{"} & +\ 3 \\
\text{"} \quad 4 & \text{"} & 6\ 10 & \text{"} & 8\ 10 & \text{"} & -16 \quad \text{"} & +10
\end{array}\right\} \qquad (A)$$

In order to increase the weight of the determination somewhat, I included not only the B stars, but also the A0–A3 stars with total proper motion below 0ʺ.100. The spectra B0–B8 and B8–A3 were treated separately at first, but as no real difference seemed to exist the results were combined. The mean right ascensions and declinations of the stars of large and small proper motion were of course found to be slightly different. The reduction to the same point was made by adopting for the vertex the provisional position 17^h48^m, $+9^\circ$. The results obtained are given in Table I.

Substituting in (3) we find

			Average δ	No. Stars	Adopted Weight	
$-48.1\,\delta\mu_\alpha$	$-\ 8.6\,\delta\mu_\delta$	$=+0.078$	$+17^\circ$	38	9	(4)
$+29.9$	$+50.2$	$=+0.485$	-27	31	6	
-70.7	-14.9	$=-0.309$	$-\ 3$	17	2	
-39.1	$+44.2$	$=+0.156$	$-\ 4$	38	4	

* The inconsistency in giving the limits of part of the regions in galactic longitude and latitude and of the other part in right ascension and declination is due to the later introduction of the third and fourth regions, which at first were left out of consideration.

A solution by least squares leads to

$$\left.\begin{aligned}\delta\mu_\alpha &= +0''.0004 \pm 0''.0019 \\ \delta\mu_\delta &= +0.0072 \pm 0.0024\end{aligned}\right\} \tag{5}$$

With respect to the probable errors (last column of Table I), this solution represents the observations satisfactorily. We thus

TABLE I

AVERAGE POSITION ANGLE OF μ

Group	α	δ	100μ	p	r	No. Stars	p_2-p_1 obs.	p_2-p_1 comp.	O−C	P.E.
1	5^h43^m	$+17°$	1″.20 2.89	162°.0 166.5	7°.5 2.0	11 27	+4°.5	− 3°.5	+ 8°.0	± 7°.7
2	7 21	−27	1.18 3.05	256.0 285.0	8.0 4.0	14 17	+29	+21.5	+ 7.5	9.0
3	4 43	− 3	0.98 3.28	182.0 164.0	20.0 13.0	7 10	−18	− 6.5	−11.5	23.8
4	6 53	− 4	1.19 3.92	215.5 224.5	15.0 6.0	15 23	+ 9	+18.5	− 9.5	±16.2
					Total	124				

find a vanishing correction for the proper motion in right ascension, but a considerable one for that in declination. Reasons will be found for assuming that the error is greatest between declinations $-15°$ and $-35°$. For these we have, exclusively by the second group, putting $\delta\mu_\alpha = 0''.000$,

$$\delta\mu_\delta = +0''.0097 \qquad (\text{Dec. } -27°) \tag{6}$$

This result, if well established, would be so important that I have sought for further evidence. For reasons already given, there will be advantages in including stars of neighboring regions and of different spectra. The following represents what I have been able to find.

a) *Astronomische Nachrichten*, **160,** 338, 1903.—From Table IV of this article we find the results in Table II. The values $\delta\mu_\delta$ Kapt. were obtained on the hypothesis adopted above. There is a very serious difference in Zone $-18°$ to $-36°$ between the proper motions from Boss and those obtained by adopting our hypothesis. For

the present purpose we derive from Table II: Correction to Boss's μ_δ, between the limits $\alpha = 0^h$ to 24^h, $\delta = -18°$ to $-36°$,

$$\delta\mu_\delta = \frac{+0''.42-(-1''.17)}{100} = +0''.016. \tag{7}$$

It is worth remarking that for Auwers' proper motions the correction is small; in fact, does not exceed the limits of its uncertainty.

TABLE II

Zone	100 $\delta\mu_\delta$		
	Kapt. − Cape	Boss − Cape	Auw. − Cape
0° to −18° ..	....	+0''.11	....
−18 " −36 ..	+0''.42±0''.24	−1.17	+0''.12±0''.17 (133)
−36 " −54 ..	−0.22±0.19	−0.27	+0.30±0.19 (100)
−54 " −72 ..	+0.37±0.22	+0.83	+0.91±0.26 (57)

b) *Groningen Publications*, No. 21, p. 40 (34).—Weersma finds, by the principle here used, for Zone $\delta = -20°$ to $-40°$

$$\mu_\delta \text{ Kapt.} - \mu_\delta \text{ Newcomb} = +0''.0087 \tag{8}$$

where μ_δ Kapt. again stands for the proper motion in δ corrected by the foregoing hypothesis. By a direct comparison of Newcomb's and Boss's catalogues between $\alpha = 4^h.5$ to $8^h.5$, $\delta = -5°$ to $-35°$ I find

$$\mu_\delta \text{ Newcomb} - \mu_\delta \text{ Boss} = +0''.0033 \text{ (34 stars)}. \tag{9}$$

From (8) and (9)

$$\delta\mu_\delta = \mu_\delta \text{ Kapt.} - \mu_\delta \text{ Boss} = +0''.0120. \tag{10}$$

c) *G, K, M stars with proper motions* $\geqq 0''.017$.—The correction (5) depends on a comparison of stars of proper motions $\leqq 0''.016$ with those having greater motion. If the correction is confirmed by stars with proper motions $\geqq 0''.017$, the probability that we have to deal, not with systematic proper motion, but with systematic catalogue error, will be enormously increased, especially if we use stars of quite different spectra. It is desirable that the investigation cover the regions (A); but it is important, first, to increase the number of stars by extending these limits, and, second, to avoid the neighborhood of the antiapex (6^h, $-34°$). I therefore

selected the region R.A. 7^h to 12^h, Dec. -15° to -40° and considered two classes of stars,

$$a)\ 0''.017 \leqq \mu \leqq 0''.050, \qquad b)\ \mu \geqq 0''.070.$$

All the Boss stars of spectrum G, K, M, and, further, those of unknown spectrum were used. I was thus led to the enormous difference:

$$\delta\mu_\delta = +0''.026 \quad (69 \text{ stars}) \tag{11}$$

No great weight can be attributed to this result, because the values of p vary so much that in a few cases there is uncertainty whether they ought not to be increased by 360°.

d) *Finally by a direct comparison* of Boss's catalogue with the fundamental catalogues of Auwers and Newcomb for the region R.A. $4^h 30^m$ to $8^h 30^m$, Dec. -5° to -35° I find

$$\left.\begin{array}{ll} \delta\mu_\delta = +0''.0015 & (33 \text{ stars) by Auwers} \\ = + \ \ .003 & (34 \text{ stars) by Newcomb} \end{array}\right\} \tag{12}$$

$$\delta\mu_\delta = +0''.002$$

Summarizing, we have Table III.

TABLE III

	Equation	Limits		$\delta\mu_\delta$
B–A3 (second group)	(6)	$7^h.0$ to $8^h.0$	-24° to -36°	$+0''.010$
All spectra	(7)	0.0 " 24.0	−18 " −36	+ .016
All spectra	(10)	0.0 " 24.0	−20 " −40	+ .012
G, K, M, and unknown	(11)	8.0 " 12.0	−17 " −40	(+ .026)
Auwers, Newcomb	(12)	4.5 " 8.5	− 5 " −35	+ .002
Boss	....	....	....	0.000

For stars in the positive declinations of the region under consideration the position angles p are little affected by a correction of μ_δ. Moreover, there is every reason to believe that the correction is small. For the declinations 0° to −20°, the last two equations of (4), putting $\delta\mu_\alpha = 0.000$, give $\delta\mu_\delta = +0''.0050$. Considering everything, I finally resolved to adopt the following values:

$$\left.\begin{array}{cc} \delta & \delta\mu_\delta \\ -20^\circ \text{ to } -40^\circ & +0''.008 \\ 0 \ \ \text{"} \ \ -20^\circ & +0.004 \\ 0 \ \ \text{"} \ \ +20^\circ & 0.000 \end{array}\right\} \tag{13}$$

Boss's proper motions in declination, thus corrected, are given in Table XXXIX and have been used in all the discussions. His proper motions in right ascension were adopted uncorrected.

5. FIRST DETERMINATION OF THE ELEMENTS

For all the B stars in each of the regions (A) averages were found for α, δ, μ_α, μ_δ, with the results shown in Table IV. A second summary including only stars with proper motions $\geqq 0''.017$ is given in Table V.

TABLE IV

AVERAGES FOR ALL B STARS

Group	α	δ	No.	μ_α (Great Circle)	μ_δ	v Vert. (14)	p obs.	p comp. Vert. (14)	p comp. Vert. (24)	λ Vert. (24)
1.......	5^h46^m	$+17°.0$	23	$+0''.0031$	$-0''.0186$	$0''.0189$	$170°.5$	$172°.5$	$180°$	$152°.0$
2.......	7 18	-27.6	25	-0.0129	$+0.0094$	0.0160	306.0	310.5	305	152.4
3.......	4 20	-4.7	12	$+0.0081$	-0.0017	0.0083	102.0	103.0	108	158.3
4.......	6 47	-5.8	18	-0.0132	-0.0063	0.0146	244.0	251.0	253	163.5

TABLE V

AVERAGES FOR B STARS HAVING $\mu \geqq 0''.017$

Group	α	δ	No.	p obs.	p comp. Vert. (14)	p comp. Vert. (24)
1...........	5^h52^m	$+17°.2$	15	$172°.6$	$176°.0$	$184°$
2...........	7 19	-27.3	15	303.4	309.0	302
3...........	4 17	-3.1	9	104.3	106.5	111
4...........	6 49	-3.7	9	249.0	242.0	245

If the stars in these regions have the same motion in space, the directions defined by the values of p must intersect in a single point. This is indeed the case with extreme approximation, the position of the antivertex being:

$$\left.\begin{array}{ll}\text{From Table IV} & 6^h\ 0^m,\ -10°.5 \\ \text{From Table V} & 5\ 57\ ,\ -10\end{array}\right\}\ \text{Adopted } 6^h0^m,\ -10° \qquad (14)$$

All the directions of Table IV pass this point within a distance of about 2°; those of Table V within 2°.5.

This close convergence to a single point makes the community of motion in the four regions highly probable. Assuming this coincidence of motion for the moment, I determined the stream-velocity from the radial velocities. On account of the scarcity of data I extended the regions a little, using the limits:

$$\left.\begin{array}{llllll} \text{Group 1} & \text{R.A.} & 5^h12^m \text{ to } 6^h30^m, & \text{Dec.} & +10^\circ \text{ to } +30^\circ \\ \text{“} \quad 2 & \text{“} & 5\ 45 \quad \text{“} \quad 7\ 45 & \text{“} & -36 \quad \text{“} \quad -20 \\ \text{“} \quad 3 & \text{“} & 4\ \ 0 \quad \text{“} \quad 5\ 12 & \text{“} & -16 \quad \text{“} \quad +15 \\ \text{“} \quad 4 & \text{“} & 5\ 43 \quad \text{“} \quad 8\ 10 & \text{“} & -19 \quad \text{“} \quad +10 \end{array}\right\} \tag{15}$$

For the average radial velocities, first applying the constant correction -4.3 km (see *Mount Wilson Contribution* No. 82, p. 28), we find the results in Table VI.

TABLE VI

RADIAL VELOCITIES

Group	α	δ	λ Vert. (24)	No.	$\rho-4.3$	ρ comp. Vert. (24)	O−C	P.E.
					km	km	km	km
1........	5^h44^m	$+18^\circ.8$	150°	9	+16.1	+17.3	−1.2	±1.8
2........	6 39	−29.4	157	9	+23.3	+18.4	+4.9	±1.5
3........	4 39	− 1.3	161	9	+14.7	+18.9	−4.2	±0.8
4........	6 42	−10.5	166	11	+21.9	+19.4	+2.5	±1.9

From this we compute the stream-velocity V by

$$V \cos \lambda = \rho, \tag{16}$$

in which λ represents the angular distance from the antivertex

$$5^h44^m, \qquad -11^\circ \tag{17}$$

which differs only slightly from (14) and will later be accepted as the definitive position. A least-squares solution leads to

$$V = 20.55 \text{ km} \quad (38 \text{ stars}). \tag{18}$$

Outside the limits (15) there are three stars in our list with known radial velocities. Including these,

$$V = 20.0 \pm 1.5 \text{ km} \quad (41 \text{ stars}). \tag{19}$$

Finally we compute the average parallax by (see *Mount Wilson Contribution* No. 82, p. 35)

$$\bar{\pi}=\frac{\bar{v}}{0.212\ V\sin\lambda}. \tag{20}$$

Adopting (17) and (19) we obtain from the data of Table IV

$$\bar{\pi}=0''.0083\pm0''.0008 \quad (78 \text{ stars}). \tag{21}$$

6. COMMUNITY OF MOTION OF THE FOUR GROUPS

It was remarked that the intersection of the four lines defined by the values of p in Tables IV and V is a strong argument in favor of the supposition that the four groups have the same motion. There are, however, other criteria. If there is community of motion, and if we may assume that the values of the mean parallax are about the same for the four groups, the values of v in Table IV must be proportional to sin λ. With vertex (14) the proportionality, as indicated by Table VII, is not close. It can be greatly

TABLE VII

Group	v obs.	$0''.0351 \sin\lambda$ Vert. (14)	O−C		
			Vert. (14)	Vert. (22)	Vert. (24)
1.	0''.0189	0''.0164	+0''.0025	+0''.0027	+0''.0026
2.	0.0160	0.0148	+0.0012	−0.0026	−0.0014
3.	0.0083	0.0154	−0.0071	−0.0006	−0.0046
4.	0.0146	0.0079	+0.0067	+0.0005	+0.0035

improved by diminishing the right ascension of the antivertex, and would be quite satisfactory (see Table VII) if we adopted

$$5^{h}12^{m}, \qquad -10^{\circ} \tag{22}$$

instead of (14). This, however, would not satisfy the directions of Tables IV and V, and we must accept a compromise between the two positions.

Adopting for each of the four regions the values

$$V=20.0\text{ km}, \quad \bar{\pi}=0''.0081, \tag{23}$$

the latter of which slightly diverges from (21) but agrees with the definitive value adopted later, we can compute for each of the four

regions a separate vertex. Table IV furnishes for the centers of each of the regions the direction of the antivertex, whereas its distance λ can be found from v by (20) and (23). I thus find for the antivertices:

Group	α	δ	No.	Deviation
1..........	$6^h\ 6^m$	$-15.^\circ8$	23	$8.^\circ0$
2..........	5 50	$-\ 9.3$	25	4.3
3..........	5 18	$-\ 7.3$	12	6.9
4..........	5 12	-16.0	18	7.3
Mean...	5 37	-12.2	....	....

The last column shows the distance of the individual antivertices from the mean of all. Taking into account the small number of stars from which these antivertices were obtained, and considering that the supposed absolute equality of π for the four groups is highly improbable, the deviations are not greater than might reasonably have been expected. Certainly they afford no sufficient reason for doubting the community of motion.

As the position (14), which rests on the directions in Table IV and is thus independent of any supposition as to the equality of the values of π, whereas this supposition is involved in the result obtained just now, I will adopt as *final* the intermediate position of the antivertex

$$5^h 44^m, \qquad -11^\circ \tag{24}$$

The values of p computed with the aid of (24) have been inserted in Tables IV and V. To my regret the computed values of p and λ given in Table XXXIX have been derived with a slightly different antivertex, viz.,

$$5^h 48^m, \qquad -9^\circ. \tag{25}$$

I have not judged it worth while to repeat the computations because the two positions probably agree within the uncertainty of (24).[1]

[1] Further, (24) is probably also to be considered as only provisional. A preliminary investigation seems to show that the community of motion extends much beyond the limits adopted in the present paper. If this proves to be true, it will be necessary later to treat all the stars with common motion as a whole, which will probably change the definitive elements a little.

7. COMMUNITY OF MOTION, CONTINUED

A more serious objection to the assumption of community of motion is furnished by the radial velocities. If from the data in Table VI we compute the stream-velocity V separately for the four regions we obtain

Group 1	$l=157°$,	$V=-18.6\pm 2.1$ km	No. = 9	(26)
" 2	205	-25.3 ± 1.6	9	
" 3	164	-15.6 ± 0.85	9	
" 4	186	-22.6 ± 2.0	11	

Comparing the results of the second and third groups we find a difference of

$$9.7\pm 1.8 \text{ km}, \qquad (27)$$

which seems real enough. At all events it becomes necessary to examine the matter more closely.

I first made a brief preliminary investigation for the region adjoining those of the present paper on the side of the small galactic longitudes (120° to 150°). It turned out that the direction of motion here passes very near the point (24), which satisfies the directions of all four regions under consideration. For the radial velocity there is a decidedly closer approach to the first and third regions than to the other two, which lie on the side of greater longitudes—see (26)—the part of the sky investigated in *Mount Wilson Contribution* No. 82. It would therefore excite no particular surprise if we found the velocity, or even all the elements of motion, to diverge somewhat from those of the other regions toward those of the region treated in *Mount Wilson Contribution* No. 82. As a matter of fact, we really find the reverse. Indeed, the observations in the second and fourth regions, badly as they are represented in radial velocity by the elements (19) and (24), are still much better represented by these elements than by those for the higher galactic longitudes, namely,

$$\text{Vertex, } 18^h 24^m, \quad +39°; \quad V=-18.0. \qquad (28)$$

The results of the comparison are

Group	p obs.	ρ obs. -4.3	Computation By				O−C			
			(19) and (24)		(28)		(19) and (24)		(28)	
			p	ρ	p	ρ	p	ρ	p	ρ
2.........	306°	+23.3	305°	+18.4	221°	+17.7	+1°	+4.9	+85°	+5.6
4.........	244	+21.9	253	+19.4	188	+15.7	−9	+2.5	+56	+6.2

The position angles are therefore not at all represented by (28), and the radial velocities markedly worse than by (19). The case therefore stands thus: If we accept the reality of the divergence of the radial velocity of Group 2, this group can be considered as local only, for it stands apart from the stars, both in lower and in higher galactic longitudes. If we do this, there is every reason for also including the fourth region in the local group, which thus becomes extensive enough for a determination of all its elements.

The directions furnished by Table IV for Groups 2 and 4 cut each other at an angle of 65°; those for Groups 1 and 3, at an angle slightly greater. If therefore we take the points of intersection as the antivertices, they must in both cases be considered well determined. Both agree almost perfectly with position (14) obtained from the four regions together. Accepting these vertices, the stream-velocity is obtained by the data of Table VI. I find

$$\left.\begin{array}{llll} \text{Groups 2 and 4} & V=23.5\pm 1.2 & \text{(20 stars)} \\ \quad\text{"}\quad\; 1\;\text{"}\;\; 3 & V=17.2\pm 1.2 & \text{(18 stars)} \\ \quad\text{Difference} & \;\;\;6.3\pm 1.7 & \end{array}\right\} \qquad (29)$$

The result thus would be that the second and fourth regions form a local group whose direction of motion coincides absolutely with that of the others but whose velocity is 37 per cent greater.

Some further criteria may be considered:

1. A local group can usually be seen as such on an ordinary star map. If on the map in *Mount Wilson Contribution* No. 82

($\mu \leqq 0''.016$) we exclude the Nebula-group (2) there seems to be indicated a group within the limits, approximately,

$$l = 180° \text{ to } 216°; \quad b = -30° \text{ to } +4°, \tag{30}$$

which covers Group 2 and the richer part of Group 4.

2. In a local group we expect the proper motions to be nearly equal. The proximity of the antivertex and the minuteness of the proper motions, which result in their being strongly influenced by observational errors, and finally the fact that if the group is not very condensed many extraneous stars will be seen projected on it, make the criterion less effective in the present case than ordinarily. If we call proper motions $\leqq 0''.016$ small, then, within the limits (30), 61 per cent of the motions are small; outside these limits, only 39 per cent.

3. In local groups we expect to find a distribution of spectra differing from that in the rest of the sky. Thus, in the Nebula-group we found a strong predominance of the Oe5–B3 stars (see Section 3). Within the limits (30) we find that 77 per cent of all the stars are Oe5–B5; outside these limits there are 59 per cent.

4. For the mean parallax of the stars within the limits (30) we find, as above, $\bar{\pi} = 0''.0076$, which scarcely differs from (21) and (23).

5. For the Nebula-group we shall find further on $\bar{\pi} = 0''.0054$. Its stream-velocity can scarcely differ from the value in (19). If at all different, it seems to be rather smaller. The elements for Groups 2 and 4 do not approach those of the Nebula-group more than they do those of Groups 1 and 3.

In conclusion, we cannot deny the possibility of a local group; still, in my opinion, the similarity in the direction of the motions outweighs all other arguments, so that the probability is not in favor of such a group. The difference (29) of 6.3 km between the stream-velocities of Groups 2+4 and 1+3, which is the main argument in favor, is indeed almost four times the probable error. But such a result is perhaps not so surprising for radial velocities as it would be for other quantities, because there must still be numerous cases of undiscovered orbital motion.

In what follows we shall assume that the stream-motions of all four groups are the same. The question can probably be settled

without difficulty by the fainter stars as soon as the *Revised Draper Catalogue* is published. Even if it should then be proved that there really is a local group, our results will be little altered. The parallaxes, and consequently the absolute magnitudes, M, in the main will not be altered, and the same, of course, holds for the luminosity curve. The computed values of p also will not change. Only the values of the radial velocities will be somewhat different, for instead of computing these with (19) we shall have to use (29).

The average values of the parallax finally adopted are in some cases slightly different from the mean in (23), because, first, our four groups do not include all the available material, and, secondly, it does not seem reasonable to adopt exactly the same value for all the partial groups. From the new computation Boss 1401, 1517, and 1994 were excluded—the first and the last because they do not seem to belong to the same system as the rest of the stars (as proved by the values of p), and No. 1517 because it has so exceptionally large a proper motion. The values thus obtained for the groups of different λ are given in Table VIII. For certain purposes they have been further contracted into the considerably overlapping groups in Table IX.

TABLE VIII

λ	Mean λ	$\bar{\pi}$	No. of Stars	Weight
145° to 149°...	147°	0″.0081	26	138
150 " 154 ...	152	0.0075	40	160
155 " 159 ...	157	0.0107	28	77
160 " 164 ...	162	0.0066	17	29

TABLE IX

λ	Mean λ	$\bar{\pi}$	No. of Stars	Weight
145° to 154°...	150°	0″.0077	66	11
150 " 159 ...	154	0.0087	68	9
152 " 167 ...	157.5	0.0083	75½	4

From Table VIII, I find, in good agreement with (23), for the general average:

$$\pi = +0''.0081 \pm 0''.0007 \quad (111 \text{ stars}). \tag{31}$$

8. INFLUENCE OF SYSTEMATIC ERROR AND SUMMARY OF DEFINITIVE ELEMENTS

On account of the excessive smallness of the proper motions and the consequently strong influence of observational errors, it will not be deemed superfluous if we try to find the effect of any remaining traces of systematic error. For this purpose I made new solutions with the data in Tables IV and VI on the supposition: (*a*) that Boss's values of μ_α require a constant correction $\delta\mu_\alpha = +0''.0030$; (*b*) that the values of μ_δ in Table XXXIX, that is, of Boss's μ_δ corrected by (13), require a further correction $\delta\mu_\delta = +0''.0030$; (*c*) that the correction (13) for μ_δ is wholly false, our new solution thus, in this case, starting with the uncorrected μ_α and μ_δ of Boss.

For this special investigation I have disregarded the values of the υ in the derivation of the vertex and adopted as antivertex the point toward which all the directions converge with the greatest approximation. I thus found

Case		Antivertex		π	V	
(1)	No correction	$6^h\ 0^m$	$-10^\circ.5$	$0''.0082$	20.8	(32)
(2)	Correction (*a*)	6 12	-8	0.0080	21.3	
(3)	" (*b*)	5 55	-4	0.0082	20.8	
(4)	" (*c*)	5 52	-18	0.0076	20.55	

In every case the four directions of Table IV intersect nearly at a single point. This is most perfectly realized in the cases (1) and (2), where none of the directions deviate from the adopted antivertex more than 2°. In cases (3) and (4) the greatest distance is 4° or 4°.5.

In view of these results I think we may conclude that, as a consequence of the remaining systematic errors and the consideration of the values of υ in Section 6, it is not impossible that there may still be an error of 3° or 4° in the position of the antivertex, but that the resulting uncertainties in V and π must be negligible.

This freedom of V from systematic error is due to the fact that the region as a whole is so near the antivertex, and to the further fact that the four separate regions lie fairly symmetrically around the antivertex. As for π, its freedom from systematic error is wholly attributable to this last circumstance. It was this consideration which in great part led to our choice of the four regions

and compelled us to introduce a supplementary group outside the limit at galactic latitude -30°.

The danger from the systematic errors being thus in great measure avoided, there is every reason to accept the probable errors as a measure of the accuracy obtained. Collecting results, we have as *definitive elements:*

$$\left.\begin{array}{l} \text{Vertex } 17^h 44^m, \quad +11^\circ \quad (\text{uncertainty } \pm 3^\circ \text{ or } \pm 4^\circ) \\ V = -20.0 \pm 1.5 \text{ km (P.E.)} \\ \bar{\pi} = 0''.0081 \pm 0''.0007 \text{ (P.E.)} \end{array}\right\} \quad (33)$$

This will not prevent the adoption for π of the separate values of Table IX. It is a very significant fact, to which I hope to revert in a later publication, that this vertex coincides so nearly with that of the first stream of all the non-helium stars, for which Eddington in *Monthly Notices*, **71**, 42, 1910, finds $18^h 3^m$, $+14^\circ.6$.

9. OBSERVED DISTRIBUTION OF THE VALUES OF v

Now that a good estimate of the average π has been obtained, we have to find the range in distance. It would be most desirable to determine the individual parallax of each star, but as this is not feasible, we will try to derive mean parallaxes for stars having given values of λ and v and, further, the probable deviation of the individual parallaxes from this mean. The amount of this deviation as a fraction of the parallax will determine the confidence with which we may use the mean parallax as a substitute for the individual parallaxes.

I will begin with the consideration of certain necessary data: first of all, with an examination of the distribution of the values of v for different values of λ. By countings in Table XXXIX, omitting stars within 15° of the antivertex and excluding Boss 1517, I find the data in Table X.

The number of stars being so small, pains have been taken to smooth thoroughly the results. We first contract the data into three partially overlapping zones:

λ	$\bar{\lambda}$	v	No.	
145° to 154°	150	$0''.0164$	66	
150 " 159	154	0.0161	68	(34)
152 " 167	157.5	0.0135	$75\frac{1}{2}$	

Even so, it is not easy to arrive at an entirely satisfactory result. The difficulty would be greatly reduced if we knew the form of the frequency curve for the v. If there were no observational errors,

TABLE X

DISTRIBUTION OF THE VALUES OF v

100 v	λ						Totals
	145°–149°	150°–151°	152°–154°	155°–159°	160°–164°	165°–167°	
6″.0 to 7″.0..	....	1	....	....	....	....	1
5.0 " 6.0..	....	....	....	....	....	....	....
4.0 " 5.0..	1	....	1	....	....	1	3
3.0 " 4.0..	5½	2	1	7	1	....	16½
2.0 " 3.0..	6½	5	4	8½	1	....	25
1.0 " 2.0..	6	3½	9	1½	4	2	26
0.0 " 1.0..	5	2½	7	9	7	½	31
0.0 " −1.0..	2	1½	1½	1	4	2½	12½
−1.0 " −2.0..	....	....	....	½	....	....	½
<−2.0....	....	....	1	½	....	....	1½
Totals......	26	15½	24½	28	17	6	117

such a form could be assigned, at least if we assumed—on grounds presently to be given—that the values of u (component of *linear* peculiar motion) are distributed according to an error-curve. For it follows from formula (20) that, for any definite value of π,

$$\pi = 0.212\, v \cdot V \sin \lambda \tag{35}$$

with individual deviations following an error-curve having the probable error

$$r_v = \pm 0.212\, \pi r_u,$$

where r_u is a constant for which later will be found the value 1.67 km. The deviations of v are thus seen to be proportional to v. From the theory developed elsewhere[1] it follows that the values of v, for the same value of λ, will be distributed according to the formula

$$\text{Prob.}\,{}_{v}^{v+\delta v} = \frac{h}{\sqrt{\pi}} \cdot \frac{\text{Mod.}}{v} e^{-h^2[\log v - M]^2} \delta v, \tag{a}$$

that is, the values log v will be distributed according to an error-curve.

[1] *Skew Frequency Curves in Biology and Statistics*, Noordhoff (Groningen), 1903, p. 21.

This, however, holds only in the absence of observational errors. Since these follow an error-curve, the distribution of the observed values of v will be something between (a) and an ordinary error-curve. In particular the real curve, unlike (a), will yield certain frequencies for negative values of v.

Now we obtain an intermediate curve if in (a) we substitute $v+K$ (K being a constant) for v. The formula then becomes

$$\text{Prob.}\,{}_{v}^{v+\delta v}=\frac{h}{\sqrt{\pi}}\cdot\frac{\text{Mod.}}{v+K}e^{-h^2[\log(v+K)-M]^2}\delta v, \tag{b}$$

which implies that the distribution of log $(v+K)$ follows an error-curve.

Led by these considerations I have tried to learn whether it is possible to determine the three constants in (b) in such a way that the observed frequencies are well represented. This being really the case, I have considered the best representation by (b) as that to be adopted. The derivation of the best-fitting values of h, K, M is very easy.[1] It is to be recommended that this be made in such a way that the arithmetical mean values $\bar{v}$ in (34) are perfectly represented. This gives as a first condition between the constants

$$\log(\bar{v}+K)=M+\frac{1}{4h^2\cdot\text{Mod.}}. \tag{36}$$

The values obtained are as follows:

$$\begin{array}{llll} \lambda & 150^\circ & 154^\circ & 157^\circ\!.5 \\ h & 9.347 & 15.274 & 6.280 \\ K & 0.0608 & 0.1095 & 0.0451 \\ M & -1.1190 & -0.9033 & -1.2464 \end{array}\Bigg\} \tag{37}$$

These being substituted in (b), we find by integration (which offers no difficulty) the values in the second, fifth, and eighth columns of Table XI. Those in the other columns will be explained in Section 13.

10. OBSERVED DISTRIBUTION OF THE VALUES OF τ

In order to increase the reliability of the conclusions, I have confined myself to the stars for which the probable error of τ does not exceed 0″.0069. The number of stars is thus somewhat diminished,

[1] *Skew Curves*, art. 18, p. 34.

but this is not so material here, because stars at arbitrarily different distances from the vertex can be combined. Boss in his catalogue gives the probable error of 100 μ_α and 100 μ_δ. The mean of the two was combined to represent 100 times the probable error of the

TABLE XI

DISTRIBUTION OF THE VALUES OF v

(Theoretical Values are for $r_u = \pm 2.5$)

λ	$\lambda=150°$			$\overline{\lambda}=154°$			$\overline{\lambda}=157°.5$		
	Obs.	Theor.	O−C	Obs.	Theor.	O−C	Obs.	Theor.	O−C
+0″.055	0.5	0.4	+0.1	0.3	0.3	0.0	1.1	0.9	+0.2
.050	0.5	0.5	0.0	0.5	0.4	+0.1	0.7	0.7	0.0
.045	0.9	0.9	0.0	0.8	0.7	+0.1	1.1	1.0	+0.1
.040	1.6	1.7	−0.1	1.6	1.4	+0.2	1.6	1.6	0.0
.035	2.6	2.8	−0.2	2.7	2.3	+0.4	2.5	2.4	+0.1
.030	4.0	4.3	−0.3	4.4	3.9	+0.5	3.6	3.4	+0.2
.025	6.1	6.2	−0.1	6.5	5.9	+0.6	5.1	4.8	+0.3
.020	7.9	8.3	−0.4	8.4	8.1	+0.3	6.8	6.6	+0.2
.015	9.2	9.8	−0.6	9.9	10.0	−0.1	8.6	8.8	−0.2
.010	10.2	10.0	+0.2	10.2	10.8	−0.6	10.0	10.7	−0.7
+0.005	9.2	8.8	+0.4	9.0	9.8	−0.8	10.5	11.5	−1.0
0.000	6.7	6.3	+0.4	6.7	7.4	−0.7	9.7	10.2	−0.5
−0.005	4.1	3.6	+0.5	4.2	4.4	−0.2	7.3	7.1	+0.2
−0.010	1.8	1.6	+0.2	2.1	2.0	+0.1	4.5	3.8	+0.7
−0.015	0.6	0.7	0.0	0.8	1.0	+0.1	1.9	1.5	+0.4
−0.020	0.1			0.3			0.5	0.6	−0.1
Total	66.0	65.9	....	68.4	68.4	....	75.5	75.6	......

proper motion in any co-ordinate, hence also of 100 τ. This is given in Table XXXIX under the heading 100 r. I have further disregarded the stars within 20° of the adopted antivertex (24), which excludes the Nebula-group. The results of the countings are given in Table XII.

TABLE XII

NUMBER OF VALUES OF τ

τ	Observed Number	τ	Observed Number
<−0″.025	0.5	+0.005 to + .010........	9.0
−0″.020 to −0″.025........	0.5	+ .010 " + .015........	8.5
− .015 " − .020........	2.0	+ .015 " + .020........	1.0
− .010 " − .015........	8.5	+0.020 " +0.025........	0.0
−0.005 " − .010........	13.5	>+0″.025	1.0
.000 " − .005........	12.0		
.000 " + .005........	15.5	Total................	72.0

The numbers fit very well an error-curve with the probable error $\pm 0\rlap{.}''0061$.

11. PROBABLE AMOUNT OF r_u AND DISTRIBUTION OF THE COMPONENTS OF THE PECULIAR MOTION

Let u and t[1] represent the components of the linear peculiar motion at right angles to the line of sight, one toward the antivertex, the other at right angles thereto; u and t subtend the angles υ and τ; $\bar{u}$ and $\bar{t}$ will denote the average values, all taken positively.

Since we suppose the directions of the peculiar motions to be distributed at random,

$$\bar{u}=\bar{t}=\text{average peculiar radial motion.} \tag{38}$$

This peculiar radial motion is the radial motion freed from stream-motion. We thus have to find[2]

$$\text{Observed radial motion}=-V\cos\lambda, \tag{39}$$

V and λ being in accordance with (33). The average of these values, all taken positively, will be $\bar{u}=\bar{t}$. Outside the Nebula-region there are 45 stars for which we have radial velocities. Of these, I exclude Boss 1761, 1817, 1935. The velocity of the first rests on measures of the bright H line, a determination, there is reason to think, not quite comparable with the others. The second is a spectroscopic binary; the velocity of the center of mass is a simple estimate which may prove to be largely in error. The velocity of the third is quite abnormal. Unless a spectroscopic binary, the star cannot belong to the same system as the other stars. For the remaining 42 stars

$$\bar{u}=5.78 \text{ km.} \tag{40}$$

Treating the stars of region (30) as a separate local group, this becomes 5.36. In *Mount Wilson Contribution* No. 82, p. 28, the corresponding value was found to be

$$\bar{u}=3.5 \text{ km} \quad \text{(61 full-weight stars)} \tag{41}$$

[1] t is the quantity for which in *Mount Wilson Contribution* No. 82 was used the somewhat less obvious notation v.

[2] The correction -4.3 km is first to be applied to the observed radial velocities.

Since the values of $\bar{u}$ depend partly on real peculiar motions and partly on observational errors, either one or the other or both of these factors must be greater now than in the earlier investigation.

The following considerations show that the difference is due to both causes. First, all results were included in (40), whereas in (41) the less reliable values were either excluded or admitted with diminished weight; secondly, (41) rests exclusively on Lick Observatory results, which do not include stars whose spectra are not susceptible of at least fairly good measurement, whereas the Mount Wilson observations, upon which (40) is partly based, include all objects on the original program, irrespective of their difficulty. If, to secure homogeneity, we limit ourselves to the values obtained by the Lick observers, and if further we exclude all values marked as uncertain, or given without any decimal, or as the estimated velocity of a spectroscopic binary, we find

$$u=4.24 \text{ km} \qquad r_u=\pm 3.6 \text{ km} \qquad (16 \text{ stars}) \qquad (42)$$

We thus approach the value (41), but the number of stars has become so small that little reliance can be put on the result.

Fortunately we can derive a much more reliable value in another way; but this requires a knowledge of the parallaxes, so that we shall have to anticipate to some extent results obtained later. With known parallaxes we can transform the τ components into linear motions by

$$t=\frac{\tau}{0.212\,\pi} \qquad (t \text{ in km per second}) \qquad (43)$$

In order to exclude completely the Nebula-group I use only stars for which $\lambda<160°$. Since for objects of small parallax observational errors in τ appear much magnified in t, I include only stars for which $\pi \geqq 0''.0070$. Finally, I exclude Boss 1517, for which the parallax is exceptionally large and uncertain.[1] For comparison the same computation was made for the stars in *Mount Wilson Contribution* No. 82. In this I avoided practically all excep-

[1] The value of τ, consequently of t, is quite normal for this star. Stars in our tables considered as not belonging to the system were, of course, also omitted; only one object, Boss 1944, was excluded on this account.

tional cases by limiting myself to the stars for which $\lambda \leqq 120°$ and galactic longitude $< 320°$, and I tried to improve the results by omitting badly observed stars for which $100\,r > 0''.80$.

Before deriving averages I tried to find the frequency-curve of t freed from observational error. For the observed values[1] the distribution is at once given by counts, but we cannot hope to pass with any precision to the distribution of the true values as long as the observational errors predominate in the observed values of τ. This is the case if we treat all the stars, but it is no longer so if we confine ourselves to the best observed stars, those for which $100\,r \leqq 0''.40$. By this limitation the number of available objects becomes small, so that a very reliable result is not to be expected, but as a roughly approximate determination is to be preferred to none at all, or to a mere supposition, I do not hesitate to communicate my results.

We may safely assume, I think, that the frequency-curves, both of the observed and of the true values of t, are symmetrical. On this assumption we find, by counting, for the stars having $100\,r \leqq 0''.40$, the results in Table XIII. For both the Orion and the Scorpius-Centaurus region the observed distributions differ little from the normal error-law, as appears if we compare them with curves having probable errors of 2.32 and 1.83 km, respectively (third and sixth columns of Table XIII). The residuals show little that is systematic; they are mostly of different sign for the two regions. We thus conclude that the observed values of t are distributed closely in accordance with the error-law.

For the stars of other types, in particular for those of type K, there appears to be a deviation from this law in the direction of an excess of large motions.[2] We do not find much evidence of such an excess here, especially if we consider that the one star in the Scorpius-Centaurus region with a large value of t, namely, Boss 3115, has a very abnormal radial velocity. Should this be confirmed by later observations, the star should be excluded from the group.

[1] By *observed* values of t, I mean the values computed by (43) with the aid of the *observed* values of τ.

[2] *Proc. Nat. Acad. Sci.*, 1, 17–18, 1915.

Admitting, therefore, a distribution of the observed values of t according to the law of error, we should infer at once a similar distribution of the true values, could we assume that the observational errors affecting t also followed that law. Now this is

TABLE XIII

DISTRIBUTION OF $t_{obs.}$ ($100\ r \leqq 0''.40$)

$t_{obs.}$	Orion No.	Orion Normal $r=\pm 2.32$	Orion O−C	Scorpius-Centaurus No.	Scorpius-Centaurus Normal $r=\pm 1.83$	Scorpius-Centaurus O−C
0 to ±1km......	9	6.4	+2.6	6	7.2	−1.2
±1 " ±2	6	6.0	0.0	7.5	6.3	+1.2
±2 " ±3	4	5.0	−1.0	6	4.8	+1.2
±3 " ±4	1	3.8	−2.8	3.5	3.2	+0.3
±4 " ±5	2	2.7	−0.7	1	1.9	−0.9
±5 " ±6	3	1.9	+1.1	....	0.9	−0.6
±6 " ±7	1	1.0	0.0	....	0.4	
±7 " ±8	0.5	0.6	−0.1	....	0.2	
±8	1.5	0.6	+0.9	1	0.1	
Totals........	28	28.0	....	25	25.0	....

certainly not rigorously the case. We may assume that the observational errors of τ—consequently, by (43), those of t—follow the error-law *for stars of the same* π. For different values of π, however, the probable value of the observational error in t will be quite different; in fact, inversely proportional to π. Nevertheless, I convinced myself that the observational errors in t follow the error-law with sufficient approximation for our present purpose, and that the corresponding probable error is ± 1.47 km for the Orion region and ± 1.50 km for the Scorpius-Centaurus stars. Consequently, the distribution of the true values of t must also follow approximately the error-law, and the probable value r_t which equals r_u will be

$$\left.\begin{array}{ll} r_t = r_u = \pm\sqrt{2.32^2 - 1.47^2} = \pm 1.80 & \text{(28 stars) Orion region} \\ r_t = r_u = \pm\sqrt{1.83^2 - 1.45^2} = \pm 1.12 & \text{(25 stars) Scorpius-Centaurus} \end{array}\right\} \quad (44)$$

The smallness of these values is the most surprising and promising fact brought to light by the present investigation. The B stars of *Mount Wilson Contribution* No. 82 with those of the present

paper comprise about 65 per cent of all the B stars. With a value of r_u as small as those just found, the determination of accurate parallaxes of all these stars becomes merely a question of securing more accurate proper motions, which is only a matter of time; and with the powerful aid of photography the interval need not be so very long.

Further, preliminary investigation has shown that a similar parallelism exists for the B stars in the Perseus region and that at least a good part of the A stars share in the motion of the helium stars.[1] It thus seems not unlikely that further investigation by these methods will give *good determinations of the parallaxes of all the B stars and of a large number of the A stars.*

For the later types the beautiful method proposed by Adams and Kohlschütter[2] and developed in detail by Adams[3] has recently opened the prospect of extensive determination of parallax by spectroscopic means. All this awakens the hope that we are at last on the way toward a wholesale determination of the third co-ordinate—distance—the lack of which has been the main obstacle in the way of substantial knowledge of the structure of the stellar system.

The extreme importance of the matter makes it desirable to obtain for the B stars the most reliable values of the parallax possible. I therefore give another solution for r_u in which the stars less satisfactorily observed are not altogether neglected.

For the Scorpius-Centaurus region all the stars were used, whatever their parallaxes; for the Orion region the stars with parallaxes $< 0''.0070$ were neglected, because it was feared that the remaining errors in the very small parallaxes would influence too greatly the values for t obtained by (43). For the first region the stars were subdivided into six groups,[4] the limits of the parallax being respectively: $100\pi \leqq 0''.49$; $0''.50$–$0''.69$; $0''.70$–$0''.86$; $0''.87$–$1''.04$; $1''.05$–$1''.27$; $1''.28$–$2''.24$. For the Orion region the limits were the same, but

[1] *Trans. Internat. Solar Union*, **3**, 220, 1911.

[2] *Mt. Wilson Contr.*, No. 89; *Astrophysical Journal*, **40**, 385, 1914.

[3] *Mt. Wilson Comm.*, Nos. 23–25; *Proc. Nat. Acad. Sci.*, **2**, 143, 147, 152, 1916.

[4] In Table XV the first two had to be combined.

the first two groups are wanting. The first four columns of Tables XIV and XV contain, respectively, the averages of $100\ \pi$, $100\ \tau$, $100\ r$, and of t computed from $\bar{\tau}$ and $\bar{\pi}$ by (43). The fifth column contains the probable amount of 100τ freed from observational

TABLE XIV

Average Values of t, etc.

Orion Region							Scorpius-Centaurus Region						
$100\ \bar{\pi}$	$100\ \bar{\tau}$	$100\ \bar{r}$	$\bar{t}$	$100\ r_\tau$	r_t	No.	$100\ \bar{\pi}$	$100\ \bar{\tau}$	$100\ \bar{r}$	$\bar{t}$	$100\ r_\tau$	r_t	No.
							(0″.32)	(0″.61)	(0″.62)	(7.66)	(0″.000)	(0.0)	(8)
							0.62	0.44	0.55	3.32	0.000	0.0	14
0″.77	0″.62	0″.48	3.70	0″.210	1.3	21	0.78	0.57	0.51	3.50	0.000	0.0	19
0.94	1.04	0.56	5.17	0.677	3.4	23	0.96	0.61	0.48	3.02	0.187	0.9	16
1.13	0.55	0.58	2.32	0.000	0.0	21	1.15	0.76	0.50	3.11	0.403	1.65	20
1.52	0.83	0.44	2.88	0.546	1.7	12	1.59	0.83	0.59	2.55	0.379	1.1	22
1.04	0.761	0.526	3.63	0.370	1.68	77	1.07	0.662	0.529	3.07	0.181	0.80	91
					1.64							0.79	

TABLE XV

Same as Table XIV but for $100\ r \leqq 0''.40$

Orion Region							Scorpius-Centaurus Region						
$100\ \bar{\pi}$	$100\ \bar{\tau}$	$100\ \bar{r}$	$\bar{t}$	$100\ r_\tau$	r_t	No.	$100\ \bar{\pi}$	$100\ \bar{\tau}$	$100\ \bar{r}$	$\bar{t}$	$100\ r_\tau$	r_t	No.
							0″.45	0″.33	0″.35	2.40	0″.000	0.0	2
0″.77	0″.58	0″.35	3.62	0″.343	2.1	10	0.79	0.28	0.31	1.72	0.000	0.0	5
0.91	0.50	0.26	2.40	0.332	1.7	5	0.96	0.30	0.32	1.45	0.000	0.0	8
1.12	0.32	0.33	1.35	neg.	0.0	6	1.13	0.53	0.32	2.22	0.314	1.3	6
1.57	0.87	0.31	2.90	0.666	2.0	7	1.89	1.45	0.37	4.02	1.168	2.9	4
1.07	0.582	0.321	2.74	0.373	1.64	28	1.07	0.538	0.329	2.18	0.314	1.38	25
					1.55							0.78	

error, as follows: Admitting that the values of τ are distributed substantially according to an error-curve (see Table XII) the probable amount $100\ r_\tau$ corresponding to $100\ \tau$ was found by multiplying the average amount by the factor 0.845. Freed from observational error we therefore evidently have

$$100\ r_\tau = 100\sqrt{(0.845\ \tau)^2 - r^2}. \tag{45}$$

In a few cases, the quantity under the radical being negative, r_τ was assumed to be zero. With the aid of r_τ we find the probable amount, $r_t = r_u$ of t, freed from observational error by

$$r_t = \frac{r_\tau}{0.212\,\pi}, \tag{46}$$

which evidently follows from (43). These quantities are given in the sixth column. In the seventh are the numbers of stars included in each average. The last line of each table shows the total averages; for r_t two values are given, the first, from the total averages of r_τ and π by (46), the second, from the separately computed values of r_t with weights proportional to the number of stars. Except for the last value in Table XV, the agreement is almost exact; I adopt the means.

Supplementing these results with those obtained by the same method when we limit ourselves still more closely to the best-observed stars, we have the following values for $r_t = r_u$:

Orion				Scorpius-Centaurus			
100 r	Reference	r_t	No.	100 r	Reference	r_t	No.
		km				km	
All values..	Table XIV	1.66	77	$\leqq 0.''80$..	Table XIV	0.80	91
$\leqq 0.''40$....	(44)	1.80	28	$\leqq 0.40$..	(44)	1.12	25
$\leqq 0.''40$....	Table XV	1.60	28	$\leqq 0.''40$..	Table XV	1.08	25
$\leqq 0.30$....	....	1.56	9	$\leqq 0.30$..	....	0.75	7

from which finally the *adopted values*

$$\left.\begin{array}{ll}\text{Orion region} & \mathbf{r_u = \pm 1.67\ km} \\ \text{Scorpius-Centaurus region} & \mathbf{r_u = \pm 1.00\ km}\end{array}\right\} \tag{47}$$

12. REMARKS

Remark 1.—It has been implicitly assumed in what precedes that the errors in the parallaxes do not sensibly influence the results for r_u. I have convinced myself that such is really the case.

Remark 2.—The accuracy of the determination of r_u certainly is still of a rather low order. I have refrained from computing

probable errors, because the data are hardly adequate for a good determination. It is easy to obtain an upper limit for r_u by neglecting the observational errors altogether. From Table XV we find, since $r_t = 0.845\,\bar{t}$,

$$\text{Upper limit for } r_u \left\{ \begin{array}{ll} \text{Orion region} & = \pm 2.3 \text{ km} \\ \text{Scorpius-Centaurus region} & = \pm 1.8 \text{ km} \end{array} \right\} \qquad (48)$$

With these numbers before us it cannot be doubted that the real values of r_u do not materially exceed the adopted values (47); the chances are that the latter are somewhat too high.

In consideration of the remaining uncertainty, however, I have carried through the computations that follow on the two extreme suppositions

$$r_u = \pm 2.5 \text{ km and } r_u = 0.0 \text{ km}. \qquad (49)$$

This will make a subsequent correction easy, if we find means of improving our values by the addition of well-observed fainter stars, by the inclusion of later observations, or by the discovery that the A stars for the region under consideration show the same motions, etc. For the present we shall definitely adopt the parallaxes corresponding to the first of (47); these will be obtained by interpolating between the two solutions based on (49).

Remark 3.—The value now derived for the Scorpius-Centaurus region is considerably smaller than $r_u = \pm 2.1$ found in *Mount Wilson Contribution* No. 82, where fortunately it does not play nearly as conspicuous a part as it will here. This was obtained in a less direct way. As far as I can see, there are two causes for the divergence, both of which tend to give too large a value: First, in *Mount Wilson Contribution* No. 82, all stars for which the probable error of the position angle of the proper motion, due to observational error, exceeds 10°, have been omitted. It can be shown that a small systematic error was thus introduced, the effect of which is an increase in the value for r_u. Secondly, in that same paper there was no limitation to stars below galactic longitude 320°. The consequence must be the introduction of a relatively larger number of stars not belonging to the group. Avoiding the first source of error by not limiting the values of r_p and excluding only the four

extreme values,[1] I find by the method of *Mount Wilson Contribution* No. 82, $r_u = \pm 1.3$ km (135 stars), which is in fairly good agreement with (47). By eliminating the second source of error as well, we should no doubt find the agreement quite satisfactory.

Remark 4.—The value (42) obtained from the best radial velocities gives for the probable amount of u, including errors of observation, ± 3.6 km. Since we adopt $r_u = 1.67$ km, the probable observational error of the radial velocities is ± 3.2 km. For the Scorpius-Centaurus region the corresponding quantity is ± 2.8 km.

[1] If we do not exclude these, the value of r becomes still smaller.

[*To be continued*]

ON THE ORBIT OF THE SPECTROSCOPIC BINARY 42 CAPRICORNI

By JOSEPH LUNT

Two plates of the spectrum of this star (H.R. 8283, $\alpha = 21^h36^m.1$, $\delta = -14° 29'$ [1900]; Mag., 5.28; Type K) taken on October 2 and October 5, 1917, gave radial velocities of -17.7 and $+7.2$ km per second, respectively. As its variable velocity appeared not to have been announced previously, fifteen further plates were secured during October and November, and these on measurement gave the results shown in Table I (p. 105).

The measures were made on the Hartmann spectro-comparator using plate 4903 of α Tauri as standard, the shift on this plate being taken as $+82.34$ km.

The observations were plotted on millimeter paper to the scale 30 mm = 1 day, 10 mm = 2 km, and the following provisional elements of the orbit were derived by graphical methods:

$$P = 13.25 \text{ days}$$
$$T = \text{J.D. } 2421525.16$$
$$\omega = 175°$$
$$e = 0.20$$
$$K = 22.75 \text{ km per second}$$
$$a \sin i = 4{,}061{,}000 \text{ km}$$
$$V_0 = -3.0 \text{ km per second}$$

Correction for solar motion $= +7.3$ km

The system is therefore receding from us with a radial velocity of $4.\dot{3}$ km, on the assumption that the solar motion is toward $18^h, +30°$ at 20 km per second.

Fig. 1 shows the observations compared with the theoretical curve. The figures beside the circles denoting the observations show the number of periods to be applied to the dates given above the curve to obtain the date of observation.

Table II shows the observations arranged in order of phase and the differences between the observed velocities and those derived

from a curve of theoretical values corresponding to the foregoing elements.

TABLE I

Plate	Date 1917	Sidereal Time	Correction to Sun	Radial Velocity
5080.....	Oct. 2	$22^h 23^m$	−22.22 km	−17.7 km
5084.....	5	22 15	23.23	+ 7.2
5090.....	17	22 31	26.81	− 0.7
5092.....	23	22 14	27.98	+ 4.0
5094.....	24	22 11	28.16	− 3.5
5096.....	25	22 31	28.38	−11.1
5098.....	26	22 27	28.54	−22.4
5100.....	27	22 27	28.69	−32.2
5101.....	31	22 44	29.29	+ 5.0
5103.....	Nov. 3	22 47	29.63	+16.4
5105.....	6	23 27	29.97	− 0.6
5106.....	7	23 35	30.04	− 7.4
5107.....	8	23 16	30.09	−22.3
5108.....	9	23 7	30.12	−29.5
5110.....	10	23 30	30.20	−23.7
5112.....	12	23 37	30.27	− 2.9
5115.....	16	23 49	30.28	+13.9

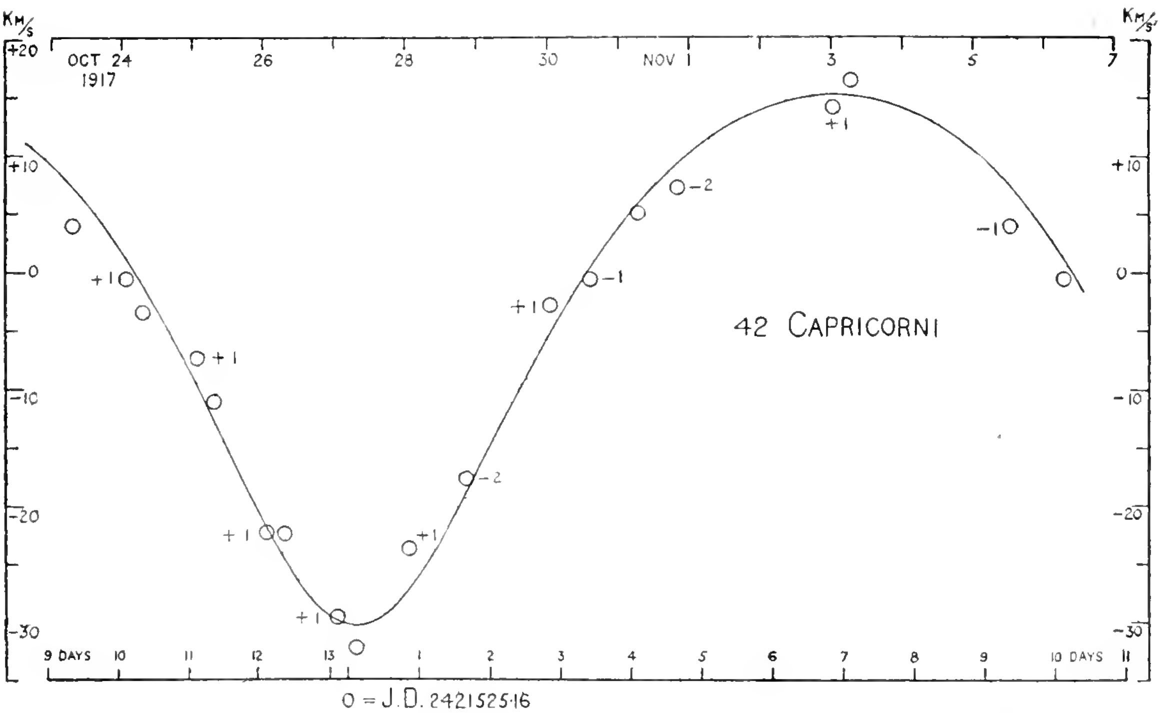

FIG. 1.—Velocity-curve of 42 Capricorni

Further observations next May are contemplated in order to determine the period more accurately, as the present observations extend over only three periods.

Campbell[1] remarks that "it is a striking fact that there are no known binaries of the G, K, and M types (excepting possibly H.R. 142, 13 Ceti) whose periods are less than twenty days." The short period (13.25 days) of this binary is therefore of interest.

TABLE II

Date 1917	G.M.T.	Phase	Observed Velocity	Computed Velocity	O−C
Oct. 27	6^h51^m	0^d1187	−32.2 km	−30.3 km	−1.9 km
Nov. 10	6 59	0.8743	−23.7	−27.2	+3.5
Oct. 2	8 26	1.6847	−17.7	−18.8	+1.1
Nov. 12	6 58	2.8736	− 2.9	− 5.2	+2.3
Oct. 17	8 34	3.4403	− 0.7	+ 0.5	−1.2
Oct. 31	6 52	4.1194	+ 5.0	+ 5.8	−0.8
Oct. 5	8 6	4.6678	+ 7.2	+ 9.3	−2.1
Nov. 16	6 54	6.8708	+14.1	+15.2	−1.1
Nov. 3	6 43	7.1132	+16.4	+15.1	+1.3
Oct. 23	6 54	9.3708	+ 4.0	+ 7.3	−3.3
Nov. 6	7 12	10.1750	− 0.6	+ 1.2	−1.8
Oct. 24	6 47	10.3663	− 3.5	− 1.2	−2.3
Nov. 7	7 16	11.1361	− 7.4	− 9.7	+2.3
Oct. 25	7 3	11.3771	−11.1	−12.8	+1.7
Nov. 8	6 53	12.1201	−22.3	−21.9	−0.4
Oct. 26	6 55	12.3715	−22.4	−24.8	+2.4
Nov. 9	6 40	13.1111	−29.5	−29.9	+0.4

The type could not be determined precisely from the limited region of spectrum photographed, but it appears to agree closely with the solar spectrum in the region examined. Harvard classifies it as of Type K.

Messrs. Woodgate and Baines assisted by exposing ten of the plates, which were taken with the four-prism spectrograph of the 24-inch Victoria telescope, using the short camera. The measures were made by the writer.

Royal Observatory, Cape of Good Hope
November 26, 1917

[1] "Second Catalogue of Spectroscopic Binary Stars," *Lick Observatory Bulletin*, No. 181, **6**, 35, 1910.

Minor Contributions and Notes

ON CHANGES OF THE WAVE-LENGTHS OF CERTAIN LINES IN STELLAR SPECTRA DEPENDING UPON THE TYPE

The object of this research was to examine to what extent stellar lines measured in spectrograms taken with the McClean telescope show relative changes, with type, of the kind already announced by Albrecht.[1]

The spectrograms were measured with Halm's wave-length machine.[2] Of the four stars, α Canis Majoris, α Canis Minoris, α_2 Centauri, and α Boötis, representing the spectral classes A, F, G, and K, respectively 10, 5, 8, and 10 plates have been measured. All the spectrograms contain a comparison spectrum of iron. One setting only was made on both the iron lines and the stellar lines. As far as possible the same iron and stellar lines were measured for each type.

The mean of the measured Fe lines for each of the four stars was formed separately and the four means were then combined into one, weighted according to the number of plates measured for every star. The differences of the observed wave-lengths were then platted in the form of a graph, and from the smoothed curve the corrections were obtained which are to be added to the observed mean Fe lines in order to reduce them to the Rowland system. The mean Fe lines for each star were then reduced to this system. The probable error of a measurement of one Fe line on a single plate was found to be ± 0.015 A.

With regard to the stellar lines, for each of the four stars separately the measured lines were first combined into means, and then the correction of every stellar line to the Rowland system was taken from the curve mentioned above. Finally the wave-lengths of the

[1] *Astrophysical Journal*, **33**, 130, 1911.

[2] *Annals of the Cape Observatory*, **10**, Part I.

λ	A	F	G	K
4191...	......	.77	.72	.79
90...	......	.50	.55	.61
99...	......	.38	.35	.37
4200...	......	......	......	.21
02...	.28	.32	.26	.27
04...	......	.20	.23	.19
06...	......	......	.85	.88
10...	......	.57	.56	.60
15...	.77	.73	.70	.83
16...	......	......	......	.36
19...	......	.56	.53	.59
22...	......	.43	.41	.42
25...	......	......	......	.60
26...	......	.89	.94	27.10
27...	.83	.64	.61	......
35...	......	.16	.38	.40
36...	.15	......	.19	.12
38...	......	.94	39.02	39.04
40...	......	.00	.03	.06
42...	.57	......	.66	......
43...	......	......	.56	.67
45...	......	.46	.48	.48
47...	......	.01	.01	.03
		47.00	46.97	46.96
47...	......	......	.62	.64
48...	......	......	......	.52
50...	......	.32	.33	.36
50...	......	.99	51.00	51.02
52...	......	......	......	.51
54...	......	.54	.56	.55
		.52	.50	.49
58...	......	......	......	.56
60...	......	......	......	.31
60...	.67	.73	.57	.81
	......	.64	.67	.72
68...	......	......	......	.13
		67.86	67.90	67.96
71...	.37	.34	.44	.55
71...	.96	.99	.95	72.07
74...	......	.98	.96	75.01
		.97	.95	.94
88...	......	.17	.20	.26
		.10	.15	.16
89...	......	......	......	.95
91...	......	......	.20	.28
94...	.33	.33	.32	.34
4299...	......	.40	.27	......
4308...	.13	.10	.10	.17
14...	......	......	......	.43
		.34	.36	.36
15...	.27	.34	.24	.21
	......	.18	.16	.15
18...	......	.85	.92	.93
21...	......	.03	.02	20.99
		.01	20.96	20.91
25...	......	.17	.27	.30
		.19	.19	.23
26...	.00	25.99	.02	.06
4333...	......	......	......	.02

λ	A	F	G	K
4335...	......	......	......	.09
		34.99	35.06	35.10
37...	......	......	.29	.29
40...	.71	.78	.73	.68
	......	.66	.66	.65
44...	......	......	......	.64
		.49	.59	.62
52...	.04	.06	.03	......
	......	.02	.01	.01
52...	......	......	.96	53.02
		.94	.96	.98
58...	......	.76	.84	.86
59...	......	.84	.80	.83
67...	......	.91	.84	......
69...	......	......	.89	.96
71...	......	......	.36	.41
76...	......	......	.12	.18
79...	......	......	......	.41
83...	.76	.76	.78	.82
84...	......	......	......	.94
95...	.24	.25	.30	.32
	......	.24	.26	.27
4399...	.98	.90	.84	......
	......	.95	.92	.89
4401...	......	......	.70	.69
04...	.97	.97	05.00	.97
07...	......	......	.84	.90
08...	......	......	.65	.61
15...	.27	.31	.33	.40
25...	......	.62	.66	......
		.59	.64	.88
27...	......	.47	.46	.48
35...	......	.17	.22	.25
		.16	.19	.24
42...	......	......	......	.59
47...	......	......	.94	.95
55...	......	.03	.99	.99
59...	......	.32	.29	.33
64...	......	.71	.77	.82
		.73	......	.90
66...	......	.75	.76	.80
68...	.71	.68	.68	.73
	......	.65	.67	.70
69...	......	.53	.56	.63
		.49	.56	.62
72...	......	.06	71.98	.03
76...	......	.26	.24	.29
81...	.45	.43	.38	......
82...	......	.38	.37	.41
4494...	......	.76	.72	.75
4501...	.51	.47	.39	.46
15...	.63	.50	.49	......
22...	.84	.84	.82	.89
28...	......	.81	.81	......
33...	......	.22	.29	.29
34...	.25	......	.33	......
36...	......	......	.02	.06
49...	.67	.83	.77	.84

stellar lines thus obtained were corrected for the radial velocity and for the earth's motion.

The table on page 138 shows the final normal wave-lengths of the stellar lines for each of the four types and in addition the wave-lengths derived by Albrecht for those lines of similar types which are common to the two investigations. Where the values are bracketed the lower ones are Albrecht's.

The agreement with Albrecht's wave-lengths is in general fairly good, the more pronounced progressions in Albrecht's values, from one type to another, being generally confirmed by the present observations.

J. Voûte

Royal Observatory, Cape of Good Hope
December 12, 1917

THE OBSERVATORY OF POULKOVA

Disquieting rumors regarding the safety of the observatory at Poulkova reached this country in brief press dispatches during the early winter. We are now happy to be able to state[1] that the dangers due to the civil strife had thus far been passed without serious damage to that famous institution. It is, however (as it should be!), very unusual for an astronomical observatory to be under artillery fire, and it can hardly be an indiscretion to give our readers the following particulars:

Rumors had reached the astronomical colony at Poulkova, which lies about 20 km southwest of Petrograd, that bodies of Cossacks were coming toward the capital to restore the ministry of M. Kerensky. The garrison from the village of Zarskoje Selo (site of the summer residence of the former emperor) presently arrived at Poulkova, having been driven out by the Cossacks. On November 11 soldiers from the garrison of Petrograd, with the Red Guard and with artillery, arrived for the relief of that body of troops.

With a zeal which we may imagine must have seemed to the astronomers in excess of its wisdom, those forces from the capital

[1] As of November 23, 1917.

surrounded the observatory on all sides and established batteries of artillery within 400 or 500 meters on both sides of the main building.

Between one o'clock and five-thirty on the afternoon of November 12 the observatory was under an intense artillery fire from the Cossacks. Fortunately, none of the instruments were damaged, although a shell burst beside the brick foundation of the dome of the large astrographic telescope. Many holes were made in the dome of the great refractor and in the roof of the director's office. The wall of the seismological station was pierced, and we may believe that the instruments registered their largest earthquake. Of course there was much damage to windows. The ground was torn up by numerous holes of a diameter of a meter. The Cossacks retired on the following day.

The director of the observatory had been able in the preceding days to foresee the dangers of the situation, and had removed the 30-inch objective, as well as the objectives of the other instruments. By a singular good fortune no one of the personnel of the observatory or colony was injured during these exciting events.

It was felt that there was no guaranty that such events might not recur under the present conditions in Russia. Men of science everywhere will certainly wish to join with us in our satisfaction that no more serious damage was done and in offering our congratulations on this escape to Director Belopolsky and his staff, with the hope that the admirable work carried on at the observatory during the last eighty years may continue unimpaired, despite the political and economic changes through which Russia is passing.

F.

March 1918

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THE EFFECT OF SIZE OF STIMULUS AND EXPOSURE-TIME ON RETINAL THRESHOLD[1]

By PRENTICE REEVES

In determining the least amount of radiant energy which would excite the retina sufficiently to give rise to a perception of light the writer used a direct method in this laboratory. It was found that for three observers the minimum radiation visually perceptible was 19.5×10^{-10} ergs per second, and for the writer alone 17.1×10^{-10} ergs per second.[2] This value represents the mean of several observations, and seems to agree with values found by other writers.[3] The writer used a "star" 1 mm in diameter viewed at a distance of 3 m, so that the stimulus may be considered as a distant point-source of light, and the given results must be understood to apply only to the radiation from such a source. In order to get further data on the subject the writer determined the absolute threshold for stimuli ranging from a 2-millimeter square to a 12-centimeter square viewed from a distance of 35 cm. Observations were also taken in the former experiment of the

[1] *Communication No. 68* from the Research Laboratory of the Eastman Kodak Company.

[2] *Astrophysical Journal*, **46**, 167, 1917.

[3] *Ibid.*, **44**, 124, 1916; **45**, 60, 1916; **46**, 296, 1917.

1-mm star at 1.5 m and 35 cm. The writer took several observations on each stimulus over a period of about a month, so that the results are directly comparable with his previous results.

The visual sensitometer was used to control the size and intensity of the stimulus. For stimuli from 2 mm to 3 cm square the apparatus was used as described by the writer, and for the larger stimuli a flood-lamp illuminated the white matte field in which the opal-glass window is set. The size of this field could be varied and the intensity of its illumination controlled by using neutral filters, whose densities were known, with the flood-lamp. Readings were taken on the 3-centimeter stimulus in both ways and were found to agree. In the sensitometer a Nernst glower was used, and in the flood-lamp a tungsten incandescent lamp.

In the experiment the observer sat facing the apparatus at the fixed distance of 35 cm. Before making any observations he first remained in total darkness from 15 to 30 minutes to counteract any previous conditions of the retina, as otherwise the results from the observer when he had been doing dark-room work would differ from the results obtained if he had previously been working in a well-lighted room. When the eye is fully adapted to darkness it can detect the minimum amount of radiation, and its sensibility is the highest when in that state. The threshold for dark adaptation gives the absolute photo-chemical reactivity of the retina. When the eye was fully dark-adapted, the observer then determined the least intensity at which the stimulus could be seen, and this procedure was repeated for all stimuli from the 2-mm to the 12-cm squares. Each result on each day was the average of several independent observations. Several series were also run for different-sized rectangular stimuli, and the averages secured. These results are shown in Table I.

When a point-source of light is used the radiant energy is concentrated on the retina and the portion of the retina stimulated is quite small. The diameter of the writer's pupil when fully dilated was previously determined to be 8.3 mm, and with a point-source at a definite distance and known intensity it is a simple matter to compute the least perceptible radiation. But when larger light-sources are used the computation becomes more com-

plex, and at present we must be satisfied with comparative results. In the experiment with the point-source the writer used the expression

$$\text{Least perceptible radiation} = SLM\frac{r^2}{R^2} \text{ ergs per sec.,}$$

where S is the area of the stimulus in square cm; L, the brightness of stimulus in lamberts; M, the mechanical equivalent of light (Ives gives this as 1.59 ergs per second per sq. cm); r, the radius

TABLE I

VARIATION OF ABSOLUTE THRESHOLD OF THE RETINA WITH SIZE AND SHAPE OF STIMULUS

Stimulus	Log. Threshold	Threshold	Energy Entering Eye
		Ml	Ergs per sec.
1-mm star at 3 m	−2.14267	0.00720	17.1×10^{-10}
" " " 1.5 m	−2.58503	.00260	24.8×10^{-10}
" " " 35 cm	−3.61979	.00024	42.1×10^{-10}
2-mm square at 35 cm	−4.54837	0.000028290	25.3×10^{-10}
5- " " "	−5.17881	6625	37.
1-cm " " "	−5.61744	2413	54.
2- " " "	−5.99012	1023	91.
3- " " "	−6.34659	450	90.7
6- " " "	−6.58872	258	208.
12- " " "	−6.75647	175	564.
Horizontal Divisions:			
$3\times\frac{1}{2}$-cm rectangle at 35 cm	−5.65694	0.00000220	73.9×10^{-10}
3×1- " " "	−5.82652	149	100.
$3\times1\frac{1}{2}$-" " " "	−5.91937	120	121.
3×2- " " " "	−6.10210	79	106.
$3\times2\frac{1}{2}$-" " " "	−6.21667	61	102.
3×3- " " " "	−6.33283	46	92.7
Vertical Divisions:			
$\frac{1}{2}\times3$	−5.57873	0.00000264	88.7×10^{-10}
1×3	−5.83239	147	98.6
$1\frac{1}{2}\times3$	−5.97840	105	106.
2×3	−5.26384	54	72.5
$2\frac{1}{2}\times3$	−5.34921	45	75.5
3×3	−5.39924	40	80.6

of the pupil, and R, the distance of the eye from the stimulus. The stimuli used with the visual sensitometer are uniformly diffusing or reflecting and may be considered as made up of numerous point-sources. In using the foregoing expression for the larger

stimuli we are able to get relative results, and while not theoretically accurate they will serve the purpose of this experiment. From the results it can be seen that as the visual angle increases the threshold decreases, but that the amount of radiant energy increases. The values of energy for the largest stimuli are probably too large, owing to unavoidable difficulties of manipulation, but at least they show the tendencies of the larger visual angles.

In the second part of this investigation the writer attempted to determine how long a time was required for the point-source of light to produce a perceptible sensation. The observer sat at a distance of 3 m with his head in a headrest and his eye fixated on the stimulus. The intensity was regulated until the observer could just see the star, and an assistant then covered the star. The observer kept his fixation; the assistant uncovered the star and recorded with a stop watch the time elapsing before the observer saw the star. At first the assistant did not notify the observer when the star was uncovered, and the watch was operated silently. Next the observer was notified, and it was found that unless the interval before silent exposure was quite long, telling the observer did not affect the results, as the observer's attention was constantly fixed on the stimulus. For the just perceptible intensity of the stimulus the perception-time recorded ranged from 1 to as much as 6 seconds, although the result of several observations gave an average of 2.2 seconds for the writer. Two other observers averaged 2.1 seconds each, although the intensity of the star in each threshold was higher than for the writer. When the intensity of the star was made just perceptibly brighter, the time for detection was lowered to 1.4 seconds for the writer and 1.2 and 0.85 seconds for the other observers. In several hundred observations on the visual sensitometer in various experiments the writer has noticed that practically all stimuli that can be perceived at all will be detected within two or three seconds. It has been found that if a stimulus could not be detected after 4 seconds' exposure it was not worth while to use longer periods of time, since the numerous factors that enter, such as attention, fatigue, eye movements, etc., make results unreliable.

As the results of this determination of perception-time were rather unsatisfactory, the writer tried another method. Carefully calibrated photographic shutters were placed before the test-spot of the visual sensitometer, and the writer determined the brightness of the test-spot necessary for it to be perceived when exposed for a given time. The test-spot was taken as a 3-centimeter square, and the observations were taken at a distance of 35 cm for a range of exposures from 0.002 seconds to 4 seconds. These results are shown in Table II. Observations made for exposures of 8, 12, and

TABLE II

VARIATION OF ABSOLUTE THRESHOLD OF THE RETINA WITH TIME OF EXPOSURE OF STIMULUS

Time	Log. Threshold	Threshold	1/T
Sec.		Ml	
0.002........	−3.44092	0.00036230	2.76
.006........	−4.01092	9752	10.3
.011........	−4.35	4458	22.4
.020........	−4.62	2394	41.7
.034........	−4.91	1228	81.5
.160........	−5.15	706	142.
.250........	−5.29	512	195.
.500........	−5.45	354	282.
1.000........	−5.58	262	382.
2.000........	−6.11	77	1300.
4.000........	−6.20	63	1588.

20 seconds almost invariably show a decrease of retinal sensibility and were not included in this paper. The effect of exposure on sensibility is shown in the column headed 1/T, as the reciprocal of the threshold is proportional to sensibility. It is seen that the sensibility increases quite rapidly up to 2 seconds.

The writer would like to see other experiments on this subject and further work on the effect of exposure-time on smaller stimuli. He wishes to acknowledge the assistance of Julian Blanchard and Milton Fillius in this experiment.

RESEARCH LABORATORY, EASTMAN KODAK COMPANY
January 30, 1918

ON PARALLAXES AND MOTION OF THE BRIGHTER GALACTIC HELIUM STARS BETWEEN GALACTIC LONGITUDES 150° AND 216°[1]—*Continued*

By J. C. KAPTEYN[2]

13. RANGE IN DISTANCE OR PARALLAX AS A FUNCTION OF λ AND v

The principle of the method followed is in the main that given by Eddington,[3] but the divergence from his treatment is sufficiently important to make a full explanation necessary. The parallaxes are derived from a comparison of the observed distribution of the v (given in Section 10) with the theoretical distribution obtained from the known distribution of u given in the preceding section.

We have (see *Mount Wilson Contribution* No. 82, p. 35)

$$v=0.212\,\pi(V\sin\lambda+u), \tag{50}$$

v and π being expressed in seconds of arc, V and u in km per second. Now, according to the preceding section, u is distributed about the central value $u=0$ in a normal error-curve having a probable error r_u. Therefore, if we consider only stars for which π and λ are practically constant, v will be distributed according to a normal curve about the central value

$$v=0.212\,\pi V\sin\lambda \tag{51}$$

with a probable error

$$r_v=\pm 0.212\,\pi r_u. \tag{52}$$

This, however, supposes that we disregard the observational errors. In the present case, where the proper motions are so very small, this is by no means permissible. But as the observational errors presumably are also distributed in a normal curve, the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 147.

[2] Research Associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

[3] *Stellar Movements and the Structure of the Universe*, p. 218.

observed values of v will have a similar distribution about the central value (51), only, instead of (52) we shall have to use

$$r_v = \pm\sqrt{(0.212\,\pi r_u)^2 + r^2} \tag{53}$$

where r is the probable observational error of a component of the proper motion in Boss's catalogue. I have used

$$r = 0''.0056 \tag{54}$$

which is the mean of the probable errors of the motions in α and δ, as given for all our stars in Boss's catalogue. Hence, finally,

$$r_v = \pm\sqrt{(0.212\,\pi r_u)^2 + 0.0056^2} \tag{55}$$

The computations were first carried through with the first value of (49), namely,

$$r_u = \pm 2.5. \tag{56}$$

Table XVI gives the theoretical distribution for v derived in accordance with (51) and (55) for $\lambda = 150°$, $V = 20.0$ km,[1] on the supposition that the constant parallax has, successively, the values 0''.0015,

TABLE XVI

DISTRIBUTION OF v ($\lambda = 150°$, $r_u = \pm 2.5$)

v	π							
	0''.0015	0''.0045	0''.0075	0''.0105	0''.0135	0''.0165	0''.0195	0''.0225
	0.000	0.000	0.000	0.003	0.025	0.097	0.216	0.355
+0''.0550								
	.000	.000	.000	.006	.031	.069	.092	.098
.0500								
	.000	.000	.002	.017	.056	.092	.108	.102
.0450								
	.000	.000	.007	.039	.087	.114	.114	.099
.0400								
	.000	.002	.021	.074	.118	.128	.111	.088
.0350								
	.001	.010	.053	.117	.142	.128	.101	.075
.0300								
	.004	.031	.102	.152	.146	.114	.084	.061
.0250								
	.017	.079	.159	.170	.134	.092	.064	.044
.0200								
	.058	.150	.191	.155	.105	.069	.045	.031
.0150								
	.129	.206	.183	.120	.072	.045	.029	.020
.0100								
	.206	.213	.140	.077	.044	.027	.017	.013
+0.0050								
	.234	.163	.083	.041	.023	.014	.010	.007
0.0000								
	.186	.092	.039	.019	.011	.006	.005	.004
−0.0050								
	.107	.039	.015	.007	.004	.004	.003	.002
−0.0100								
	0.058	0.015	0.005	0.003	0.002	0.001	0.001	0.001

[1] In accordance with the definitive elements (33).

0."0045, 0."0075, 0."0105, ... 0."0225.[1] As a matter of fact, the computation was made for all the following values of π: 0."0005, 0."0015, 0."0025, ... 0."0235, and the totals in Table XVIII are the values obtained by this more elaborate computation. Indeed it is only in order to avoid typographical difficulties that the abridgment has been made. The abridged calculation is hardly less accurate, however, as will be seen if, in Table XVIII, the totals are formed for each line. Multiplied by 3, these differ but little from the totals obtained by the more elaborate computation.

Fig. 1 gives a comparison of the observed (see Table XI for $\lambda=150$) and theoretical distribution of v for $\pi=0."0077$, which,

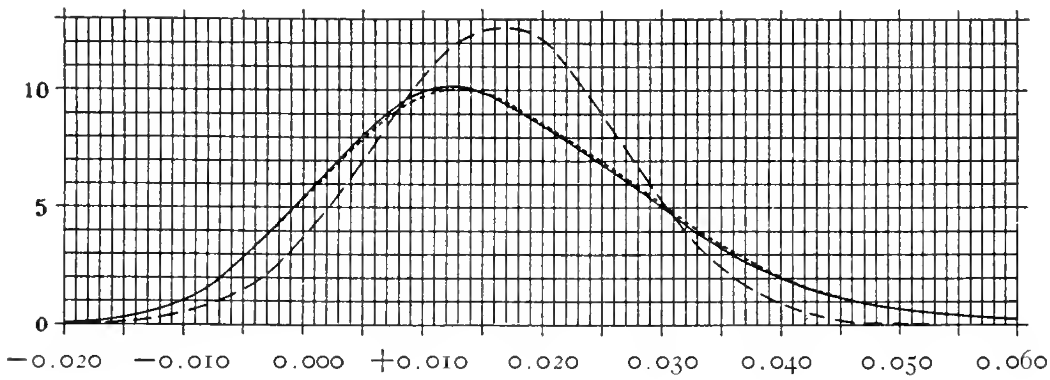


FIG. 1.—Distribution of the values of v for $\lambda=150°$

———— Observation
— — — — Theory, constant parallax, $\pi=0."0081$
· · · · Definitive theory, interpolation from Tables XI and XXII

according to Table IX, is the mean parallax of the stars at $\lambda=150°$. It shows clearly that the observations are not represented at all. We conclude that the assumption of a constant π for all the stars must be abandoned. The deficiency of the theoretical curve in large values of v can evidently be remedied only by the addition of stars having larger parallaxes, while the deficiency in small values of v calls for stars of smaller parallax. Let us suppose, therefore, that there are (in fractions of the whole) n_5 stars of

[1] Stated as a formula, the tabulated quantity is

$$\text{Prob. } v_1 \text{ to } v_2 = \frac{1}{\sqrt{3.141}} \int_{H(v_1-0.212\,\pi V \sin\lambda)}^{H(v_2-0.212\,\pi V \sin\lambda)} e^{-z^2} dz$$

where $H=0.4769/r_v$ and r_v has the value given by (55).

parallax 0."0005; n_{15} stars of parallax 0."0015, etc. We must find n for different values of the parallax, that is, we must determine $n = \phi(\pi)$ in such a way that the mixture of all these stars gives a distribution for v agreeing with the observed distribution.[1]

Thus we might imagine each column of Table XVI to be multiplied by an indeterminate factor and then find the numerical values of these factors by equating the sums of the several horizontal lines to the corresponding numbers of the observed distribution. For instance, the sixth line of Table XVI, compared with the observed number from Table XI, would give:

$$0.001\, n_{15} + 0.010\, n_{45} + 0.053\, n_{75} + \ldots\ldots + 0.075\, n_{225} = \frac{4.0}{66.0} = 0.061\,.$$

We could thus determine all the constants, provided they were not more numerous than the equations of condition; but the solution would be neither complete—on account of the restricted number of constants—nor satisfactory. There are conditions which certainly, or at least very probably, must be satisfied if the solution is to be acceptable. As such we may specify:

$$\left.\begin{array}{l} \phi(0) = 0 \qquad \phi(\infty) = 0 \\ \phi(\pi) \text{ shall have but one maximum and vary continuously;} \\ \quad \pi = \text{known value}^{2} \end{array}\right\} \qquad \text{(B)}$$

I therefore proceeded somewhat differently. Starting from a first approximation[3] for $\phi(\pi)$, which satisfies the conditions (B), I computed the theoretical distribution. The divergences of this from

[1] Put in the form of an integral equation: if $\psi(v)\delta v$ represent the fraction of the stars having their v between v and $v + \delta v$, we have

$$\psi(v) = \frac{1}{\sqrt{3.1416}} \int_0^\infty H\phi(\pi) e^{-H^2(v - 0.212\,\pi\, V \sin\lambda)^2} \delta\pi$$

where

$$H = \frac{0.4769\ldots}{\sqrt{(0.212\,\pi r_u)^2 + r^2}}\,. \qquad \text{In this equation: given } \psi(v), \text{ required } \phi(\pi).$$

[2] This condition might have been omitted. The method will itself furnish the value of π. I have preferred, however, to use the values already found.

[3] It is not of much importance what form we choose as a first approximation for $\phi(\pi)$. Still it will shorten matters not a little if it is not too far in error, and it is not difficult to find a fairly plausible beginning. Let $\psi(M)\delta M$ be the fraction of all the stars having absolute magnitude between M and $M + \delta M$; $D(\pi)$ = star density =

the observed distribution were then diminished by a local correction of $\phi(\pi)$, in which I took pains to keep (B) always satisfied. The process requires some care; but I was not long in obtaining a solution,[1] which, if it is not the best that can be obtained, meets the requirements in a very satisfactory manner. The solutions for $\lambda=154^\circ$ and $\lambda=157^\circ.5$ were carried through in exactly the same manner. The results for $\phi(\pi)$ are in Table XVII. The computation of the theoretical distribution for v with the help of these values $\phi(\pi)$ is shown for $\lambda=150^\circ$ in Table XVIII. The second column is obtained by multiplying the second column of Table XVI by 0.027, i.e. (see Table XVII), by the value of $\phi(\pi)$ for $\pi=0''.0015$, and similarly for the other columns. These values of $\phi(\pi)$ are entered in the last line of the heading of Table XVIII. The theoretical distribution of v is given in the column headed Σ, which is the sum of all the numbers (for all the parallaxes $0''.0005$, $0''.0015$, $0''.0025$ . . . .) in the several horizontal lines. The last two

total number of stars per unit of volume at parallax π; m = apparent magnitude. Then it is easily found that

$$\phi(\pi)=\frac{A}{\pi^4}D(\pi)\psi(m+5+5\log\pi). \qquad (a)$$

It seems probable that we shall obtain a fair approximation if we assume the star-density to be constant, and if for $\psi(M)$ we use

$$\psi(M)=\frac{h}{\sqrt{\pi}}e^{-h^2(M-K)^2}$$

which was found for all the stars (see *Groningen Publication*, No. 11), and if finally for m we adopt the mean apparent magnitude of the Boss stars. We thus find

$$\phi(\pi)=\frac{B}{\pi^4}e^{-\lambda^2(G+5\log\pi)^2} \qquad (b)$$

in which there are two relations between the three constants expressing

$$\int_0^\infty\phi(\pi)\delta\pi=1; \qquad \int_0^\infty\pi\phi(\pi)\delta\pi=\pi=\text{known quantity.}$$

The third constant is easily obtained from the observed distribution for v by successive approximations. For stars of every spectrum considered together, *Groningen Publications*, No. 11, p. 11, gives in each column of Table II the $\phi(\pi)$ for a determined value of the apparent magnitude m. I find that these are in fact represented by formula (b) with surprising accuracy.

[1] It turns out that rather different solutions for π represent the distribution of v with almost equal precision; in other words, the solution for $\phi(\pi)$ is necessarily inaccurate. This does not mean that the determination of π as a function of v and λ, although based on $\phi(\pi)$, is not fairly reliable. In fact it is little affected by small changes in $\phi(\pi)$.

columns will be explained presently. Repeating the computation for $\lambda=154°$ and $\lambda=157°.5$, and multiplying the frequencies into the totals of stars observed, we have, for comparison with the observed numbers, the theoretical values in Table XI. The agreement, I think, may be considered satisfactory. For $\lambda=150°$ the theoretical distribution is shown by the dotted line in Fig. 1.

TABLE XVII

$\phi(\pi)$ FOR $r_u = \pm 2.5$

π	λ 150°	154°	157°.5	Mean
0″.0005	0.015	0.011	0.052	0.026
.0015	.027	.026	.087	.047
.0025	.044	.043	.087	.058
.0035	.066	.062	.079	.069
.0045	.084	.075	.071	.077
.0055	.093	.084	.063	.080
.0065	.095	.091	.056	.081
.0075	.096	.095	.053	.081
.0085	.093	.094	.050	.079
.0095	.087	.090	.047	.075
.0105	.078	.082	.044	.068
.0115	.066	.072	.042	.060
.0125	.054	.059	.040	.051
.0135	.041	.045	.037	.041
.0145	.028	.032	.033	.031
.0155	.019	.020	.031	.023
.0165	.010	.010	.028	.016
.0175	.003	.005	.024	.011
.0185	0.001	.003	.021	.008
.0195	....	0.001	.018	.006
.0205	....	....	.015	.005
.0215	....	....	.011	.004
.0225	....	....	.008	.003
0.0235	....	....	0.003	0.001
π	0″.0079	0″.0083	0″.0083	....

The values of $\phi(\pi)$ having thus been found, the derivation of the average parallax of all the stars having the same v and λ offers no difficulty.[1] Take for instance $\lambda=150°$. From Table XVIII we

[1] We have:

$$\pi_{v,\,\lambda}=\frac{\int_0^\infty H\pi\phi(\pi)e^{-H^2(v-0.212\,\pi V\sin\lambda)^2}\delta\pi}{\int_0^\infty H\ \phi(\pi)e^{-H^2(v-0.212\,\pi V\sin\lambda)^2}\delta\pi}$$

where $$H=\frac{0.4769\ldots}{\sqrt{(0.212\,\pi r_u)^2+r^2}}\,,\quad r=0''.0056\,.$$

find for the interval $v = 0''.0350$ to $0''.0300$, that is, practically for $v = 0''.0325$,

$$\pi = \frac{0''.0045 \times 0.0008 + 0''.0075 \times 0.0051 + \ldots + 0''.0165 \times 0.0013}{0.0008 + \qquad 0.0051 + \ldots \qquad + 0.0013} = 0''.0116.$$

The more complete computation, carried through with three times the number of parallaxes, furnishes the more accurate value $\pi = 0''.0107$ entered in the last column of Table XVIII. All other cases were treated in the same way.

TABLE XVIII

THEORETICAL DISTRIBUTION OF v TOGETHER WITH VALUES OF π
(All Numbers in Units of the Fourth Decimal)

v	π AND $\phi(\pi)$							Σ	$\Sigma(\pi \div \text{tab.})$	π
	15	45	75	105	135	165	195			
	270	840	960	780	410	100	0			
	0	0	0	2	10	10	0	65	0.05	146
+550..										
	0	0	0	5	13	7	0	74	1.01	136
500..										
	0	0	2	13	23	9	0	140	1.81	129
450..										
	0	0	7	30	36	11	0	250	3.08	123
400..										
	0	2	20	58	48	13	0	420	4.85	115
350..										
	0	8	51	91	58	13	0	659	7.05	107
300..										
	1	26	98	119	60	11	0	943	9.27	98
250..										
	5	66	153	133	55	9	0	1,253	11.14	89
200..										
	16	126	183	121	43	7	0	1,484	11.79	79
150..										
	35	173	176	94	30	5	0	1,523	10.76	71
100..										
	56	179	134	60	18	3	0	1,335	8.36	63
+ 50..										
	63	137	80	32	9	1	0	953	5.27	55
0..										
	50	77	37	15	5	1	0	548	2.82	51
− 50..										
	29	33	14	5	2	0	0	244	1.11	46
−100..										
	16	13	5	2	1	0	0	110	0.46	42
Totals	271	840	960	780	411	100	0	10,001	79.73	79

Table XIX gives the complete results for $\lambda = 150^\circ$, $\lambda = 154^\circ$, and $\lambda = 157^\circ.5$. For convenience I have added the values for $v = -0''.0125$, as found by extrapolation.

The argument of this table is the *observed* value of v, which of course is the argument needed for the computation of the parallaxes. For certain purposes, however, another table is desirable with the *true* value of v (freed of observational errors) as argument. This for instance would be useful for stars whose proper motions

are known with extreme precision. To calculate it we have only to use formula (52) instead of (53).[1] The results are given in Table XX.

TABLE XIX

$\pi = F(v, \lambda)$ $(r_u = \pm 2.5)$ OBSERVED VALUES OF v

v obs.	λ		
	150°	154°	157°.5
+0″.0525	0″.0135	0″.0142	0″.0180
.0475	.0129	.0134	.0170
.0425	.0122	.0127	.0160
.0375	.0115	.0119	.0150
.0325	.0107	.0112	.0139
.0275	.0098	.0103	.0124
.0225	.0089	.0094	.0107
.0175	.0079	.0085	.0090
.0125	.0071	.0076	.0073
.0075	.0063	.0068	.0061
+0.0025	.0055	.0062	.0052
−0.0025	.0051	.0057	.0045
0.0075	.0046	.0053	.0042
−0.0125	0.0042	0.0050	0.0042

TABLE XX

$\pi = F(v, \lambda)$ $(r = \pm 2.5)$ TRUE VALUES OF v

True v	λ		
	150°	154°	157°.5
+0″.070	0″.0171	0″.0187	0″.0199
.060	.0159	.0175	.0195
.050	.0146	.0161	.0187
.040	.0132	.0142	.0173
.030	.0115	.0121	.0151
.020	.0092	.0099	.0119
.010	.0061	.0069	.0076
+0.005	.0043	.0058	.0046
0.000	.0052	.0055	.0034
−0.005	.0104	.0083	.0125
−0.010	0.0136	.00124	0.0154

The divergences of Table XX from Table XIX are in a direction easy to foresee. The last shows the curious fact that, as v decreases beyond a certain value, π begins to increase, and soon

[1] The substitution in the denominator of H of the value (52) for that of (53) is the only change in the formulae in the footnotes.

increases at a rapid rate. This, too, is in accordance with what might have been expected, as is perhaps best seen by comparing the mean parallax of stars (all at the same distance λ from the vertex) having $v = 0''.000$ with that of stars having a definite negative value of v. For stars at all distances, the probability of the value $v = 0''.000$ is the same, for it is simply the probability that the linear velocity component u is equal, but opposite in sign, to the stream-velocity component $V \sin \lambda$. In order that v shall be negative, the linear velocity component u must be greater than (and opposite to) the stream component $V \sin \lambda$. In addition, the more distant stars, in order to show the same negative value of v, must have the greater negative value of u. A greater negative value of u, however, means a less probable one. Consequently, whereas near and distant stars contribute equally in the production of $v = 0''.000$, the nearer stars predominate in the formation of a negative value of v. Therefore, the mean parallax will be greater in the latter case.[1] This assumes that there are no observational errors. That Table XIX, in which these errors are taken into account, must show quite another behavior is clear. Indeed it is easily seen that, without such errors, practically no negative values of v would occur, at least for stars at some distance from the vertices.

We now return to the parallaxes obtained by taking the observational errors into account. In Section 17 (*a*) evidence is given which indicates that it would have been preferable to have adopted the *same* values of $\phi(\pi)$ for all values of λ. I accordingly repeated the computation of the theoretical distribution of v and of the parallaxes, using in each case the mean values of $\phi(\pi)$ in the last column of Table XVII. These are direct means, notwithstanding the fact that the values for $\lambda = 154°$ depend partly on stars also

[1] In order to show more in detail the way in which π varies with v, I computed the following table on the supposition that half the stars have $\pi = 0''.0000$, the other half $\pi = 0''.0100$, and further that $V = 20.0$ km, $\lambda = 150°$, $r_u = \pm 2.5$ km.

v	$\bar{\pi}$	v	$\bar{\pi}$
$+0''.030$	$0''.0100$	$+0.005$	0.0073
0.025	0.0099	0.000	0.0080
0.020	0.0091	-0.005	0.0092
0.015	0.0078	-0.010	0.0099
$+0.010$	0.0072		

used for $\lambda=150^\circ$ and $\lambda=157^\circ.5$. The weight of the determination for $\lambda=150^\circ$ is undoubtedly considerably greater than that for $\lambda=157^\circ.5$. As the values for $\lambda=154^\circ$ are practically identical with those for $\lambda=150^\circ$, the manner of forming the means is sensibly equivalent to giving double weight to $\lambda=150^\circ$, which cannot be far wrong. Since the differences between the observed and the theoretical distribution were increased in every case by this procedure, the indication alluded to is not confirmed by the observed distribution of v. The safest course seems to be to adopt the arithmetic means of the parallaxes obtained on the two hypotheses, (*a*), $\phi(\pi)$ different for λ different; (*b*), $\phi(\pi)$ constant for all λ.

Having thus obtained the definitive parallaxes (for $r_u=\pm 2.5$) for $\lambda=150^\circ$, 154°, and $157^\circ.5$, we consider less frequently occurring values of λ. For $\lambda=140^\circ$, 145°, 165°, 180° I adopted the mean $\phi(\pi)$ in Table XVII and derived Table XXI, from which the individual parallaxes for $r_u=\pm 2.5$ were obtained by simple interpolation.

TABLE XXI

$\pi_{2.5}=\phi(v, \lambda)$ ADOPTED, FOR $r_u=\pm 2.5$ KM

v	λ							
	140°	145°	150°	154°	157°.5	160°	165°	180°
+0″.0525	0″.0143	0″.0144	0″.0145	0″.0151	0″.0172	0″.0171	0″.0170	....
.0475	.0133	.0135	.0137	.0142	.0162	.0161	.0160	....
.0425	.0124	.0126	.0128	.0133	.0151	.0151	.0150	....
.0375	.0114	.0116	.0119	.0123	.0141	.0140	.0139	....
.0325	.0104	.0107	.0110	.0115	.0130	.0129	.0127	0″.0139
.0275	.0093	.0097	.0100	.0105	.0117	.0116	.0115	.0123
.0225	.0082	.0086	.0090	.0094	.0103	.0103	.0103	.0108
.0175	.0071	.0075	.0079	.0085	.0089	.0090	.0092	.0094
.0125	.0060	.0065	.0070	.0075	.0074	.0077	0083	.0084
.0075	.0051	.0056	.0061	.0066	.0064	.0068	.0075	.0077
+0.0025	.0043	.0048	.0053	.0060	.0056	.0060	.0069	.0074
−0.0025	0.0035	.0042	.0048	.0054	.0050	.0055	.0065	.0074
0.0075	....	0.0038	.0043	.0049	.0047	.0053	.0065	.0077
0.0125	....	....	0.0038	0.0046	0.0046	0.0052	0.0065	.0084
−0.0175	....	....	....	....	....	....	....	0.0094

14. CASE $r_u=0.0$

According to Section 12, Remark 2, the calculations have now to be repeated for the value $r_u=0.0$. Tables XXII, XXIII, and XXIV, corresponding to Tables XI, XVII, and XXI, unchanged

but for the value of r_u, embody the results. Comparison of Tables XI and XXII shows that the observed distribution of v is about equally well represented in the two cases. The values of $\phi(\pi)$ in Tables XVII and XXIII show large differences. The remarks in the footnotes of Section 13 also hold here.

TABLE XXII*

DISTRIBUTION OF v (THEORETICAL NUMBERS ARE FOR r_u=0.0)

v	$\lambda=150°$			$\lambda=154°$			$\lambda=157°.5$		
	Obs.	Theor.	O−C	Obs.	Theor.	O−C	Obs.	Theor.	O−C
	0.5	0.3	+0.2	0.3	0.2	+0.1	1.1	0.1	+1.0
+0″.055									
	0.5	0.5	0.0	0.5	0.3	+0.2	0.7	0.5	+0.2
.050									
	0.9	0.9	0.0	0.8	0.5	+0.3	1.1	0.8	+0.3
.045									
	1.6	1.8	−0.2	1.6	1.5	+0.1	1.6	1.9	−0.3
.040									
	2.6	2.9	−0.3	2.7	2.7	0.0	2.5	3.0	−0.5
.035									
	4.0	4.6	−0.6	4.4	4.4	0.0	3.6	4.4	−0.8
.030									
	6.1	6.4	−0.3	6.5	6.4	+0.1	5.1	5.6	−0.5
.025									
	7.9	8.1	−0.2	8.4	8.4	0.0	6.8	6.9	−0.1
.020									
	9.2	9.4	−0.2	9.9	10.1	−0.2	8.6	8.4	+0.2
.015									
	10.2	9.6	+0.6	10.2	10.5	−0.3	10.0	10.0	0.0
.010									
	9.2	8.6	+0.6	9.0	9.4	−0.4	10.5	10.9	−0.4
+0.005									
	6.7	6.4	+0.3	6.7	7.0	−0.3	9.7	10.0	−0.3
0.000									
	4.1	3.8	+0.3	4.2	4.2	0.0	7.3	7.2	+0.1
−0.005									
	1.8	1.8	0.0	2.1	1.9	+0.2	4.5	3.8	+0.7
0.010									
	0.6	0.8	−0.1	0.8	0.9	+0.2	1.9	1.7	+0.2
0.015									
	0.1			0.3			0.5	0.4	+0.1
−0.020									
Totals	66.0	66.0	......	68.4	68.4	......	75.5	75.6	......

* Same as Table XI except that r_u=0.0 instead of ±2.5.

The parallaxes obtained by interpolation from Table XXI ($r_u=\pm 2.5$) appear in Table XXXIX under the head $\pi_{2.5}$; those

from Table XXIV ($r_u=0.0$), under the head $\pi_{0.0}$. Finally the adopted π, corresponding to the adopted value $r_u=\pm 1.67$ (47), was obtained from

$$\pi=\frac{2\,\pi_{2.5}+\pi_{0.0}}{3}. \tag{57}$$

TABLE XXIII*

$\phi(\pi)$ FOR $r_u=0.0$

π	λ 150°	λ 154°	λ 157°.5	Mean
0″.0005.....	0″.040	0″.032	0″.119	0″.064
.0015.....	.066	.058	.115	.080
.0025.....	.074	.066	.094	.078
.0035.....	.077	.069	.072	.073
.0045.....	.077	.073	.054	.068
.0055.....	.075	.075	.043	.064
.0065.....	.072	.072	.039	.061
.0075.....	.070	.070	.036	.059
.0085.....	.067	.067	.035	.056
.0095.....	.063	.063	.034	.053
.0105.....	.058	.058	.033	.050
.0115.....	.052	.052	.032	.045
.0125.....	.046	.046	.030	.041
.0135.....	.039	.039	.028	.035
.0145.....	.032	.032	.027	.030
.0155.....	.023	.029	.026	.026
.0165.....	.018	.024	.025	.022
.0175.....	.013	.019	.024	.019
.0185.....	.010	.016	.023	.016
.0195.....	.009	.013	.022	.015
.0205.....	.007	.011	.020	.013
.0215.....	.006	.008	.019	.011
.0225.....	.004	.006	.016	.009
.0235.....	0.002	0.002	.014	.006
.0245.....	....	....	.010	.003
.0255.....	....	....	.007	.002
0.0265.....	....	....	0.003	0.001
π	0″.0079	0″.0085	0″.0083	

* Same as Table XVII except $r_u=0.0$ instead of ± 2.5.

For Boss 1517, for which v is far beyond the limits of the tables, see the footnote to Table XXXIX. For one or two stars there has been a slight extrapolation. For all stars outside the Nebula-group within 15° of the antivertex the values of π have been marked uncertain (:).

TABLE XXIV*

$\pi_{0.0}=\phi(v, \lambda)$ ADOPTED, FOR $r_u=0.0$

v	λ 140°	145°	150°	154°	157°.5	160°	165°	180°
+0″0525	0″0176	0″0186	0″0197	0″0203	0″0224	0″0226	0″0226	0″0081
.0475	.0160	.0171	.0182	.0191	.0212	0214	.0216	.0081
.0425	.0142	.0153	.0165	.0177	.0199	.0200	.0202	.0081
.0375	.0126	.0136	.0148	.0162	.0183	.0184	.0186	.0081
.0325	.0109	.0119	.0130	.0143	.0165	.0166	.0169	.0081
.0275	.0093	.0102	.0111	.0122	.0142	.0145	.0151	.0081
.0225	.0077	.0085	.0093	.0102	.0116	.0120	.0129	.0081
.0175	.0062	.0069	.0076	.0084	.0091	.0097	.0109	.0081
.0125	.0049	.0055	.0061	.0068	.0068	.0075	.0089	.0081
.0075	.0038	.0043	.0048	.0053	.0051	.0058	.0072	.0081
+0.0025	.0029	.0033	.0037	.0043	.0039	.0045	.0059	.0081
−0.0025	0.0026	0.0026	.0030	.0034	.0032	.0037	.0048	.0081
0.0075	....	....	.0025	.0027	.0028	.0034	.0045	.0081
−0.0125	....	....	0.0022	0.0023	0.0026	0.0032	0.0043	0.0081

* Same as Table XXI except $r_u=0.0$ instead of ± 2.5.

15. PROBABLE ERRORS OF THE PARALLAXES

We again consider separately the two cases $r_u=\pm 2.5$ km and $r_u=0.0$ km, beginning with $r_u=\pm 2.5$. The value of π adopted for any one star is really the mean π of the parallaxes of all stars having the same v and λ. Let π_t be the true parallax. It is easy to find the probable value of the deviations $\pi_t-\pi$, if for the moment we assume that they follow the error-law. Take for instance a star for which $\lambda=150°$, $v=0''0325$. Table XVIII shows that for $r=\pm 2.5$ the adopted value of π ($=0''0107$) is the mean built up from

0.0008 stars having $\pi=0''0045$, therefore $\pi_t-\pi=-0''0062$
0.0051 " " " $=0.0075$, " " $=-0.0032$
0.0091 " " " $=0.0105$, " " $=-0.0002$
0.0058 " " " $=0.0135$, " " $=+0.0028$
0.0013 " " " $=0.0165$, " " $=+0.0058$

Hence, for a total of 0.0221 stars, the sum of the residuals, all taken positively, is

$$0.0008\times 0.0062+0.0051\times 0.0032+ \dots\dots =0.00004688.$$

Therefore

Average value of $\pi_t-\pi=0''00212$
Probable value of $\pi_t-\pi=0''00179$

The more refined computation, based on a table such as Table XVIII, but extended to all the parallaxes, 0″0005, 0″0015, 0″0025, ... gives

$$\text{Probable value of } \pi_t - \bar{\pi} = \pm 0''00191.$$

With similar computations for other values of v and λ, and a repetition for $r = 0.0$, I find finally, interpolating for $r_u = \pm 1.67$, the results in Table XXV, which include the effect of observational errors in the proper motions. Roughly speaking, we may say that for $\lambda < 156°$

$$r_\pi = \pm 0''0021. \tag{58}$$

For $\lambda > 156°$ the error rapidly increases.

TABLE XXV

PROBABLE ERRORS OF π

v	$\lambda = 150°$	$\lambda = 154°$	$\lambda = 157°5$
+0″0325	0″0020	0″0021	0″0029
0.0225	0.0021	0.0022	0.0030
0.0125	0.0020	0.0022	0.0029
+0.0025	0.0019	0.0021	0.0025
−0.0075	. . . .	. . . .	0.0022

But these probable errors have been computed on the supposition that the deviations follow the error-law. This may answer for the larger values of v, but it gives only a rough idea of the deviations for the smaller values of v. Indeed it is seen from Table XVIII, in which each horizontal line gives the frequency of the various values of π for stars of the same λ and v, that the distribution becomes very skew indeed for the vanishing values of v. The skewness is still more marked for the higher values of λ, the tables for which are not given here.

It is evident that this must be so. None of the parallaxes is negative; therefore, as an example, in the case of $\lambda = 150°$, $v = 0''0025$, where Table XVIII gives $\bar{\pi} = 0''0055$, deviations below $-0''0055$ are impossible, whereas no such impossibility exists for deviations greater than $+0''0055$. In fact, according to Table XVIII, the number of such deviations is by no means negligible.

In *Groningen Publications*, No. 8, p. 21, I assumed—in a very similar case and for quite the same reason—that not π but log π is distributed according to a normal error-curve. Schwarzschild[1] demonstrated theoretically that this must really be so, or rather that it is consistent with what we know or accept in connection with the distribution of the stars in space.

Table XVIII and others similar to it for the case of vanishing observational errors seem to offer an opportunity for testing the validity of the assumption directly by what we know from observation, for these tables are really derived from the observed distribution of v. It may be sufficient to state that, on trial, I found the result somewhat inconclusive. For $\lambda = 157\overset{\circ}{.}5$ the representation of the difficult cases by the logarithmic curve is satisfactory; for $\lambda = 150°$ it is not. More extensive data, less strongly influenced by observational errors, are required to make the test decisive.

16. APPLICATION OF THE METHOD TO THE VALUES OF τ

It seems natural to attempt to use the component τ of the proper motion as we have used v in what precedes. Since τ is independent of the stream-velocity, we avoid all the difficulties presented by this motion. We need not consider groups within narrow limits of λ; errors in its velocity V are of no consequence, etc.

We compare the observed distribution, which has been given in Table XII, with the theoretical distribution. To determine the latter we have, as before,

$$\tau = 0.212 \pi t \tag{59}$$

(τ and π in seconds of arc; t in km per second). The distribution of t (Section 11) is practically according to an error-curve, the central value being $t = 0$, with a probable error

$$r_t = r_u = \pm 1.67 \text{ km} \tag{60}$$

Hence, the values of τ for stars of the same parallax will also be distributed according to the error-law. The central value will be zero, and the probable error $\pm 0.212 \pi r_u$. The observational

[1] *Astronomische Nachrichten*, **190**, 361, 1912.

errors will not disturb the distribution in an error-curve; the only effect will be to increase the probable error to

$$R = \pm \sqrt{(0.212\,\pi r_u)^2 + r^2}$$

In Section 10 values of τ with probable errors exceeding $0''.0069$ were neglected. Consequently r is now found to be smaller than in the preceding section, namely,

$$r = \pm 0''.0047. \tag{61}$$

Hence

$$R = \pm \sqrt{(0.354\pi)^2 + (0''.0047)^2} \tag{62}$$

In analogy with Section 13, we might now compute the distribution of τ for a series of consecutive values of π and then try to find the relative numbers of stars with these parallaxes necessary to produce a distribution of τ agreeing with that observed—in other words, find values of $n = \phi(\pi)$ which will harmonize theory with observation.[1] But it appears that such an attempt is doomed to failure, for, whether we assume all the stars to have the same parallax, or whether we assume π to be distributed according to $\phi(\pi)$ in Tables XVII and XXIII, we find practically the same distribution for τ. The solution is so little sensitive to changes in $\phi(\pi)$ that the determination of this function in the manner proposed is hopeless.

For example, let I represent the distribution of τ resulting from the known distribution of t, when we adopt for all the stars the parallax $0''.0081$. The distribution of τ then follows an error-curve with a probable error found by (62) to be $R = \pm 0''.0055$. Similarly let II represent the distribution of τ, resulting when we assume for $\phi(\pi)$ the values found for $\lambda = 150°$ by interpolation between Tables XVII and XXIII. Finally let III represent the same distribution, when for $\phi(\pi)$ we adopt the result of interpolating between the

[1] This would be equivalent to finding the solution of the integral equation:

$$\text{Prob.}\ {}_{\tau}^{\tau+\delta t} = \frac{1}{3.14\ldots}\delta\tau \int_0^\infty H\phi(\pi)e^{-H^2\tau^2}\delta\pi$$

where

$$H = \frac{0.4769\ldots}{\sqrt{(0.212\,\pi r_u)^2 + r^2}}.$$

The first member being given, we would have to find $\phi(\pi)$.

mean values of Tables XVII and XXIII. The method of obtaining II and III is so analogous to that followed for v in Section 13 that it need not be described. The results (total number of stars 72) are given in Table XXVI. They are so nearly identical that, whatever

TABLE XXVI

τ	Frequencies			No. of Stars		
	I	II	III	I	II	III
$> \pm$ 0″025.	0.002	0.000	0.006	0.1	0.0	0.4
$\pm$0″025 to $\pm$0.020.	.012	.014	.018	0.9	1.0	1.3
$\pm$0.020 " $\pm$0.015.	.052	.052	.056	3.7	3.7	4.0
$\pm$0.015 " $\pm$0.010.	.154	.152	.154	11.1	10.9	11.1
$\pm$0.010 " $\pm$0.005.	.320	.318	.314	23.0	22.9	22.6
$\pm$0.005 " $\pm$0.000.	0.460	0.464	0.454	33.1	33.4	32.7
Totals.	1.000	1.000	1.002	71.9	71.9	72.1

the observed distribution, a choice between the three solutions is impossible. All three agree equally well or equally badly.[1] Actually the agreement is in no case perfect, for solution I (therefore also II and III) represents the distribution in an error-curve with probable error 0″0055, while the observed distribution is sen-

[1] In the case of data a hundred times richer, such differences as we find between the several solutions might lie well outside the limits of uncertainty of the observed numbers. But even then the solution for $\phi(\pi)$ would have to be considered illusory, at least in the present state of science. To show this clearly, let us assume our data to be so numerous that the frequencies could be considered reliable to the third decimal inclusive, and further, that the observed frequencies agreed absolutely with the theoretical frequencies of solution II. Would this prove that solution II for $\phi(\pi)$ would be the correct one? By no means, for we are far from certain that the linear motion t for the distribution of which we assumed the error-law, really follows this law with a precision of a few thousandths in the frequencies. Indeed we know that, at least for a mixture of stars of all spectra, there are very appreciable deviations from this law (see Schwarzschild, *Astronomische Nachrichten*, **190**, 376, 1912; Kapteyn and Adams, *Proc. Nat. Acad. Sci.*, **1**, 18, 1915). It even seems questionable whether the accidental observational errors in t follow the error-law with such nicety. The real frequency law of the linear velocities might therefore be such that if we reduce to angular motion on the assumption of the same parallax for all stars, and add the effect of the accidental errors of observation, we would find, instead of I, such a distribution as II or III or any other distribution diverging from I by quantities of the same order. In case we found exactly the distribution II we should of course conclude, instead of inferring that $\phi(\pi)$ had the values given above for II, that all the stars had the same parallax.

sibly in an error-curve with the probable error $\pm 0''.0061$ (Section 10). Were the difference well established it would have to be interpreted, either (*a*) as an error in the adopted mean parallax $0''.0081$, or (*b*) as an error in the adopted probable amount ± 1.67 km for t, or (*c*) as a combination of both (*a*) and (*b*). On the first hypothesis the mean parallax should be $0''.0110$; but the evidence for the change is almost negligible.

17. ADDITIONAL EVIDENCE AS TO THE VALUES OF THE PARALLAXES

The difficulty of the question and particularly the relative crudeness of the data seem to justify the accumulation of as many checks as can be obtained. I have therefore brought together whatever further evidence I have been able to find, not hesitating to include some which is only partly independent of what precedes.

a) *The range in parallax.*—Though the values of τ do not afford a useful determination of the parallaxes, they may serve indirectly by rendering possible the elimination of observational errors. For this purpose I will compare the components υ and τ for stars at the same, or nearly the same, distance λ from the vertex. The values of υ contain the component of the stream-motion (in angular measure) and the peculiar proper motion; τ contains only the latter. If all the stars had the same parallax, the angular stream-motion in υ would be the same for all. As the peculiar motions are supposed to be random in direction, the common angular stream-motion would be the arithmetic mean $\bar{\upsilon}$ of υ. Hence $\upsilon-\bar{\upsilon}$, as well as τ, would include only the peculiar motion. Both would have all sorts of values, but the average amounts (all taken positively) and similarly the probable amounts would be equal.

If, on the contrary, the parallaxes are unequal, the stream-motion involved in the υ will not be the same for all stars. The quantities $\upsilon-\bar{\upsilon}$ will include the peculiar motions, and also an element dependent on the stream-motion, which will be the greater, the greater the diversity of parallaxes.

The average value of $\upsilon-\bar{\upsilon}$ (disregarding signs) and its probable value will now be greater than the corresponding values for τ. The difference will be a measure of the diversity of the parallaxes, that is, of the range in distance.

In the elaboration of this idea we must consider the observational errors. Since the component of the stream-motion at right angles to the line of sight for stars at the distance λ from the vertex is $V \sin\lambda$ km per second, or $0.212\ V \sin\lambda$ astronomical units per year, we shall have, if we call α and β the peculiar angular motions in the direction of υ and τ respectively,

$$\left.\begin{aligned} \upsilon &= 0.212\ \pi V \sin\lambda + \alpha + \text{obs. err. in } \upsilon \\ \tau &= \beta + \text{obs. err. in } \tau \end{aligned}\right\}. \tag{63}$$

For a great number of stars the arithmetic means of α, of β, and of the observational errors in υ and τ will be zero. Hence

$$\left.\begin{aligned} \upsilon - \bar{\upsilon} &= 0.212\ V \sin\lambda\ (\pi - \bar{\pi}) + \alpha + \text{obs. err. in } \upsilon \\ \tau &= \beta + \text{obs. err. in } \tau^{1} \end{aligned}\right\}. \tag{64}$$

Squaring and forming the means for all available stars, we find, putting

$$\pi - \bar{\pi} = \Delta\pi \tag{65}$$

$$\left.\begin{aligned} \frac{1}{n}\Sigma(\upsilon-\bar{\upsilon})^2 &= (0.212\ V \sin\lambda)^2\overline{(\Delta\pi)^2} + \frac{1}{n}\Sigma\alpha^2 + \frac{1}{n}\Sigma(\text{obs. err. in } \upsilon)^2 \\ &\qquad + \text{double products} \\ \frac{1}{n}\Sigma\tau^2 &= \frac{1}{n}\Sigma\beta^2 + \frac{1}{n}\Sigma(\text{obs. err. in } \tau)^2 \\ &\qquad + \text{double products} \end{aligned}\right\} \tag{66}$$

The double products will nearly disappear, because the factors are as frequently positive as negative. Further, $\Sigma\beta^2$ will approximately equal $\Sigma\alpha^2$. The terms depending on observational errors may differ slightly on account of possible differences of accuracy in the proper motions for α and δ; but as these differences are generally small, and greatly diminished in υ and τ, we assume these terms also to be equal. Hence, if we write

$$S^2 = \frac{1}{n}\Sigma(\upsilon-\bar{\upsilon})^2 - \frac{1}{n}\Sigma\tau^2 \tag{67}$$

we obtain

$$(0.212\ V \sin\lambda)^2\ \overline{(\Delta\pi)^2} = S^2 \tag{68}$$

where S is a quantity given by the observations.

[1] It is slightly more correct to use $\tau - \bar{\tau}$ than τ. This refinement was indeed introduced; $\bar{\tau}$, however, is so small that the correction is without influence.

Applying this formula to narrow zones of λ, I find

λ		No. Stars	Weight	
145° to 149°	$\overline{(\Delta\pi)^2}=+0.000011$	26	15	(69)
150 " 154	$+\ .000042$	40	12	
155 " 159	$+\ .000025$	28	4	
160 " 164	$+0.000017$	17	1	

from which

$$\overline{(\Delta\pi)^2}=+0.000024\pm0.000005 \quad (111 \text{ stars}). \tag{70}$$

From the preceding section, for $\lambda=150°$ and $\lambda=157°.5$, the two being practically independent,

$\lambda=150°.0$	$\overline{(\Delta\pi)^2}=0.0000135$
$\lambda=157.5$	0.0000337
Mean	0.000024

The agreement of the two mean values is beyond expectation. As $\overline{(\Delta\pi)^2}$ is preponderatingly dependent on the extreme parallaxes, both large and small, I see in this agreement a good test of the correctness of the total range in distance. On the other hand, we here find a difference for high and low λ which is not confirmed by (69). As this difference is due to the differences shown in Tables XVII and XXIII between the values $\phi(\pi)$ for different values of λ, we obtain an indication that it would have been preferable to adopt for the different zones of λ values for $\phi(\pi)$ which more nearly agree. This is the reason why I definitively adopted in the preceding section, for $\lambda=150°$, $154°$, and $157°.5$, values of $\phi(\pi)$ intermediate between those found directly and the mean of all.

b) Test of the moderate and large parallaxes.—Since v is the angle subtended at the earth by the projection on the sphere of the stream-velocity increased by the component u (u counted positively in the sense of the stream-motion), it will, on the whole, be somewhat greater for the stars having positive values of u, and smaller for those having negative values of u. In other words, there must be a certain degree of correlation between v and u. The coefficient of this correlation ρ necessarily lies between zero and unity, $\rho=0$ corresponding to an absence of correlation. For this case the average of the several u's corresponding to any given v would be

zero; for $\rho=1$, u is a pure function of v alone, and a value of v fully determines u, which I will call u_v. According to (50) the two extreme cases give

$$\rho=0; \qquad \pi_0=\frac{v}{0.212\ V\sin\lambda}{}^{*} \tag{71}$$

$$\rho=1; \qquad \pi_1=\frac{v}{0.212\ (V\sin\lambda+u_v)}\,. \tag{72}$$

As the truth lies between the two, we thus obtain two limits for $\overline{\pi}$.

The value of u_v for any value of v is easily obtained. It will be necessary, however, first to derive from Table XI, which gives the distribution of the observed v, one giving the distribution of the true v (free from observational error). Professor Eddington[1] has given an extremely elegant and convenient method, but the observed frequency-curves are here so nearly error-curves that an even more convenient method is available.

Thus, for instance, the observed frequency-curve of Table XI for $\lambda=150°$ is represented approximately by an error-curve with the central value 0″0162 and the probable error 0″00917. As the probable error of observation is ±0″0056 (54), the frequency-curve of the v, freed from observational error, will evidently be an error-curve with the same central value, $\nu=0''0162$, and the probable error $\sqrt{0.00917^2-0.0056^2}=\pm 0''00725$. Proceeding in the same way for the other values of λ we have:

	Central Value	P.E. Free from Obs. Error
$\lambda=150°$	0″0162	±0″00725
$\lambda=154$	0.0159	0.00705
$\lambda=157.5$	0.0135	±0.00693

* This is only approximately true and assumes that (with the exception of a vanishingly small number of extreme values of u) u is small as compared with $V\sin\lambda$. For in this case we may write, neglecting squares of small quantities,

$$\pi=\frac{v}{0.212\ (V\sin\lambda+u)}=\frac{v}{0.212\ V\sin\lambda}\left(1-\frac{u}{V\sin\lambda}\right)$$

from which, because $\bar{u}=0$,

$$\pi=\frac{v}{0.212\ V\sin\lambda}\,.$$

[1] *Monthly Notices*, **73**, 359, 1913.

With the aid of these numbers we find by the ordinary theory of the distribution of observational errors that the following fractions of all the stars entered in Table XXVII have values of v exceeding given limits.

TABLE XXVII

FREQUENCY OF v EXCEEDING CERTAIN LIMITS

v	$\lambda=150°$	$\lambda=154°$	$\lambda=157°.5$
>0″.030......	0″.100	0″.089	0″.054
.025......	.206	.192	.132
.020......	.362	.347	.263
.015......	.544	.535	.442
.010......	.718	.714	.633
0.005......	0.851	0.851	0.906

Now it is evident that if u is a function of v alone, as it is in the extreme case under consideration, it increases and diminishes with v, and the number of values of v and of u exceeding specified corresponding limits must be the same, and conversely. That is, the value u corresponding to any particular value of v will be that value of u which is as frequently exceeded as is the particular value of v. Therefore, to take an instance, since by Table XXVII, for $\lambda=150°$, v exceeds 0″.030 in 0.100 of all the cases, and since in the theory of observational errors we find that an error exceeding $+1.90$ times the probable error (which for u is ± 1.67) occurs in just 0.100 of all the cases, therefore $u_{0''.030}=+1.90\times 1.67=+3.2$, and similarly in other cases, thus giving the results in Table XXVIII. With the aid of these data, (71) and (72) lead to Table XXIX.

TABLE XXVIII

u_v $(r_u=\pm 1.67)$

v	λ		
	150°	154°	157°.5
	km	km	km
0″.030........	+3.2	+3.4	+4.0
.025........	+2.0	+2.2	+2.8
.020........	+0.9	+1.0	+1.6
.015........	−0.3	−0.2	+0.4
.010........	−1.4	−1.4	−0.85
0.005........	−2.6	−2.6	−3.3

TABLE XXIX

LIMITS FOR π

v	$\lambda=150°$			$\lambda=154°$			$\lambda=157°.5$		
	$\bar{\pi}_0$	π	$\bar{\pi}_1$	$\bar{\pi}_0$	π	$\bar{\pi}_1$	$\bar{\pi}_0$	π	$\bar{\pi}_1$
0".030...	0".0141	0".0124	0".0107	0".0161	0".0134	0".0116	0".0185	0".0162	0".0121
.025...	.0118	.0109	.0098	.0134	.0119	.0108	.0154	.0142	.0113
.020...	.0094	.0093	.0087	.0107	.0102	.0097	.0124	.0120	.0103
.015...	.0071	.0076	.0073	.0081	.0084	.0082	.0093	.0097	.0088
.010...	.0047	.0056	.0055	.0054	.0064	.0064	.0062	.0071	.0069
0.005...	0.0024	0.0037	0.0032	0.0027	0.0048	0.0038	0.0031	0.0041	0.0054

With these limits we have to compare the values of π corresponding to the true values of v as argument. For $r_u = \pm 2.5$ km, π has been given in Table XX; for $r=0.0$ it is to be computed by

$$\pi = \frac{v}{0.212\, V \sin\lambda} = \frac{v}{4.24 \sin\lambda}\,.$$

Interpolating for $r_u = \pm 1.67$, we find the values in Table XXIX under the heading π. Our test is that π ought to be included between the limits π_0 and π_1.

In making the comparison we must consider that the values of u_v in Table XXVIII depend in the last instance on observations, and as such are liable to error. The limit π, itself, is therefore liable to such error. Further, from (72) we see that for values of u_v not too far from $-V \sin\lambda$ any error in u_v appears in $\bar{\pi}$ enormously exaggerated. For this reason it is not surprising that for values of v as small as 0".015 or 0".010 the values of π in Table XXIX are sometimes slightly beyond the limit $\bar{\pi}_1$. For v still smaller, the present test of the parallaxes is, I think, of little value. Taking all this into consideration there seems to me every reason to be satisfied.

c) Test for the distant stars.—I will here use only the B0–B5 stars and will consider that fifth of them for which v is smallest—on the whole, the more distant objects. They are

for $\lambda = 145°$ to $154°$ those for which $v < +0".0048$
for $\lambda = 155$ to 165 those for which $v < +0.0015$.

Stars still nearer to the antivertex will be disregarded, as will also the few stars for which $\lambda < 145°$. It is my aim to find something

like a lower limit for the parallax π_d of these stars which for brevity will be called the great-distance stars.

For this purpose I will make use of the luminosity-curve of the Bo–B5 stars, for which a fairly reliable curve has been found in *Mount Wilson Contribution* No. 82. A formula was also derived (*op. cit.*, p. 64) by which the mean parallax can be found for a group of stars which is complete down to a specified magnitude m_0, when the average apparent magnitude $\overline{m}$ of the whole group has been observed. For $m_0 = 5.81$, which is the Harvard magnitude limit to which Boss's catalogue is complete, the formula becomes

$$\overline{m} = -4.115 - 5 \log \pi - 0.691 \frac{e^{-P^2}}{\frac{1}{\sqrt{\pi}} \int_{-\infty}^{P} e^{-z^2} \delta z} \tag{73}$$

where[1]

$$P = +4.055 + 2.045 \log \pi . \tag{74}$$

In order to eliminate to a great extent the errors of the adopted luminosity-curve, I will apply the formula, not only to the great-distance stars, but also, for comparison, to the entire group of Bo–B5 stars, adopting for the mean parallax of the latter $\overline{\pi} = 0''.0081$. The method thus becomes a more or less differential one.

If to the limit 5.81 mag.

$\overline{m}_1$ = average mag. of all the stars

$\overline{m}_2$ = average mag. of the great-distance stars

then (73) and (74) show that

if $\overline{m}_2 - \overline{m}_1 =$		$0^m.00$	then π_d		$= 0''.0081$
"	"	$+0.10$	"	"	0.0069
"	"	$+0.20$	"	"	0.0057
"	"	$+0.30$	"	"	0.0046
"	"	$+0.40$	"	"	0.0036

From Table XXXIX I find that in reality

$\overline{m}_1 = 4.78 \pm 0.06$ (86 stars)

$\overline{m}_2 = 4.81 \pm 0.14$ (14 stars)

[1] The formula supposes that the range in π is not excessive. I feel confident that we may safely apply it in the present case. But, in case of doubt, the method which is fully explained in *Mount Wilson Contribution* No. 82, pp. 65–68, can be applied.

Therefore $m_2 - m_1 = +0.03 \pm 0.15$, and according to the preceding table

$$\pi_d = 0''.0077 \pm 0''.0018. \qquad (75)$$

Since the average value of v for the stars of great distance is found to be $-0''.0016$, interpolation between Tables XXI and XXIV gives for the mean of $\lambda = 150^\circ$, 154°, and $157^\circ.5$

$$\pi_d = 0''.0046. \qquad (76)$$

The difference is considerable, but so also is the probable error in (75). In fact the difference is 1.7 times the probable error.

The conclusion from this test would seem to be that the number of stars is too small to lead to any well-founded result. The result as it stands would indicate that the adopted parallaxes of the distant stars are too low rather than too high. Any material change in this direction seems pretty well excluded, however, by the more reliable tests (*a*) and (*b*).

d) *Test by binary systems.*—Professor Hertzsprung courteously furnished me with the following hypothetical parallaxes, derived by a method which allows of a fairly good determination of parallax for any double star as soon as it shows sensible orbital motion. He will no doubt soon publish his method. In addition I quote the parallax of δ Orionis according to Stebbins.[1] For comparison the parallaxes obtained in the present paper are added.

	Hertzsprung and Stebbins	Kapteyn
ζ Orionis = Boss 1398 . .	0″.0082	0″.0057 neb.
σ " = " 1389 . .	0.0060	0.0057 neb.
15 Monocerotis = " 1706 . .	0.0089	0.0057
32 Orionis = " 1331 . .	0.0138	0.0156
δ " = " 1339 . .	0.0032	0.0057 neb.
Mean	0.0080	0.0077

18. THE NEBULA-GROUP. ITS PARALLAX

In this group the proper motions are so exceedingly small that the method for finding the stream-elements and the parallax used

[1] *Astrophysical Journal*, **42**, 145, 1915.

above fails altogether. Fortunately I had at my disposal for this region and its surroundings, namely,

$$\text{R.A. } 5^h0^m \text{ to } 6^h0^m, \quad \text{Dec. } -10° \text{ to } +20°, \tag{77}$$

the spectra for the A and B stars obtained by Miss Cannon for the *Revised Draper Catalogue*. In Table XXX the numbers of these spectra have been summarized. In order to be sure of including only the densest part of the Nebula-group, I narrowed the limits (2) of this group[1] to

$$\text{R.A. } 5^h20^m \text{ to } 5^h40^m, \quad \text{Dec. } -7° \text{ to } +2°, \tag{78}$$

an area of 45 square degrees.[1] For comparison I included the stars of region (77) well outside the Nebula-group. For these I adopted the limits:

$$\left.\begin{array}{llll} 5^h\ 0^m & \text{to } 5^h20^m & +3° & \text{to } +20° \\ 5\ 20 & \text{"}\ \ 5\ 60 & +5 & \text{"}\ \ +20 \\ 5\ \ 0 & \text{"}\ \ 5\ 10 & -10 & \text{"}\ \ +3 \\ 5\ 50 & \text{"}\ \ 5\ 60 & -10 & \text{"}\ \ +5 \end{array}\right\} \tag{79}$$

containing in all 305 square degrees. The space between these regions was not used in order to be independent of any uncertainty in the exact limits of the Nebula-group.

The last line but one in Table XXX gives the mean of the observed magnitudes. The last line gives the ratio of star density in nebula region to star density outside this region. We thus find that the nebula region is fully twelve times richer in B0–B5 stars than are the surroundings. For later types this ratio rapidly diminishes. The Nebula-group thus contains both B and A stars, but, as compared with the surrounding sky, it is very much richer in spectra of the former class.

The striking fact in the numbers of Table XXX is that the magnitude of greatest frequency is different for the nebula and the region outside. This at once promises a determination of the ratio of the parallaxes of the two groups of stars. For the absolute magnitudes of stars which are practically at the same distance

[1] This narrowing of the limits is the reason why the number of bright B stars in the Nebula-group, as shown by Table XXX, is smaller than that in Table XXXIX. Moreover the Oe5 stars have been omitted from Table XXX.

TABLE XXX

NUMBER OF STARS IN "REVISED DRAPER CATALOGUE"

Magnitude Limits	Mean Mag.	B3		B5		B0–B5		B5–B9		B8–B9		A	
		Neb.	Outside	Neb.	Outside	Neb.	Outside	Neb.	Outside	Neb.	Outside	Neb.	Outside
9.05–9.55	9.3	0	0	0	0	0	0	12	7	12	7	48	87
8.55–9.05	8.8	0	0	2	0	2	0	52	31	50	31	43	207
8.05–8.55	8.3	3	0	3	3	6	3	26	45	23	42	21	99
7.55–8.05	7.8	3	3	3	2	6	6	18	31	15	29	6	35
7.05–7.55	7.3	3	0	1	0	4	0	5	9	4	9	2	13
6.55–7.05	6.8	1	1	2	2	3	3	4	11	2	9	4	8
6.05–6.55	6.3	4	3	4	....	11	4	7	4	3	4	0	13
5.55–6.05	5.8	4	1	....	....	6	3	....	7	....	7	2	2
5.05–5.55	5.3	0	1	....	....	3	2	....	3	....	3	....	8
4.55–5.05	4.8	2	2	....	....	3	2	....	1	....	1	....	0
4.05–4.55	4.3	....	1	....	....	0	3	....	....	....	....	....	1
3.55–4.05	3.8	....	....	....	....	1	0	....	....	....	....	....	0
3.05–3.55	3.3	....	....	....	....	0	0	....	....	....	....	....	0
2.55–3.05	2.8	....	....	....	....	0	0	....	....	....	....	....	1
2.05–2.55	2.3	....	....	....	....	2	0	....	....	....	....	....	....
1.55–2.05	1.8	....	....	....	....	1	1	....	....	....	....	....	....
1.05–1.55	1.3	....	....	....	....	....	....	....	....	....	....	....	....
0.55–1.05	0.8	....	....	....	....	....	....	....	....	....	....	....	....
0.05–0.55	0.3	....	....	....	....	....	....	....	1	....	1	....	....
Total no.		20	12	15	7	48	27	124	150	109	143	126	474
Mean mags.		6.79	6.17	7.50	7.73	6.42	6.06	....	....	....	....	....	....
Density Nebula / Density outside		11.3		14.5		12.0		5.6		5.2		1.83	

differ from the apparent magnitudes by a constant, $5+5\log\pi$.[1] Consequently the frequency-curve of the apparent magnitudes differs from the luminosity-curve (which is the frequency-curve of the absolute magnitudes) merely in that there is a displacement of $5+5\log\pi$ mags. between the two. For two groups with parallaxes π_1 and π_2 the frequency-curves of the apparent magnitudes will thus be the same, with the exception of the displacement

$$5\log\frac{\pi_2}{\pi_1}\text{ magnitudes.} \tag{80}$$

Consider the region of the nebula and that defined by (79) lying outside the nebula. As to the former, the lateral dimensions being small, it is all but certain that the depth also will be small. For the outside stars the distances must vary to a much greater extent,

[1] See, for instance, *Groningen Publications*, No. 11, p. 12.

and the frequency-curve of the apparent magnitudes will here consist of the superposed frequency-curves for the different distances. If we suppose the curve of the absolute magnitudes to be an error-curve (see Section 27), the frequency-curve of the apparent magnitudes for stars at the average distance will be an error-curve with its maximum at a certain magnitude. For the stars of smaller parallax the curve will be shifted to the fainter magnitudes; for the stars with greater parallax, to the brighter magnitudes. As long as the range in distance for the great majority of the stars is moderate, as is here the case, the superposition of all these curves will still be practically an error-curve, with its maximum at nearly the same magnitude as for the stars of average parallax. Only the probable error of the curve will have increased.

This is just what is seen in Table XXX. Take first the B0–B5 stars. Owing to the limited data the numbers run very irregularly. There is, however, the fortunate circumstance that for both regions the number of stars fainter than 9.0 is zero; for the outside stars there is none fainter than 8.5. I conclude that, in the latter case at least, we have before us the *complete* frequency-curve. In the nebula region this is less certain; still the number of stars fainter than 9.0 is certainly small. It seems reasonable to assume that the true number of stars fainter than 9.05 (adopted average magnitude 9.3) is between zero and six. Now, if the luminosity-curve is an error-curve, or any symmetrical curve, the lateral shift will be that of the center of gravity or of the arithmetic mean. The mean of the magnitudes for the outside stars is 6.06. For the Nebula-group it is 6.42 and 6.74, respectively, for the two limits adopted. The displacement therefore lies between 0.36 and 0.68 mags. As the whole difference corresponds to only $0\overset{''}{.}0010$ in the parallax we may adopt

B0–B5 stars Displacement = 0.52 mag. (81)

For the B8–B9 stars the matter is more difficult. But I think we may assume with some confidence that the numbers for magnitudes 9.0 and brighter are complete: for, (*a*) the whole number of stars for which Miss Cannon has determined the spectrum is 240,000,[1] which, according to *Groningen Publications*, No. 18, involves average

[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **51**, 85, 1916.

completeness to magnitude 9.5 (Harvard Scale); (*b*) the B and A stars, being very white, must in general have been observed to a fainter limit (visually) than the others; (*c*) according to Table XXX so many stars fainter than 9.05 have been observed in the regions now under consideration that it is improbable that the brighter stars should be seriously incomplete.

On the contrary, the numbers between 9.05 and 9.55 in Table XXX are almost certainly incomplete. To be comparable with the rest they must be multiplied by a factor ϵ, greater than unity. For the nebula region, ϵ cannot well exceed 2, as is shown by the A stars, which at magnitude 9.3 must be near the maximum of their frequency-curve of apparent magnitude; the total number, therefore, from 9.05 to 9.55 can scarcely be more than double that between 8.55 and 9.05. If we admit this the uncertainty about the true value of ϵ is of little consequence, for in attempting to fit an error-curve to the observed number of the B8–B9 stars in the nebula region, we find the maximum at practically 8.8, both for $\epsilon = 1$ and $\epsilon = 2$.

For the outside region it seems better to ignore the stars fainter than 9.05 altogether. From the others it appears that the maximum cannot lie far from 8.30. We are thus led to a displacement of 0.50 mag. Another estimate of this quantity made in a somewhat different way gave 0.37 mag.[1] I finally adopt:

$$\text{B8–B9 stars} \qquad \text{Displacement} = 0.44 \text{ mag.} \tag{82}$$

The displacements (81) and (82) require a correction. In the case of types B0–B5, the stars within the limits (78) of the Nebula-group are twelve times more crowded than those in the surrounding regions. We conclude that a twelfth part of the B0–B5 stars within these limits do not belong to the group, but are merely seen projected on it. These can have no displacement relative to the outside stars, to which they really belong. For the others, it must

[1] In the countings of the Pickering-Cannon stars, there is, as in other investigations of the kind, an obvious excess of stars of magnitude 9.0, 8.5, 8.0, and a deficiency for magnitudes 9.1, 8.9, 8.1, 7.9. I also carried through a computation in which this error was corrected; as the corrections for the numbers of stars within the half-magnitude intervals of Table XXX are insignificant, I will not burden this paper with the computation.

be increased by an eleventh in order to correct the displacement (81). For similar reasons the displacement (82) must be increased by $\frac{1}{4.2}$ its amount.

Hence:

B0–B5 stars: corrected displacement = 0.57 mag. (weight 2)
B8–B9 stars: corrected displacement = 0.54 mag. (weight 3)

The weights are estimates; the weighted mean is

$$\text{Displacement} = 0.55 \text{ mag.} \tag{83}$$

and by (80)

$$\frac{\pi \text{ Outside Stars}}{\pi \text{ Nebula-group}} = 1.288 \tag{84}$$

It remains to find π for the stars of magnitude 7.8, which is the average of the Cannon stars. For the Boss stars, of average magnitude 4.7, $\pi = 0''.0081$. From the Table XXXIX I find[1]

Average Mag.	π	No.
2.73	0''.0075	7
4.16	0.0083	14
5.04	0.0086	43
5.81	0.0083	38

Apparently there is no change with the magnitude; but, as it is improbable that there should be no change at all, I adopt as a conservative estimate,

$$\bar{\pi} = 0''.0077 \ (m = 7.8, \text{ outside stars}), \tag{85}$$

with the aid of which by (84)

$$\pi \text{ (Nebula-group)} = 0''.0060. \tag{86}$$

I think that the error of the displacement (83) cannot well be greater than half its amount and that its probable amount will not exceed a fourth of the whole. The probable error of the difference of the average parallaxes of the two groups of stars here considered ought not, therefore, to exceed 0''.0005 and, as the probable error of

[1] Preliminary values of the parallaxes were used. For the present purpose these are sufficient.

the average parallax of the outside Boss stars is, by (33), $\pm 0''.0007$, the probable error of (86) may be estimated at $\pm 0''.0009$. It is found in Section 26, however, from a discussion of the luminosity-curve of the A stars, that the value (86) is very probably somewhat high, and I finally adopted

$$\bar{\pi}\ (\text{Nebula-group}) = 0''.0054 \pm 0''.0009. \tag{87}$$

The lateral dimensions of the Nebula-group, according to Section 3, are about 7° in right ascension and 14° in declination. If, as a crude approximation, we suppose that the group is roughly spherical with a diameter of 10°.5, the parallaxes will range from 0''.0049 to 0''.0060. For that half of the stars for which the proper motions are smallest I assume the parallax to be 0''.0051, and for the others, 0''.0057; that is,

$$\left.\begin{array}{ll} \text{for } 100\,\mu \geqq 0''.8 & \pi = 0''.0057 \\ \text{for } 100\,\mu < 0''.8 & \pi = 0''.0051 \end{array}\right\} \tag{88}$$

As already mentioned, one-twelfth of the B0–B5 stars within the limits (2) do not belong to the physical group. Within these limits there are in Table XXXIX 26 stars, practically all B0–B5 stars. Two or three are, therefore, probably outside the group, but I have assumed that the four, Boss 1302, 1343, 1376, and 1382, the only ones having $\mu \geqq 0''.020$, are in this class. The last has a large negative value of v and perhaps ought to be excluded altogether. The motions of the others can be satisfactorily explained as the result of traces of stream-motion, peculiar motion, and observational error.

Five stars, Boss 1454, 1277, 1435, 1512, and 1297, having spectra B0–B5, are within 5° of the adopted limits (2) of the Nebula-group. Their proper motions are nearly vanishing. It is doubtful whether they belong to the Nebula-group. As their parallaxes computed on the hypothesis that they are group stars differ in no case more than 30 per cent from those obtained on the assumption that they are outside stars, I adopt the means for the two suppositions.

19. TESTS FOR THE PARALLAX OF THE NEBULA-GROUP

a) By the luminosity-curve of the B0–B5 stars.—From the third column of Table XXX, we have concluded that the maximum of

the frequency-curve for the B0–B5 stars in the Nebula-group lies between

$$m=6.42 \quad \text{and} \quad m=6.74. \tag{89}$$

In *Mount Wilson Contribution* No. 82, p. 43, the absolute magnitude corresponding to this maximum was found to be

$$M=0.885. \tag{90}$$

If, therefore, we adopt the mean of (89) we find from

$$M=m+5+5 \log \pi, \tag{91}$$

$$\pi=0''.0072. \tag{92}$$

I have not used this for the improvement of the parallax (87) because it is preferable to follow the inverse course, that is, use (87) for converting the apparent magnitudes of Table XXX into absolute magnitudes, which will be employed to improve the luminosity-curve.

b) *Binary systems.*—Among the binaries of Section 17, the three marked "neb." belong to the Nebula-group. In the mean they give $\pi=+0''.0058$, agreeing almost perfectly with the value $0''.0057$ derived from (87). That these binaries were also used in the test of the parallax of the outside stars is justified by the fact that our determination of the parallax of the Nebula-group is based on that of the outside stars.

20. MOTION OF THE NEBULA-GROUP

Considering only the best-observed stars ($100\, r<0''.040$) within the limits (2) of the Nebula-group, we derive from 17 stars the averages:

$$\alpha=5^h 25^m; \quad \delta=-1^\circ.4; \quad 100\, \tau=+0''.03; \quad 100\, \bar{\upsilon}=+0''.02 \tag{93}$$

Including further all the stars of known radial velocity, we find from 13 stars:

$$\alpha=5^h 26^m; \quad \delta=-2^\circ.2; \quad \rho_{\text{corr.}}=\rho-4.3=18.46 \text{ km} \tag{94}$$

The value (93) of $\bar{\upsilon}$ being absolutely insensible, I conclude that the antivertex must be not far from the point

$$5^h 25^m, \qquad -1^\circ.4. \tag{95}$$

How far away can it be placed without conflicting with the observations? I think it can safely be said that, in an average of 17 well-observed stars, a value of 100 $\bar{v}$ as high as $0''.5$ is irreconcilable with (93). It seems worth while in this connection to remark that the 12 outside stars, for λ between 170° and 178°, give

$$100\,\bar{v}=+0''.46 \text{ from which } \pi=0''.0083. \tag{96}$$

Notwithstanding the extremely unfavorable circumstances we still find a parallax agreeing perfectly with the final result (33). Now, from (51) and (16), eliminating V,

$$\tan\lambda=\frac{v}{0.212\,\rho\pi}, \tag{97}$$

from which, if we put $\pi=0''.0054$; $v=+0''.0050$; $\rho=+18.46$, we obtain

$$\lambda=13^\circ. \tag{98}$$

It is to be considered extremely probable, therefore, that the antivertex of the Nebula-group lies within 13° of (95). The antivertex of the outside stars is at

$$5^{h}44^{m}, \qquad -11^\circ \tag{99}$$

which is 10°.7 from (95). The radial velocity of the outside stars at the point (94), which is 9°.9 from (99), is 20.0 cos 9°.9 = 19.7, which differs only 1.24 km from (94). It is very possible, therefore, that both the direction and the velocity of the Nebula-group are identical with those of the outside stars. At all events the difference must be relatively small.

[*To be concluded*]

ON THE CAUSE OF THE DISTANCE-VELOCITY EQUATION IN STELLAR MOTIONS

By C. D. PERRINE

A series of dependences of radial velocities upon the magnitudes, spectral classes, size of proper motions, and distances of the stars have been found by various investigators in recent years which have proved most puzzling. All of these relations have been shown with more or less certainty by various methods of discussion, in addition to the two-stream or preferential motion.

The finding of what is believed to be evidence of the action of cosmical matter as a factor in the spectral conditions of the stars has led me to consider the bearing of such a hypothesis on the motions of the stars. Before discussing this bearing, however, it is necessary to attempt to determine whether these relations are to distance alone or to spectral class and absolute magnitude also. I have investigated to some extent these apparent relations, particularly the dependence upon apparent brightness, and accepted the evidence of these relations until quite recently, when I found[1] that if the ellipsoidal motion was taken into account simultaneously practically all of the other peculiarities could be satisfied upon the hypothesis that the more distant stars, as inferred from their proper motions, were in general moving more slowly than the near stars.

Adams and Strömberg[2] find that when the apparent magnitudes of F, G, K, and M stars were reduced to absolute magnitudes, by means of parallaxes measured directly, spectroscopically, or derived through proper motions, there was shown a strong relation of the radial velocities to these absolute magnitudes and a consistent difference between the velocities of the F and G stars and the K and M stars. As the element of distance which exists in the apparent brightnesses is eliminated in the absolute magnitudes, the conclusion that the dependence is upon absolute magnitude and

[1] *Astrophysical Journal*, **46**, 266, 1917. [2] *Ibid.*, **45**, 293, 1917.

spectral class appears to conflict with my own, stated above, viz., that the dependence is really upon distance after the ellipsoidal or two-stream motion has been allowed for.

Adams and Strömberg did not take into account directly stream-motion but believe that such influence upon their results from absolute magnitude is slight.[1] In my own investigation the effect of stream-motion in the nearer stars and an eccentricity or skewness in the distant stars showed so prominently that I am still in doubt whether these effects may not be sensible unless special precautions are taken to eliminate them. Some indication of such a dependence when stream-motion is not taken into account is found in my own investigation, which otherwise shows no such dependence, and will be referred to later.

The strongest evidence for doubt as to there being a dependence of velocity upon absolute magnitude is found in Tables VIII and IX of my article[2] where the data are in a form to show at once any considerable effect of absolute magnitude. These tables are reproduced in the present article as I and II. In the stars of small μ (Table I) the average μ is not given for the different groups of magnitudes, but it will probably not differ very greatly, as the same superior limit was adopted for all. ρ_2 shows no dependence upon magnitude. ρ_1 shows for the group of brighter stars a decidedly smaller value than for the fainter groups, but in the face of the other groups with many more stars can scarcely have much weight.

Assuming that the distances are essentially the same for all of these groups, the grouping by apparent magnitude becomes a grouping according to *relative absolute magnitude* also.

In Table II the range of values of μ is considerable and is taken into account. If we assume that the distances for such groups as these are proportional to their proper motions, then the values of ρ_2' and ρ_1' become the values reduced to a common unit for the three classes of apparent brightness, and therefore equivalent to *relative absolute magnitudes*.

These groupings leave the effects of two factors undetermined, viz., spectral class and the effect of the solar motion when classified according to the ellipsoidal axis. The distribution of the

[1] *Loc. cit.*, p. 302. [2] *Astrophysical Journal*, **46**, 278, 1917.

spectral types is probably sufficiently uniform, with the possible exception of the stars of 2.9 and brighter in Table I where there is probably an excess of stars of class B. In this connection it is to be noted that, in the investigation already referred to, no dependence upon spectral class was shown when streaming was directly allowed for. The effect of the neglected solar motion may be appreciable on the separate values of ρ but is not likely to be in the separate groups under either ρ_2 or ρ_1, which in the present case is the important consideration.

TABLE I*

CLASSIFICATION ACCORDING TO BRIGHTNESS

SMALL μ

	ρ_2	ρ_1	$\rho_2 : \rho_1$
	km	km	
$2^{M}.9$ and brighter...	(34) 13.5	(34) 6.2	2.18
3.0 to $5\frac{1}{2}$.........	(297) 13.6	(317) 11.8	1.15
$5\frac{1}{2}$ to $6\frac{1}{2}$..........	(179) 12.4	(169) 10.4	1.19

TABLE II*

CLASSIFICATION ACCORDING TO BRIGHTNESS

LARGE μ

	ρ_2		ρ_2'	ρ_1		ρ_1'	$\rho_2 : \rho_1$	$\rho_2' : \rho_1'$
	km	μ	km	km	μ	km		
$2^{M}.9$ and brighter.	(15) 16.6	0.″42	39.5	(28) 13.8	0.″62	23.1	1.28	1.71
	(13) 15.9	.29	54.8	(26) 13.6	.38	35.8	1.17	1.53
3.0 to $5\frac{1}{2}$.......	(96) 26.4	.38	69.3	(212) 17.6	.44	40.2	1.57	1.72
$5\frac{1}{2}$ to $6\frac{1}{2}$........	(31) 45.2	.64	70.6	(49) 22.3	.56	39.8	2.03	1.77

* ρ_2 represents the major axis of the ellipsoid, the limits being 50° from either vertex. ρ_1 represents the minor axis, the limits being from 60° to 90° from the vertices. ρ_2' and ρ_1' are the corresponding quantities reduced to a common value of μ. ρ_2/ρ_1 and ρ_2'/ρ_1' are the prolatenesses of the respective ellipsoids.

All of the fainter stars and one of the groups of brighter stars, comprising 95 per cent of the whole number (1457), show no increase of velocity with decreasing brightness, nor upon the foregoing assumptions, with decreasing absolute magnitude. The brighter stars of large μ and one group of the brighter stars of small μ are smaller than fainter stars of their respective classifications. The small number of such stars, 5 per cent of the whole, make their evidence doubtful in the face of the other, which is very definitely to the contrary.

A rough recombination of this data to neutralize the classification by stream-motion, according to the number of stars and on the assumption that such a grouping gives velocities according to absolute magnitude, shows an increase of velocity amounting to 1 km per absolute magnitude between the first and second groups, but no appreciable increase between the second and third groups. This result, although not of great weight, shows that a classification of these stars in which the peculiarities of preferential motion have not been definitely allowed for may produce a dependence upon absolute magnitude.

Further examination of Adams and Strömberg's results shows an apparent relation of velocity to size of parallax in the group of largest parallax in both the F and G group and the K and M group, as well as indications of some effect of a small number of large velocities among the absolutely fainter stars. They state:[1] "The increase in the value of the radial velocity with the absolute magnitude is shown clearly for essentially all of the groups in Table I. It is most marked in the case of the stars of large parallax, since these show the largest range in absolute magnitude, and their parallaxes are most accurately determined." From this I understand that most confidence is placed in the evidence from the stars of larger parallax, a confidence which, I think, will be shared by all investigators.

Now it is precisely in these two groups of their Table IV where the evidence seems to be doubtful. In the first place there is a small but well-defined progression in the mean parallaxes of the groups *in the same direction as the progressions in velocity. If the values of V' are reduced to unity upon the assumption of a linear relation between π and V' the continuity of the progression in the F and G stars is greatly weakened and the progression in the K and M stars destroyed.* The results are given in Table III and Fig. 1.

The method of geometric means used by Adams and Strömberg avoids, as they point out, the undue effect of large individual values of velocity. Forty-two of such inclusions appear to have been used by them, of which 31 are of absolute magnitude 3.3 and fainter, or 36 of absolute magnitude 2.5 and fainter. It will be seen that the greater portion of such stars of large velocity are of absolutely faint

[1] *Astrophysical Journal*, **45**, 295, 1917.

magnitude, and that the total number of such stars is only about 3 per cent of the whole. Some such method appears to be excellent for using all data and justified, at least in principle. If, however,

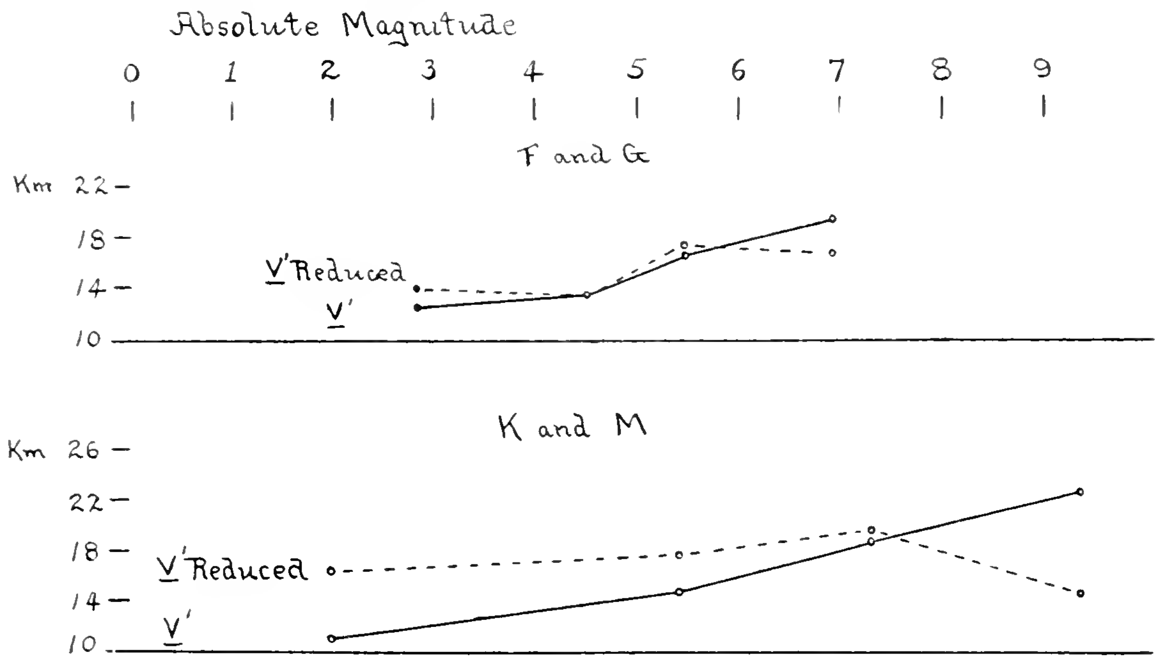


FIG. 1

TABLE III*

π	$\overline{M}$	No. of Stars	$\overline{V'}$	$\overline{V'}$ Reduced	$\underline{V'}$	$\underline{V'}$ Reduced
			F and G Stars			
			km	km	km	km
0."076	2.9	41	15.3	16.7	12.7	13.9
.082	4.47	35	16.8	17.0	13.4	13.5
.080	5.35	33	16.0	16.6	16.9	17.5
.094	6.92	33	21.2	18.8	19.2	16.9
			K and M Stars			
0."066	1.95	19	17.2	25.3	11.1	16.3
.081	5.38	38	25.7	30.7	14.8	17.8
.092	7.27	24	27.5	29.0	18.8	19.8
.149	9.34	27	28.4	18.5	22.2	14.5

* A bar above the symbol denotes the arithmetical mean. A bar below the symbol denotes the geometric mean.

the inclusion of such a small percentage of the whole has a material effect on the reality of any phenomena its inclusion in that particular case would seem not to be warranted. After the finding of the

peculiar effect in the two groups of large parallax, noted above, I tried the effect of limiting a test to the remaining 97 per cent as given in their values of $\overline{V}'$ in Table IV, where these large values had been excluded. The reduced values for the groups of large parallax are given in column 5 of Table III and the curve in Fig. 2. In neither of these does the evidence appear to be any stronger of a dependence upon M than upon π.

They conclude also[1] that "There seems to be little evidence of a decrease in velocity with distance for a constant absolute magnitude, and it appears certain that the effect if present must be relatively slight." This statement probably refers more particularly to Table IV, but seems to me to be in conflict with the results of Tables V and VI, where strong progressions are shown in π for both spectral groups. Indeed the changes in π in these tables appear to be fully as well marked as the changes in M. These characteristics as well as an abrupt change at about the middle of the groups and differences in the small π-bright M and the large π-faint M stars are better shown in the cutves in Figs. 3 and 4 which have been prepared from Table V. On account of the abrupt change mentioned and their different characteristics, the two portions of the curves were made discontinuous at the middle.

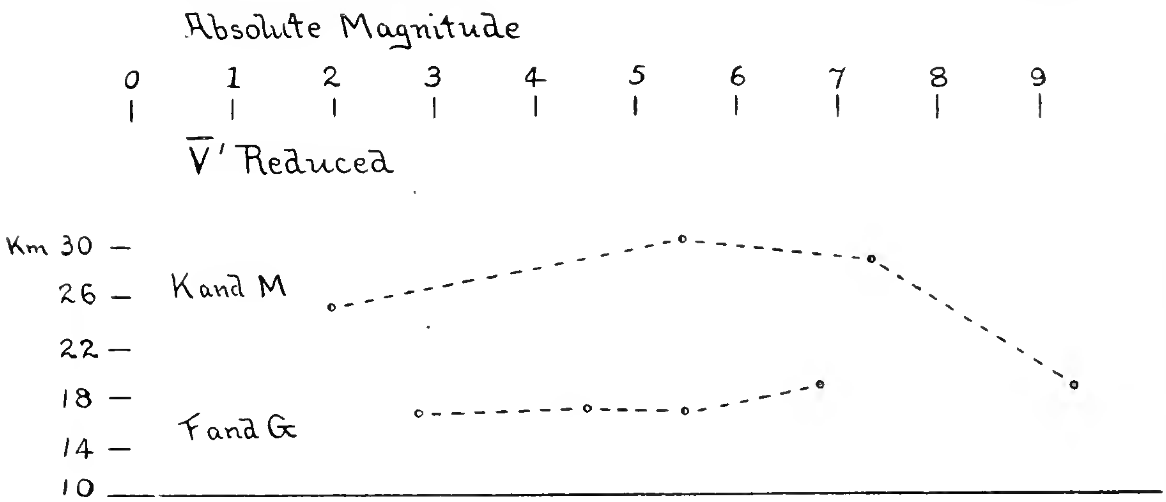


FIG. 2

These curves show a striking similarity in all parts, a similarity which is undoubtedly significant. *Both spectral groups show increases of velocity with increasing π and decreasing M for the small π*

[1] *Astrophysical Journal*, 45, p. 302.

and bright M groups and no certain change of velocity in the large π and faint M groups.

A direct interpretation of the mere fact of the progressions of π and M of Tables V and VI suggests that distance and absolute magnitude are equally concerned. The peculiarities of the curves of Figs. 3 and 4 are better satisfied, however, in my opinion by the conclusion that the dependence, whatever it is, is not a *continuous* function. This seems very clearly indicated by the curves. If

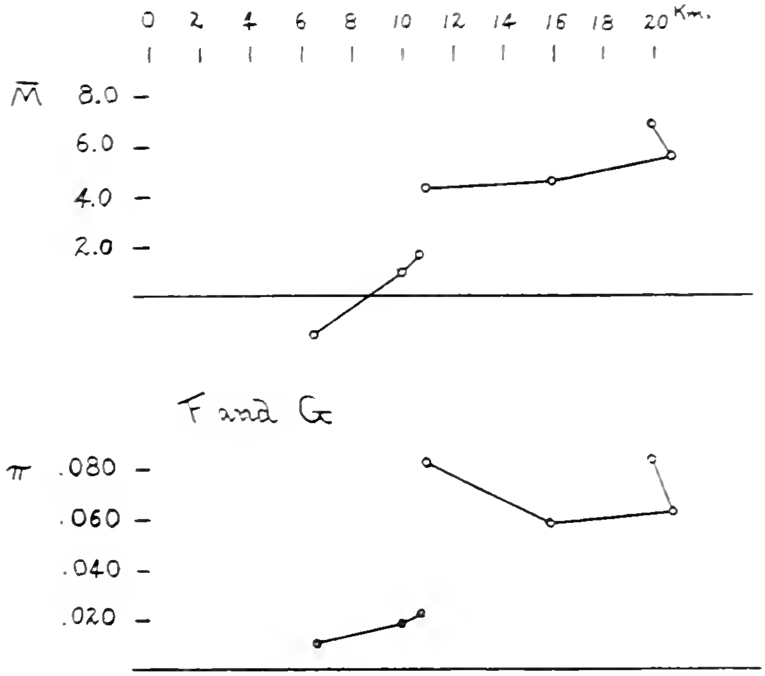


FIG. 3

such a discontinuity should be confirmed from more extensive data it would appear to be difficult to find an adequate explanation in a dependence wholly upon absolute magnitude. Some hypothesis in which distance is concerned, probably rather in the nature of regions, seems to me to offer a more satisfactory explanation of the observed facts. Taking all of these into consideration, I have no hesitancy in expressing the belief that distance in some manner plays a considerable part in the phenomena in question. If my hypothesis is correct the chief factor is distance or region, and the absolute magnitudes, spectral classes, and changes of velocity follow from the different conditions existing in the near and distant regions of our stellar system. This would explain also the very close relation of all of these factors.

The radial velocities of the stars appear to be so closely entangled with their magnitudes, spectral classes, distances, and streaming that the only hope of disentangling them completely seems to be by classifying the data simultaneously according to all of the factors concerned, including preferential motion. The considerable number of these factors necessitates so many subdivisions of the data

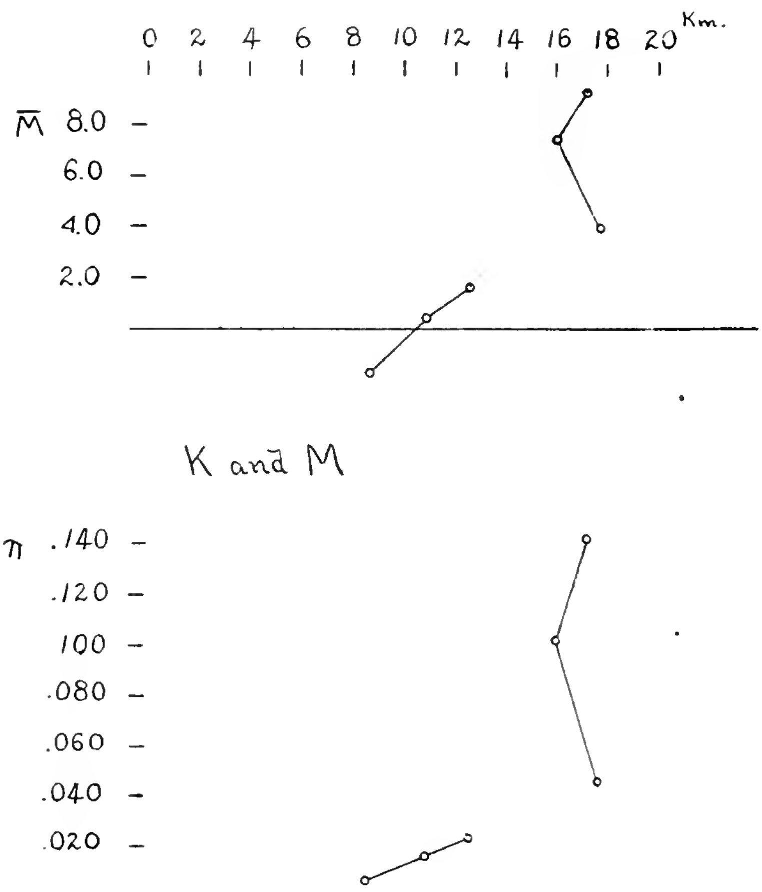


FIG. 4

as to make it desirable that the conclusion arrived at—that the dependence of velocity is primarily upon distance and streaming—should be confirmed by much more extensive data than is available at present.

Before going farther in explanation it is necessary to attempt to determine whether or not the smaller velocities for the early type, distant stars is a general phenomenon, i.e., spatial, or whether it may be explained by assuming that the line of sight may make a large angle with the direction of a stream. At present the only

evidence that I can discover which bears on this is that of the motions in the stars (the nearer ones) which show the strong preferential motion. This applies, of course, only by extrapolation.

If it is a case of stream-motion entirely and the direction of the line of sight with respect to the axis of the stream, the observer being in the stream in the one case (the near stars) and looking at the stream at a considerable angle, from a distance, in the distant stars, then the radial velocities at right angles to the stream-motion in the near stars (which show preferential motion) should be the same as the average radial velocities for the distant stars. An examination of Tables I and II shows a well-marked increase of ρ_1, (the minor axis) for the near stars.

So far as this data is competent, therefore, it indicates that the decrease of radial velocity in the distant stars is general and not due entirely to the direction which the line of sight makes with stream-motion. This appears to be independent of whatever other relations there may be between proper motion and radial velocity, so long as the larger proper-motion stars are also nearer.

The hypothesis based upon the assumption of finely divided cosmical matter in the relatively distant galactic regions, which has been suggested to account very largely for the observed differences of spectral type, also appears competent to explain the smaller velocities of the distant stars, whether this dependence is entirely upon distance or partially upon absolute magnitude and spectral class. This hypothesis may be stated briefly as follows:

The spectral condition of a star depends chiefly upon its size and mass and the external conditions of density of cosmical matter and relative velocities of star and matter. This matter is assumed to be most dense in the relatively distant regions in the direction of the galaxy.

The theory of such a change of velocity is that the cosmical matter acts as a resisting medium and *slows down* the higher velocities. This hypothesis points to a reversal of the direction of change of velocity; that instead of increasing from the early to the late types the velocities of the later type (and nearer stars) have been retarded. The operation is so obvious in principle that no more detailed explanation seems called for at this time.

No attempt is made to account for the present (higher) velocities of the nearer- and later-type stars, nor to explain the greater evolutionary changes in the motions of our stellar system. The present investigation is limited to a single, perhaps transient, phase of the great evolutionary process.

This hypothesis suggests the question, Have the stars grown up by accretion from very small beginnings, their velocities gradually slowing down during the process? If this is so, then we should expect the stars of small mass to have greater velocities than those of large mass, thus conforming to an absolute magnitude progression.

As far as the B-type stars, the reduction of velocity appears to be more or less systematic. We know little as yet of the radial velocities of the Wolf-Rayet stars and have too few velocities of the novae to make it possible to discuss these two classes with profit at present. But there are sufficient observations of the velocities of the planetary nebulae to show that the velocities of these bodies are undoubtedly large, perhaps even larger on the average than the later-type stars. These high velocities appear to be required by the hypothesis[1] put forth to account for these objects, viz., that the outburst was of unusual violence on a *single body*.

The large, irregular gaseous nebulosities appear to be nearly at rest with respect to the stellar system. But probably too few of them have been observed to serve as a sufficient basis for discussion as yet. It may not be out of place, however, to consider the cause on the assumption that low velocities should prove to be the rule. If the suggestion which has been made to account for these bodies should be confirmed, low general velocities might be expected. The suggestion was based upon the observed fact that many of the largest of these bodies have two or more bright stellar bodies more or less centrally located in the mass and was that the nebulosity had been thrown off in a collision or near-collision among these bodies. The resultant of such opposing velocities as here indicated would probably be low.

Observatorio Nacional Argentino, Córdoba
November 30, 1917

[1] "On the Cause Underlying the Spectral Differences of the Stars" (in press), *Astrophysical Journal*, 1918.

RADIAL VELOCITY AND ABSOLUTE MAGNITUDE

COMMENTS ON PROFESSOR PERRINE'S ARTICLE[1]

By WALTER S. ADAMS and GUSTAF STRÖMBERG

Professor Perrine has been so kind as to send us the manuscript of his article in advance of publication, and it will perhaps aid the reader to form a clearer idea of the points at issue if we comment briefly upon certain matters involved. The question is primarily one of the interpretation of the increase of radial velocity with proper motion. Perrine considers this as mainly an effect of distance from the sun, while we regard it as dependent upon the absolute magnitudes of the stars considered.

In Tables I and II of his article Perrine has divided the stars into two groups defined by proper motions less than 0."1 and greater than 0."1, respectively. Within these groups he arranges the stars according to apparent magnitude and gives the radial velocities along the axes of the ellipsoid. Of the stars in Tables I and II he says: "In the stars of small μ (Table I) the average μ is not given for the different groups of magnitudes, but it will probably not differ very greatly, as the same superior limit was adopted for all. . . . . In Table II the range of values of μ is considerable and is taken into account." As a result of this argument Perrine concludes that the grouping by apparent magnitude becomes a grouping according to relative absolute magnitude as well.

To this mode of treatment of the stars in Table I we are obliged to object seriously. The dispersion in distance among these stars is enormous, much greater than that for the stars in Table II, if we assume, as Perrine does, that distance is inversely proportional to proper motion. Since B stars are included by him in Table I, we may have a range of from 0."001 to 0."100 in the proper motion, or a range in distance of 100 to 1 on this basis. It would, in fact, be more correct to assume that the stars within the zone of proper motions larger than 0."1 are all at the same distance than to assume

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 146.

this for the more distant stars without proof. It is therefore not possible to conclude that the grouping of stars according to apparent magnitude in Table I corresponds to a grouping according to absolute magnitude.

A second very natural objection to Perrine's method of applying to the radial velocities factors which will reduce all the stars to the same proper motion (and distance) is the nature of the consequences involved. If a group of stars with an average proper motion of 0.″3 has one-half the average radial velocity of a group with a proper motion of 0.″6, the velocity of a group with proper motion of 0.″005 will be far less than any average observed radial velocity. It would be necessary to assume that the change of velocity with distance is greatest near the sun. This involves the remarkable assumption that the sun is a singular point in space (at the present time), and also it is opposed to Perrine's hypothesis that cosmical matter acts as a resisting medium, since he supposes this matter to lie in relatively distant galactic regions.

In a discussion of our results Perrine concludes that in the two groups of stars of types F and G, and K and M, which have the largest parallaxes, there is a "small but well-defined progression in the mean parallaxes of the groups in the same direction as the progression in velocity." This is quite true, but this progression corresponds to an exceedingly small range in distance when compared with that of the more distant stars. The two groups to which he refers have parallaxes larger than 0.″05. If velocity is inversely proportional to distance within this comparatively narrow zone, we should have a velocity approaching zero at a relatively small distance, a conclusion which is contradicted by observation.

Perrine selects from our results only two groups of stars from a total of thirteen, since the parallaxes are most reliable in the case of the larger values. No doubt the distances of the nearer stars are known more accurately, but it is quite inadmissible to neglect the evidence afforded by the stars of small parallax. One of the principal advantages of the spectroscopic method of deriving parallaxes is that it is independent of their absolute values. Accordingly the relative determinations of distance, even among the stars

of smallest parallax, should be entitled to considerable weight. We find, however, that the average radial velocity for a group of over 200 stars with an average parallax of about 0".010 is practically the same as that for a similar group with a parallax of 0".050 of the same absolute magnitude.

Two other statements by Professor Perrine require a word of comment. His reference to Tables V and VI of our previous article as indicating increase of radial velocity with increase of parallax is due to a misconception of their purpose. Among the very distant stars we cannot observe stars of very low absolute magnitude, since such stars are apparently too faint. Accordingly Tables V and VI naturally show a progression of magnitude with increase of parallax and radial velocity. The attempt to separate the effect of distance from that of absolute magnitude is confined to the previous discussion, and the results are contained in Table IV.

As regards the effect of stream-motion upon our velocities, no attempt was made to eliminate it completely from our earlier results. There can, however, be no doubt that its influence was reduced greatly by our inclusion of velocities at right angles to the line of sight, and the confirmatory evidence obtained from these values is of considerable importance. An extended investigation of stream-motion as related to absolute magnitude has been made by Strömberg and is now in press. Its results are in agreement with our previous conclusions.

In general, it may be said that the principal cause of difference between Professor Perrine and ourselves is one of point of view. We have attempted in our discussion of stellar motions to find a fundamental relationship which would apply over the entire range of distance and absolute magnitude which has been investigated. For the reasons already given, it does not seem possible to conclude that a law making radial velocity inversely proportional to distance from the sun can hold over more than a very limited range. Outside of a comparatively narrow limit it would give values of the radial velocity much too high or too low, unless the law were modified greatly. Moreover, such a relationship would assign to the sun an extraordinary position in the stellar system, which we have no reason for believing that it holds. It is a star of very

moderate size and mass, and is known to lie at a great distance from the center of the galaxy. It would seem far more probable that any relationship between velocity and distance would be one in which the latter was measured from some much more fundamental reference point than our sun.

Mount Wilson Solar Observatory
February 1918

ORBIT OF THE SPECTROSCOPIC BINARY 32 θ_2 CYGNI

By J. B. CANNON

32 θ_2 Cygni ($\alpha = 20^h\ 12^m$, $\delta = +47°24'$; mag., 5.15; type G5) was announced a binary in *Lick Observatory Bulletins*, **4**, 96, from four measures giving a range of 30 km. Later in *Astronomische Nachrichten*, **198**, 409, 1914, Küstner published three measures of plates taken in 1908, 1909, and 1911, having a range of 40 km. The star was under observation here during the years 1914, 1915, 1916, and 1917, during which time 117 plates were taken.

The following determination of the elements must be considered as preliminary. The period is so long, 1170 days, that the measures were only beginning to repeat themselves at the end of the fourth year. More observations several years hence may change the period by some days, but the other elements as determined are probably fairly accurate. Five plates taken in 1908 helped in the determination of the period.

The wave-lengths adopted for the lines used are given in Table I.

TABLE I

4571.896	4395.461	4289.872
4549.646	4351.674	4282.858
4523.052	4340.705	4271.888
4501.865	4326.076	4215.906
4415.200	4304.891	4128.108
4404.042	4294.514	4101.653

The plates were grouped into twelve normal places found in Table II.

A curve running through the normal places has some similarity to that which would be given by the blending of the lines of the primary star by those of a secondary star, but the deviation from the elliptic form takes place unsymmetrically in such a way as to be hardly thus accountable. Also there appears to be no widening

of the lines at any place in the orbit and it seems as though the irregularities in the curve are due to actual variations in the motion of the light-giving body. It was therefore attempted to run an elliptical curve as closely as possible through the normal places and

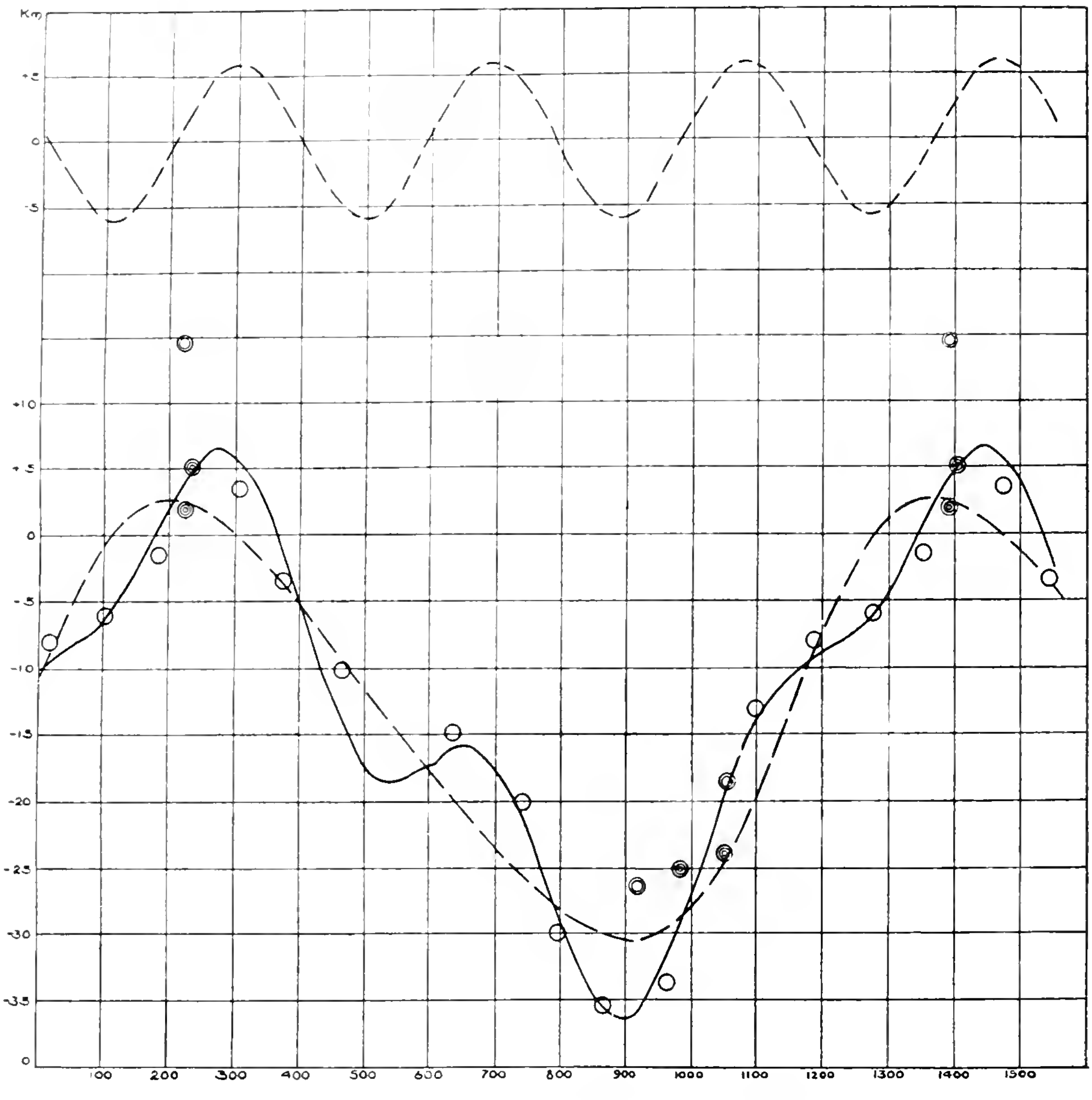


Fig. 1.—Radial velocity-curve of 32 θ_2 Cygni

apply a secondary correction to this by introducing a circular curve of one-third the period (Fig. 1).

A least-squares solution was carried through, which resulted in slight corrections to the elements and a reduction of the value of Σpvv from 75 to 60. The preliminary and final values of the

TABLE II

No.	Julian Day	Phase	Velocity	Weight	Residual
			km		km
1	2420510.835	795.708	−30.1	4	−1.6
2	543.658	867.018	−35.4	3	+0.2
3	495.178	964.788	−33.6	1	−2.1
4	629.599	1099.209	−13.1	1.5	+0.8
5	716.627	16.237	− 7.8	2	+1.3
6	801.337	100.947	− 5.9	2	±0.0
7	882.701	182.311	− 1.4	1	−2.1
8	1007.588	307.198	+ 3.7	1	−1.9
9	1076.982	376.592	− 3.4	1.5	−1.7
10	1166.664	466.274	−10.1	1	+4.5
11	0272.465	635.711	−14.9	2	+1.6
12	1443.212	742.822	−20.1	2	+1.6

elements are given below. Probable errors were computed and are appended to the final values.

Element	Preliminary	Final
P	1170 days	1170 days
e	0.2	0.182 ± 0.053
ω	280°	$281^\circ.05 \pm 4^\circ.8$
K	16 km	16.64 ± 0.93 km
T	2420700.25 J.D.	$2420700.39 \pm 18^d.7$
γ	−14.6 km	-14.35 ± 0.65 km
K_1	6 km	5.86 ± 0.90 km
T_1	2420515 J.D.	$2420515.821 \pm 12^d.4$
$a \sin i$		263250000 km
$\frac{m^3_1 \sin^3 i}{(m+m_1)^2}$		0.53 ⊙

To explain the curves one would have to consider the system as consisting of a light-giving body revolving about another body in a circular orbit in 390 days, and these two about a third in an elliptic orbit in 1170 days (Fig. 2).

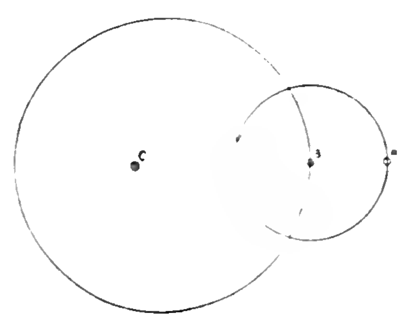


FIG. 2

In Fig. 1 the single circles represent the Ottawa normal places, the double circles Küstner's observations, and the triple circles the Lick observations.

DOMINION OBSERVATORY, OTTAWA, CANADA
February 1918

ON THE ORBIT OF THE SPECTROSCOPIC BINARY ALPHA PHOENICIS

By JOSEPH LUNT

The variable velocity of this star ($\alpha = 0^h 21^m.3$, $\delta = -42°51'$ [1900]; magnitude $= 2.44$; type K) was announced in *Lick Observatory Bulletin*, No. 75 (**3**, 110) by Wright. Five plates had then been secured in 1903–4 showing a range of 5.5 km. The star has been under observation here from November 27, 1903, to December 28, 1914, giving an observed range of 11.2 km.

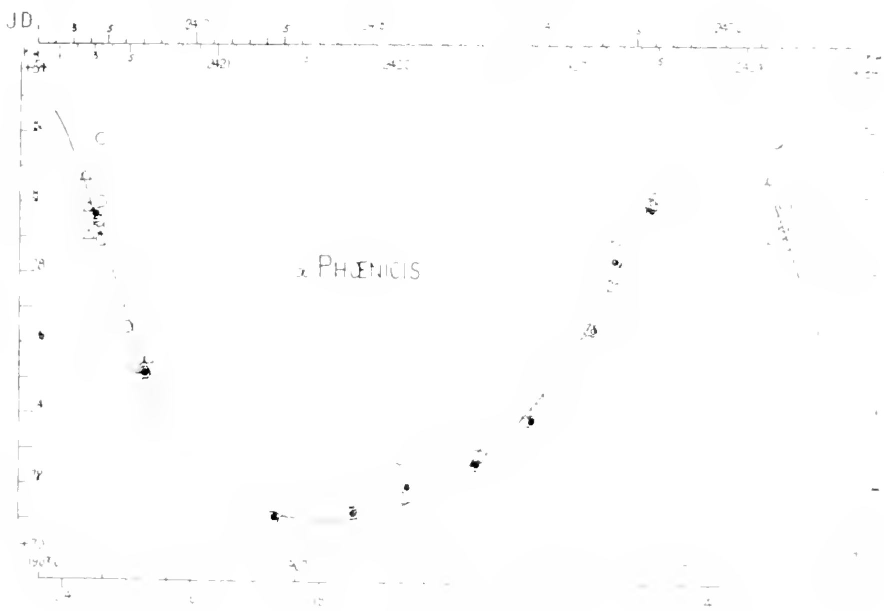
The last plate showed that a cycle had been completed, and as no further observations or orbit have been published, so far as the writer is aware, it seems desirable to publish the results obtained and to give a preliminary orbit which fits the present observations satisfactorily. It is regretted that observations were interrupted after July 18, 1912, when the maximum velocity was approaching, and the peak of the curve was not observed. A single plate on December 28, 1914, however, has served to indicate the period, and the following orbit was derived by graphical methods.

$$P = 3880 \text{ days}$$
$$= 10.62 \text{ years}$$
$$T = \text{J.D. } 2416185$$
$$= 1903 \text{ March } 11$$
$$\omega = 20°$$
$$e = 0.32$$
$$K = 6.0 \text{ km}$$
$$V_0 = +75.76 \text{ km}$$
$$a \sin i = 308{,}000{,}000 \text{ km}$$

α Phoenicis is a remarkable star in some respects. If we include visual binaries, it appears to be second only to σ Puppis[1] in

[1] See *Astrophysical Journal*, **44**, 260, 1916.

To accompany article in the April number by Joseph Lunt entitled "On the Orbit of the Spectroscopic Binary Alpha Phœnicis."



Velocity-curve of the spectroscopic binary α Phoenicis

(To face page 107 in the April number.)

velocity of the center of mass, to β Capricorni in size of orbit, and to Polaris in length of period. The data compare as follows.

	P	e	$a \sin i$	K	V_0
	Years		km	km	km
α Phoenicis......	10.62	0.32	308,000,000	6.0	+75.8
σ Puppis........	0.706	0.20	62,570,000	18.0	+88.5
Polaris*.......	11.9	0.35	166,800,000	2.98	−14.8
β Capricorni.....	3.77	0.44	377,000,000	22.2	−18.8

* Center of mass of short-period binary system.

The orbit given indicates that the radial velocity is now at a minimum (January 1918) and increasing to a maximum in February 1924. The velocities given in the column of remarks of Campbell's second list of binaries, viz., 1903.7, +80 and 1907.8, +70, are half a kilometer and a kilometer lower, respectively, than the present orbit demands but agree within the errors of observation.

Probably other observations have been made at Santiago and it will be of interest to fill in the lacunae.

Table I shows the observations compared with the computed velocities. Of the 44 plates considered, including 5 Santiago plates, 25 give residuals from 0.0 to 0.5; 14 from 0.6 to 1.0; and 5 above one kilometer. The measures were made by the writer with the Hartmann spectrocomparator and solar standards (daylight spectra) after 1908, and α Bootis, Plate No. 1191, before the end of that year.

The plates marked * were also measured by Dr. Halm, using α Tauri, Plate No. 2145, as standard, and the means of his measures and the writer's were adopted.

The upper scales of years and Julian days in the figure refer to the epoch of observation and the lower ones to the curve regarded as an ephemeris commencing with the periastron of October 24, 1913. The black-centered circles refer to means of groups of plates. The triangles represent Santiago measures.

TABLE I

Plate No.	Date	J. D. 2400000 +	Phase Days	Radial Velocities Observed	Radial Velocities Computed	Residuals O−C
				km	km	km
—‡	1903 Sept. 15*	16373	.188	+80.7	+80.5	+0.2
—‡	Oct. 1‖	389	204	79.0	80.2	−1.2
—‡	5*	393	208	79.8	80.2	−0.4
§ 615	Nov. 27	446	261	79.3	79.3	0.0
§ 616	Dec. 2	451	266	79.5	79.3	+0.2
§ 617	12	461	276	80.0	79.1	+0.9
§ 618	15	464	279	81.8**	79.1	+2.7
§ 619	17	466	281	78.9	79.0	−0.1
§—‡	1904 Aug. 3¶	696	511	75.2	75.8	−0.6
§ 697	26	719	534	74.8	75.5	−0.7
§—‡	Sept. 10	734	549	75.5†	75.3	+0.2
§ 915	1906 May 29	17360	1175	70.5	71.4	−0.9
§1021	Oct. 22	506	1321	71.8	71.1	+0.7
§1022	23	507	1322	71.0	71.1	−0.1
§1287	1907 Nov. 5	885	1700	73.3**	71.1	+2.2
§1294	8	888	1703	71.6	71.1	+0.5
§1348	Dec. 17	927	1743	70.4	71.1	−0.7
1824*	1908 Aug. 13	18167	1982	72.8	71.4	+1.4
1871*	Sept. 1	186	2001	70.6	71.5	−0.9
1881*	3	188	2003	72.7	71.5	+1.2
1885*	4	189	2004	71.7	71.5	+0.2
2081*	Dec. 17	293	2108	72.1	71.7	+0.4
2494	1909 Sept. 8	550	2365	72.8	72.6	+0.2
2535	Oct. 14	594	2409	72.3	72.7	−0.4
2548	22	602	2417	73.2	72.8	+0.4
2553	29	609	2424	71.8	72.8	−1.0
2584*	Nov. 17	628	2443	72.9	72.9	0.0
2762	1910 July 26	879	2694	73.4	74.1	−0.7
2765	27	880	2695	73.9	74.1	−0.2
2768	31	884	2699	74.1	74.1	0.0
2837	Oct. 4	949	2764	74.6	74.5	+0.1
2842	7	952	2767	73.5	74.5	−1.0
3220	1911 July 9	19227	3042	76.2	76.6	−0.4
3225	16	234	3049	76.5	76.6	−0.1
3308	Oct. 10	320	3135	76.7	77.4	−0.7
3355	Nov. 16	357	3172	77.7	77.8	−0.1
3378	Dec. 6	377	3192	78.8	78.0	+0.8
3387	9	380	3195	77.9	78.0	−0.1
3399	19	390	3205	79.0	78.0	+1.0
3651*	1912 June 17	571	3386	80.0	80.0	0.0
3665*	23	577	3392	79.2	80.1	−0.9
3682*	July 1	585	3400	80.0	80.2	−0.2
3720*	18	602	3417	80.2	80.3	−0.1
4467*	1914 Dec. 28	20495	0455	76.5	76.8	−0.3

* Curtis. † Correction +1.1 km applied. ‡ D. O. Mills Expedition.
§ From *Annals of the Cape Observatory*, **10**, 52. ‖ Palmer. ¶ Wright. ** Weight ½.

ROYAL OBSERVATORY, CAPE OF GOOD HOPE
February 5, 1918

Minor Contributions and Notes

DESIRABILITY OF OBSERVATIONS OF THE SOLAR ROTATION AT THE TIMES OF ECLIPSE

In the lines of spectra from the solar limb there is blending of such nature as to affect the measurements of the velocity of the sun's rotation from the displacements of these lines. The magnitude of the effect varies from line to line. Various factors contribute to the blending of spectra.[1] The factors operating between the moon and the point of observation are nearly eliminated just before and after totality, when only the limb under observation is visible. To investigate the problem a powerful spectroscope (producing spectra of scale about 1 mm to 1 A) is necessary and a large image of the sun preferred. The region of the *b*-group is suggested because the effect of blending is known for the *b*-lines and others near them, and because iodine and chlorine comparison spectra are available. It is very desirable that such observations be made at the eclipse of June 8, so that terrestrial sources of blended spectra may be removed in order to investigate the solar and interplanetary sources. The observations need not interfere with those during the precious seconds of totality.

At least one party is planning[2] to make determinations of the speed of rotation of the corona from displacements of the coronal lines at the time of total eclipse on June 8. In view of the fact that Deslandres, Campbell, and Bosler have obtained values of this speed of from 3 to 4 km per second, it is very important that every precaution be taken to keep check on the errors that beset such measurements. In this regard I would suggest that the spectroscope be used to determine the velocity of rotation of the reversing layer by observations of the east and west limbs of the

[1] *Journal of the Royal Astronomical Society of Canada*, **10**, 201–219, and 345–357, 1916; *Astrophysical Journal*, **44**, 177–189, and 198–199, 1916.

[2] Edwin B. Frost, *Popular Astronomy*, **26**, 110, 1918.

sun before and after totality of eclipse, using a comparison spectrum and employing precisely the same method in determining the velocity from the coronal lines. In this way one can judge as to the reliability of the latter. If it is impossible to use an arc or a spark comparison spectrum, the stronger iodine absorption lines or the narrow strips of continuous spectrum between them may be found suitable in the neighborhood of λ 5303.

RALPH E. DE LURY

DOMINION ASTRONOMICAL OBSERVATORY, OTTAWA
April 2, 1918

THE
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AND ASTRONOMICAL PHYSICS

VOLUME XLVII — MAY 1918 — NUMBER 4

THE RADIAL VELOCITIES OF 60 SOUTHERN STARS

By JOSEPH LUNT

At the beginning of the year (1917) a new observing program was commenced, embracing all stars south of the equator of types F, G, K, and M, down to magnitude 5.5 in the *Harvard Revised Photometry* (50) for which radial velocities had not been published by Campbell[1] or Adams[2] in their lists of 915 and 500 stars, respectively, or previously observed here. Five stars from Campbell's *Second Catalogue of Spectroscopic Binary Stars*[3] have been included, as well as 22 others subsequently announced in *Lick Observatory Bulletins* to be variable in velocity.

The 24-inch refractor, in conjunction with the four-prism star spectrograph, was employed, the short camera of 16-inch (40.6 cm) focus being used for the first time. The prisms are of light flint and give spectra of approximately the same linear scale as is given by the two dense prisms and short cameras used with the 60-inch reflector at Mount Wilson.

[1] The Radial Velocities of 915 Stars, *Lick Observatory Bulletin*, No. 229.

[2] The Radial Velocities of 500 Stars, *Mt. Wilson Contr.*, No. 105.

[3] *Lick Observatory Bulletin*, No. 181.

TABLE I

H.R. No.	Star's Name	R.A. 1920	S. Decl. 1920	Mag.	Type	No. of Plates	Epoch 1917+	Radial Velocity	Solar§ Motion Correction	Corrected Radial Velocity	Range
								km	km	km	km
3	33 Piscium‡	0^h 1^m	6° 0′	4.68	K	3	.888	− 17.2	− 1.2	− 18.3	0.7
37	*Cetus*	0 8	18 23	5.47	K	2	.928	− 8.0	− 3.7	− 11.7	4.2
412	46 Ceti	1 22	15 1	5.19	K	3	.011	− 23.1	− 8.4	− 31.5	2.3
500	*Cetus*	1 39	4 5	5.27	K	2	.022	− 33.5	− 7.9	− 41.4	1.0
539	ζ Ceti‡	1 48	10 44	3.92	K	2	.495	+ 8.2	− 9.5	− 1.4	1.7
994*	15 Eridani	3 15	22 48	5.05	K	3	.068	+ 25.1	−15.9	+ 9.3	2.4
1787	p Orionis	5 20	0 58	5.15	K	3	.112	+ 21.6	−17.2	+ 4.3	4.4
1836	λ Doradus	5 25	58 59	5.06	K8	2	.144	+ 10.5	−17.4	− 6.9	3.2
2087	ξ Columbae‡	5 53	37 8	5.02	K	2	.147	+ 56.9	−19.8	+ 37.0	3.1
2140*	*Lepus*	6 0	26 17	5.18	G5	3	.123	+178.9	−20.0	+158.9	3.0
2275*	*Orion*	6 16	2 54	5.18	Ma	3	.137	+ 47.4	−17.8	+ 29.7	1.5
2469	*Monoceros*	6 38	9 5	5.32	Kp	3	.205	+ 3.9	−18.5	− 14.6	3.4
2574*	θ Can. Maj.	6 50	11 56	4.25	K5	5	.159	+ 98.7	−18.6	+ 80.1	2.7
2652	23 Carinae	6 59	51 18	5.02	Mb	2	.147	+ 3.8	−18.3	− 14.5	2.0
2701	20 Monocerotis	7 6	4 7	5.02	G5	2	.202	+ 78.8	−17.3	+ 61.6	1.0
2934*	Q Carinae	7 34	52 21	4.92	K5	3	.202	+ 62.2	−17.6	+ 44.6	5.2
2959	*Puppis*	7 37	15 4	5.15	K	2	.250	+ 4.8	−17.9	− 13.0	3.1
3123	12 Puppis	7 56	23 5	5.22	G5	2	.208	+ 11.7	−17.9	− 6.2	2.8
3229	20 Puppis	8 10	15 32	5.05	K	2	.324	+ 18.1	−16.8	+ 1.4	4.0
3484	D Hydrae‡	8 43	13 15	4.44	G5	2	.257	− 9.5	−15.1	− 24.6	0.3
3681*	23 Hydrae	9 13	6 1	5.40	K5	3	.307	− 9.0	−12.5	− 21.6	2.0
3765	ε Antliae‡	9 26	35 35	4.64	K2	3	.307	+ 22.0	−14.6	+ 7.4	3.8
4159	r Carinae‡	10 33	57 9	4.54	K5	2	.348	+ 8.4	−11.9	− 3.5	2.7
4499	*Centaurus*‡	11 37	61 39	4.88	G	2	.451	+ 15.5	− 9.6	+ 5.8	2.6
4523	*Centaurus*	11 43	40 4	5.04	G	2	.465	+ 17.3	− 7.4	+ 9.8	1.1
4682	F Centauri‡	12 15	54 42	4.98	Ma	2	.481	− 3.1	− 7.5	− 10.6	2.6
4699*	*Corvus*	12 17	13 7	5.36	K	3	.432	+ 16.2	− 1.1	+ 15.2	3.5
4955	49 Virginis	13 4	10 18	5.26	K	2	.440	− 7.4	+ 2.9	− 4.6	3.5
5095*	74 Virginis	13 28	5 51	4.83	Mb	3	.494	+ 17.9	+ 5.4	+ 23.3	4.3
5172	M Centauri‡	13 42	51 2	4.68	K	2	.465	+ 0.3	− 3.1	− 2.9	0.2
5301	*Virgo*	14 7	15 56	5.10	Ma	4	.522	+ 18.0	+.6.0	+ 24.0	3.9

5371	*Centaurus*‡	14 17	58 5	4.89	G	2	.539	+ 10.0	− 3.4	+ 6.6	5.7
5444	*Lupus*	14 31	45 54	5.41	K2	3	.570	− 55.6	+ 0.2	− 55.4	0.9
5535*	11 Librae	14 47	1 58	5.00	K	3	.520	+ 84.9	+11.2	+ 96.1	0.6
5777	37 Librae	15 30	9 47	4.83	K	3	.448	+ 48.2	+11.8	+ 60.1	2.5
5824	42 Librae	15 36	23 34	5.06	K	3	.610	− 18.5	+ 8.8	− 9.7	5.6
6017	*Scorpio*	16 6	29 12	5.16	G	3	.652	− 24.9	+ 8.4	− 16.5	1.0
6048*	χ Scorpii	16 9	11 38	5.50	G5	3	.599	− 22.2	+13.0	− 9.2	3.2
6128*	*Ophiuchus*	16 23	7 26	5.45	Ma	2	.641	+ 99.8	+14.4	+114.2	1.4
6166	H Scorpii	16 31	35 5	4.30	Ma	3	.580	− 1.3	+ 7.4	+ 6.1	5.5
6196	24 Ophiuchi	16 37	17 35	5.04	K	4	.615	− 23.1	+12.4	− 10.6	3.4
6424	*o* Ophiuchi	17 13	24 12	5.39	K	3	.666	− 27.4	+11.4	− 16.0	1.7
6510	*Ophiuchus*	17 26	0 59	5.34	G	2	.692	− 70.7	+17.0	− 53.8	1.4
6693	*Sagittarius*	17 54	30 14	5.27	K5	2	.662	− 17.8	+ 9.9	− 7.9	0.8
6761*	ι Pavonis	18 3	62 1	5.48	G	3	.702	+ 26.3	− 0.7	+ 25.6	1.3
6801	1 Sagittarii	18 7	23 43	5.13	K	3	.655	+ 4.9	+11.8	+ 16.7	2.2
6896	21 Sagittarii‡	18 21	20 35	4.96	G5	2	.699	− 10.3	+12.6	+ 2.3	1.9
7050	μ Coronae Aust.	18 42	40 30	5.28	G	2	.680	− 20.2	+ 6.5	− 13.7	3.4
7116*	ν′ Sagittarii	18 47	22 51	4.96	G5	2	.747	− 12.0	+11.7	− 0.3	1.9
7127	ω Pavonis	18 52	60 19	5.14	K	2	.761	+177.0	− 0.3	+176.6	0.0
7304	d Sagittarii	19 13	19 6	5.03	K5	3	.696	+ 16.4	+12.3	+ 28.7	1.8
7515*	f Sagittarii	19 42	19 57	5.06	K	4	.712	+ 20.9	+11.3	+ 32.2	0.9
7597	ω Sagittarii	19 51	26 31	4.81	G5	2	.706	− 14.0	+ 9.3	− 4.7	1.8
7604	b Sagittarii‡	19 52	27 23	4.62	K2	3	.688	− 17.6	+ 9.0	− 8.6	0.6
8283	42 Capricorni	21 37	14 24	5.28	K	17	.826	− 3.0†	+ 7.3	+ 4.3†	48.7
8311*	c Capricorni	21 41	9 27	5.28	K	2	.880	− 5.0	+ 8.1	+ 3.1	2.8
8644	ρ Gruis	22 39	41 50	4.89	K	3	.829	+ 29.0	− 2.2	+ 26.8	5.1
8841*	ψ′ Aquarii	23 12	9 32	4.46	K	3	.811	− 22.9	+ 1.9	− 21.0	1.1
8866	94 Aquarii	23 15	13 54	5.27	K	3	.881	+ 5.6	+ 0.9	+ 6.5	4.7
9103	3 Ceti	24 0	10 57	5.16	K	3	.912	− 40.5	− 1.9	− 42.4	3.5
Mean											2.6

* Check stars.

† Velocity of the center of mass of the system (see *Astrophysical Journal*, **47**, 134, 1918).

‡ Previously announced as variable in velocity (*Lick Observatory Bulletin*).

§ The solar apex assumed as 18^h, +30°, and velocity 20 km per second.

‖ Difference between extremes of separate plates.

The linear scale of the spectra from the following iron lines toward the red was:

λ	A per mm
4247.6	17.2
4340.6 (Hγ)	19.3
4376.1	20.1
4528.8	24.1

The measures were made with the Hartmann spectrocomparator in the region between the foregoing lines, with use of the 45 mm objectives, without extension tubes, and the low-power eyepiece. The magnification employed is 20 diameters, and the elevation scale-reading (W) is 13.

TABLE II

Comparison of Results* for 16 Stars

H.R. No.	Mag.	Type	No. of Plates	Radial Velocities			Differences	
				Lunt (Cape) (1)	Campbell (Lick) (2)	Adams (Mt. Wilson) (3)	(1)−(2)	(1)−(3)
				Km	Km	Km	Km	Km
994	5.05	K	3	+25.1	+26.3	....	−1.2	....
2140	5.18	G5	3	+178.9	+183	....	−4.1	....
2275	5.18	Ma	3	+47.4	....	+48.3	....	−0.9
2574	4.25	K5	5	+98.7	+96.7	....	+2.0	....
2934	4.92	K5	3	+62.2	+61.1	....	+1.1	....
3681	5.40	K5	3	−9.0	....	−7.3	....	−1.7
4699	5.36	K	3	+16.2	....	+12.5	....	+3.7
5095	4.83	Mb	3	+17.9	+19.2	−19.1†	−1.3	−1.2
5535	5.00	K	3	+84.9	+83.2	....	+1.7	....
6048	5.50	G5	3	−22.2	....	−26.3	....	+4.1
6128	5.45	Ma	2	+99.8	....	+97.1	....	+2.7
6761	5.48	G	3	+26.3	+31	....	−4.7	....
7116	4.96	G5	2	−12.0	−12.0	....	0.0	....
7515	5.06	K	4	+20.9	+23‡	+16.5‡	−2.1	+4.4
8311	5.28	K	2	−5.0	....	−6.5	....	+1.5
8841	4.46	K	3	−22.9	−26.9	−28.4	+4.0	+5.5
Mean							−0.5	+2.0
Excluding 8841, possibly variable							−0.9	+1.6

* Adams has compared results for 26 stars with Lick values and obtains, for Lick − Mt. Wilson,
F and G types, 14 stars, +1.6
K and M types, 12 stars, +0.4 *Mt. Wilson Contr.*, No. 105, p. 14.

† Sign of velocity taken as plus.

‡ Difference suspected as due to variability by Adams.

Under these conditions the same mirror strips between the prisms can be used as were employed with the spectra obtained

with the long camera. The micrometer screw is of half-millimeter pitch and one division on the head (0.01 rev.), approximately 1/5000 of an inch, is equivalent to a radial velocity shift of 6.6 km per second in the mean of the 12 settings. The mean range, difference between highest and lowest value of velocity given by separate plates, is 2.6 km per second. Plate 4903 of α Tauri was used as standard plate throughout, the shift being taken as +82.34 km per second. As a check on the results a number of stars for which radial velocities have been published by Campbell or Adams or both were included, and 16 of these have been observed. Table II shows a comparison of the results obtained. The Cape results appear to lie between those of Lick and Mount Wilson.

Table I gives the provisional velocities of 60 stars for which two or more spectra have been measured. Only one of these stars shows distinct evidence of variable radial velocity during the period of observation, viz., 42 Capricorni, and for this a preliminary orbit has been computed (*Astrophysical Journal*, **47**, 134, 1918).

Other stars may prove to be variable in velocity when the period of observation is extended, and it is noteworthy that 12 of these stars, marked ‡, are variables previously announced. All the computations were made to the second decimal place in kilometers per second and rounded off to a tenth in the tables.

In addition to those stars already known to have high radial velocities the following may be noted:

H.R. No.	Star's Name	Magnitude	Type	Radial Velocity	Solar Motion Correction	Corrected Radial Velocity
				Km	Km	Km
7127.....	ω Pavonis	5.14	K	+177.0	− 0.3	+176.6
2701.....	20 Monocerotis	5.02	G5	+ 78.8	−17.3	+ 61.6
6516.....	*Ophiuchus*	5.34	G	− 70.7	+17.0	− 53.8

These stars have small proper motions.

Messrs. Woodgate and Baines took part in exposing the plates. The measures were made by the writer.

Royal Observatory, Cape of Good Hope
December 31, 1917

THE GENERAL MAGNETIC FIELD OF THE SUN

APPARENT VARIATION OF FIELD-STRENGTH WITH LEVEL IN THE SOLAR ATMOSPHERE[1]

By G. E. HALE, F. H. SEARES, A. VAN MAANEN, AND F. ELLERMAN

The preliminary results of a study of the Zeeman effect due to the general magnetic field of the sun have been given in a previous paper.[2] With the aid of suitable polarizing apparatus, used in conjunction with the 75-foot spectrograph of the 150-foot tower telescope, four lines in the third-order spectrum of an excellent Michelson grating were found to show displacements corresponding in sign and agreeing closely in magnitude with theoretical values calculated for a uniformly magnetized sphere. The extreme minuteness of the displacements, usually less than a thousandth of an angstrom, led us to defer final acceptance of the provisional conclusions until they could be rigorously tested by additional measures. The present paper contains results which amply confirm those previously published, and reveal a relation between the intensity of the sun's general field and the character of the spectral lines used for its determination. The stronger lines give smaller values of the field-strength, and, since the intensity of a line depends upon the level in the solar atmosphere at which it originates, it is natural to interpret these differences in field-strength as a consequence of differences in level.

In the original paper only four lines were shown to have displacements attributable to the general field of the sun. A number of others, mainly stronger lines known from laboratory investigations to have large Zeeman separations, showed no corresponding solar displacements. Fortunately an explanation of this apparent contradiction was offered in the circumstance that the displaced lines probably originate at a low level in the solar atmosphere, while

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 148.

[2] Hale, *Mt. Wilson Contr.*, No. 71; *Astrophysical Journal*, **38**, 27, 1913. See also Hale, *Terrestrial Magnetism*, **17**, 173, 1912.

the others correspond to higher levels where the field is too weak to be detected.

Although the hypothesis that the general field decreases rapidly in intensity with increasing elevation has usually proved a reliable guide in the choice of additional lines, caution must be exercised in accepting this interpretation of the differences in field-strength, inasmuch as the measures may perhaps be affected by systematic errors depending upon line-intensity. The question is one of much complexity, and the relevant evidence now available is presented in a later section. Although a final answer cannot now be given, this uncertainty in no wise affects the two main results of the investigation. So far as there may be doubt, it is associated with the interpretation of the second result—the relation between field-strength and line-intensity—and, perhaps involved with this, is the puzzling circumstance that certain lines having large displacements in the third-order spectrum show little or no displacement in the second order.

Before proceeding to a discussion of the observations, certain details concerning the measurement of the displacements require consideration.

1. CONFIRMATION OF DISPLACEMENTS BY OTHER OBSERVERS

The observations and measures have always been arranged so as to avoid vitiating influences that might arise from a knowledge of the observing conditions. Nevertheless we have considered it of the utmost importance to obtain all possible confirmation of the measured displacements, which are so small that it is difficult to obtain definite evidence of their reality. The lines are wide in comparison with their shifts, and when measures are first undertaken the accidental errors are usually so large as to mask completely the quantities to be observed, which even after much practice remain for many observers below the limit of perception. Thus five members of the Observatory's staff have made more or less extensive series of measures without obtaining a positive result. On the other hand an equal number of other members of our staff have produced evidence of the objectivity of the displacements and of their agreement with the

hypothesis that the sun behaves approximately as a magnetized sphere.

Besides those whose measures were described in *Mount Wilson Contribution*, No. 71, Miss Richmond and Miss Felker of the Com-

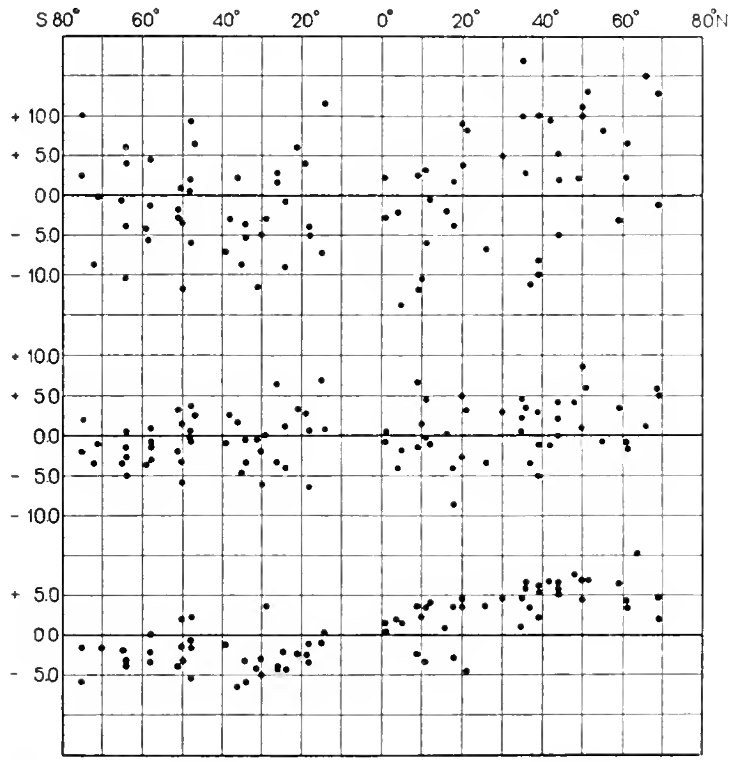

FIG. 1.—Measures by Miss Richmond (upper series), Miss Felker (middle series), and van Maanen on λ 5856.312. Abscissae are heliographic latitudes; ordinates are values of the displacements, the unit being 0.001 mm. The number of settings of the micrometer in the middle series is twice that for the upper series.

TABLE I

SUMMARY OF MEASURES ON λ 5856.312

Limits of Latitude	Mean Latitude	Mean Δ		
		Richmond	Felker	van Maanen
69° N–30° N....	47° N	+3.6±1.1 (24)	+1.8±0.5 (24)	+5.2±0.3 (23)
26 N–29 S....	1 S	−1.2±0.8 (28)	0.0±0.5 (28)	0.0±0.4 (28)
30 S–65 S....	49 S	−2.4±0.7 (29)	−1.2±0.3 (29)	−2.7±0.3 (21)

puting Division have made trial series which confirm the general character of the results. These measures, which were on λ 5856.312 (Fe, 2), are illustrated in Fig. 1, and a summary is given in Table I.

Appended to each mean displacement in the table is its probable error, the unit being 0.001 mm; the number of values included in the mean is added in parentheses. The accidental errors are large (Miss Richmond's series includes half as many settings as that by Miss Felker) and the mean displacements are systematically smaller than those by van Maanen; but for the 45° regions the algebraic signs are all correct, and the means themselves are sufficiently in excess of their probable errors to make the results of significance.

2. MEASUREMENT OF DISPLACEMENTS WITH KOCH'S REGISTERING MICROPHOTOMETER

In the earlier work two forms of measuring machine were employed: a comparator of the ordinary type, with fixed cross-hair, and a parallel-plate micrometer, with which adjacent sections of the displaced line can be brought into alinement by inclining a plane-parallel strip of glass. At the outset the results of different observers with the ordinary micrometer were frequently in disagreement, and it was in the hope of avoiding such discrepancies that the parallel-plate machine was tried. This has proved so satisfactory that all of van Maanen's measures have been made with it. His results for the spectra measured by Miss Lasby with the comparator agree well with hers in sign, although there is a marked systematic difference in magnitude.[1]

A promising means of avoiding systematic errors of measurement is offered by Koch's registering microphotometer, which automatically records the distribution of density in the photographic image of a spectral line. The photograph of the line is moved at a uniform rate across a narrow slit, through which light from a constant source falls upon the sensitive electrode of a photo-electric cell. The variations in electromotive force, caused by changes in the intensity of the transmitted light, produce horizontal displacements of the filament of a string electrometer connected with the other electrode. The image of a small section of the filament projected on a photographic plate, moved vertically by the same clock that carries the negative across the slit, traces a record of the intensity-curve of the line (see Fig. 2). The records may be used to determine

[1] *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, **38**, 87, 1913.

the relative positions of spectral lines, as well as the distribution of intensities within them, and thus afford a means of measurement independent of the personal errors that affect observations made with a comparator.

The instrument employed, which was kindly loaned by Professor Röntgen from the collection of the University of Munich for use in Pasadena during Professor Koch's visit in 1913, was not designed for the measurement of extremely small displacements, and it was doubtful whether the necessary degree of precision in the relative motion of the negative and the recording plate could be counted upon.

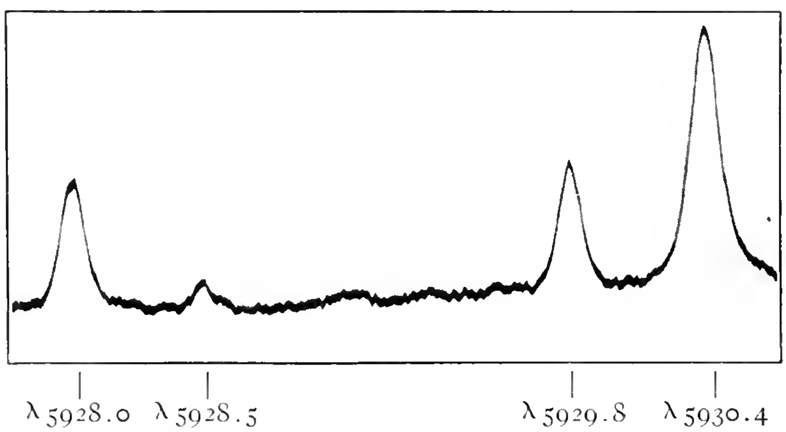

FIG. 2.—Intensity-curve registered by the Koch microphotometer used in measuring the displacements of λ 5928.013 and λ 5929.898 with respect to λ 5930.406, which is not displaced by the sun's general field. The atmospheric line λ 5928.510 is not suitable as a reference line because of its low intensity. Cut is 0.72 original curve.

Three solar lines were first studied, two of which (λ 5928.013 and λ 5929.898) had been found to be subject to displacement, while one (λ 5930.406) was not appreciably affected and thus furnished a reference mark to which the other lines could be referred.[1] Although the range of spectrum is only 2.4 A, Professor Koch was compelled to limit the ratio between the movements of the negative and the recording plate to 7.65, instead of using the more advantageous value of 46.4 applicable to very small fields. Curves were measured for two general-field plates representing direct and reverse records of each of four adjoining strips of the

[1] The atmospheric line λ 5928.510 would be preferable for this purpose, but its intensity-curve cannot be satisfactorily measured.

quarter-wave plate. One of these curves is reproduced in Fig. 2. Pairs of abscissae corresponding to ordinates increasing by one millimeter were measured to hundredths of a millimeter. Two reductions were made. In the first, all the measures were used, averaging fifteen pairs for λ 5928, thirteen for λ 5929, and thirty-one for λ 5930; in the second, only the ten pairs corresponding to ordinates common to all three lines were employed. The means of the abscissae give the positions of the lines, from which the displacements in Table II were derived.

TABLE II

COMPARISON OF MEASURES

Line	Plate	Koch's Registering Photometer		van Maanen Parallel-Plate Micrometer
		First Reduction	Second Reduction	
λ 5928.013.......	T'193*a*	−2.7 μ	−5.9 μ	− 0.7 μ
	193*b*	+9.3	+8.4	+ 2.6
λ 5929.898.......	193*a*	−7.3	−7.9	−10.3
	193*b*	+6.6	+7.2	+ 6.2

The differences between the two reductions indicate the uncertainty of the results obtained with the registering photometer. For the shorter interval (λ 5929.898) the measures are in approximate agreement with each other and with those of van Maanen. For the longer interval the inaccuracies of the driving mechanism are naturally reflected in the results.

As a further test, measures were made with the Koch photometer on λ 6302.709 (Fe, 5), which was referred to the neighboring atmospheric line λ 6302.975. Here it was possible to use the larger magnification of 46.4, although owing to the inherent uncertainties it is doubtful if any greater precision was obtained.

Twenty density-curves covering four adjacent spectra were registered from plate T'337*a*. As before, two reductions were made, one including all the abscissae for each curve, the other only those for ordinates common to the two curves. The results are

First reduction, $\Delta = +1.9\ \mu$
Second reduction, $\Delta = +3.0$

From the original negative van Maanen had previously found $+2.8\,\mu$. The uncertainty of the results from the Koch machine is rather large in this case, and the agreement with van Maanen's value, though probably of no real significance, is better than might have been expected.

These preliminary results indicate that a specially designed microphotometer should afford measures of great value, particularly for the study of systematic personal errors. As soon as opportunity permits, the instrument recently constructed in our shops will be used for a study of this question.

3. THE OBSERVATIONS

The measures considered in the present discussion depend on the groups of plates designed as Series IV, V, VI, VII, and VIII. The details, as well as the general method of observation, were the same as those previously described. The photographs of the third-order spectrum were made by Ellerman personally or else under his immediate supervision with the 75-foot spectrograph of the 150-foot tower telescope, the 43-cm solar image being used throughout. In view of the high excellence required for successful use, plates have frequently been rejected without measurement; insufficient or excessive contrast and irregularities in density were the most common reasons for rejection.

The record of observations for Series IV was published in *Mount Wilson Contribution*, No. 71, the original investigation having been partly based upon this series. Since Series V–VIII usually include only two days (λ 5247, Series VII, and λ 4406 and λ 4418, Series VIII, were observed on three days, and λ 4421, Series VIII, on four days), the chronological list of plates for these series is omitted, the essential data being collected in Tables III–VIII.

The observations given in this paper were not intended for an exhaustive study of the sun's magnetic field, but rather as a means of checking the earlier results and of selecting lines suitable for investigating the position of the magnetic axis and other related questions. The measures have therefore been restricted to the regions of maximum displacement near 45° N. and S. latitude. Most of the lines combine the characteristics of low or moderate

solar level and large laboratory separation, although several showing displacements in the sun have been included whose laboratory separations have not yet been determined.

TABLE III

LINES SHOWING MAGNETIC DISPLACEMENT

Series	Rowland λ	Rowland El.	Rowland Int.	Plates	Dates of Observation	*D*	Algebraic Sign of Δ: Right	Algebraic Sign of Δ: Wrong	Zero Values
					1913				
IV......	5831.821	Ni	1	T′255–290	Jan. 29–Feb. 18	−6°4	94	28	3
	5856.312	Fe	2	264–290	Jan. 31–Feb. 18	−6.6	66	8	1
	5928.013	Fe	2	258–290	Jan. 30–Feb. 18	−6.5	75	18	2
V.......	6007.540	Ni	1	318–333	June 19–20	+1.7	54	17	1
	6039.953	V	0	"	"	"	79	6	0
	6079.227	Fe	2	"	"	"	94	12	2
	6111.290	Ni	2	324–333	"	"	53	13	0
	6119.740	V	1	318–333	"	"	74	9	0
	6129.190	Ni	1	"	"	"	67	13	0
	6149.458	Fe*	2	"	"	"	65	10	1
	6173.553	Fe	5	"	"	"	85	5	0
VII.....	5247.737	Cr	2	424–440	Sept. 9–11	+7.2	171	12	1
	5250.817	Fe	3	431–436	Sept. 9–10	"	48	6	0
	5253.633	Fe	2	"	"	"	49	5	0
	5263.486	Fe	4	"	"	"	47	5	2
	5300.929	Cr	2	430–435	"	"	45	4	1
	5304.355	Cr	0	"	"	"	36	8	3
	5328.515	Cr	2	"	"	"	46	4	1
	5329.329	Cr	3	"	"	"	42	8	2
	5329.975	Cr	0	"	"	"	40	8	0
	5340.639	Cr	0	"	"	"	37	8	0
	5348.511	Cr	4	"	"	"	46	6	1
VIII....	4406.810	V	2	463–464	Nov. 10	+3.3	30	3	1
	4418.499	Ti	1	479–480	Dec. 1, 5	+0.5	36	0	0
	4421.733	V	0	463–480	Nov. 10–Dec. 5	......	49	19	0
	4430.785	Fe	3	463–480	Nov. 10–Dec. 5	......	23	2	1
	4438.006	V	0	463–464	Nov. 10	+3.3	14	4	1

* Unidentified by Rowland

The twenty-six new lines for which appreciable displacements have been detected are listed in Table III, with dates of observation, limiting plate numbers, etc. The table also contains λ5831.821; the general behavior of this line was indicated in *Mount Wilson Contribution*, No. 71, but the individual displacements are given in this paper for the first time. The last three columns of Table III show that a large majority of the algebraic signs accord with the

hypothesis that the displacements are caused by a general magnetic field. Indeed, but few of the disagreeing cases can be said to present contradictions, for frequently the real displacements are so small that a measured value may show a wrong sign without deviating excessively from the truth.

TABLE IV

LINES FROM SERIES IV–VIII WHICH SHOW NO DISPLACEMENT

Series	Rowland			Algebraic Sign of Δ		Zero Values
	λ	El.	Int.	Right	Wrong	
IV	5804.681	Fe	0	18	20	0
	5838.592	Fe	1	19	19	0
	5848.342	Fe	3	15	16	1
	5892.920	Fe	00	7	9	0
	5905.895	Fe	4	15	18	6
	5916.475	Fe	3	25	31	3
V	5991.600	....	2	22	25	4
	5998.002	Fe	2	23	26	3
	6005.770	Fe	1	31	38	4
	6012.450	Ni	1	22	16	1
	6042.315	Fe	3	31	26	3
	6081.665	V	0	21	24	1
	6142.700	....	1	24	25	8
VI	6455.820	Ca	2	32	38	2
	6496.688	Fe	2	41	34	3
	6597.807	Cr	1	32	44	0
VII	5224.471	Ti	0	22	21	3
VIII	4404.433	Ti	1N	10	9	0

All the measures discussed have been made by van Maanen with the parallel-plate micrometer. The results, collected for each line according to latitude, are given in Tables V–VIII. Those for Series VIII are less numerous than those for the other series, but the photographs are generally of excellent quality, and compensate in some degree for the smaller number of displacements measured. The results for λ 4421 from three plates of this series (sixteen values of Δ) were, however, rejected after measurement because of discordances due probably to lack of proper density. The line is weak and generally difficult of measurement. The rejected values do not appear in Table VIII.

TABLE V

DISPLACEMENTS FOR LINES OF SERIES IV

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 5831.821			λ 5831.821—*Cont.*			λ 5831.821—*Cont.*		
N. 71°...	276*a*	+0.7	0°...	290*a*	−1.0	S. 60°...	255*a*	−3.0
70....	274*a*	+1.3	0....	290*b*	0.0	60....	255*b*	−4.3
68....	272*a*	−1.2	S. 3....	290*a*	+0.7	60....	264*a*	−1.7
68....	273*b*	+2.0	3....	290*b*	+2.0	60....	264*b*	−2.3
64....	271*b*	+4.0	7....	290*a*	+4.3	62....	261*a*	−0.7
63....	274*a*	+3.0	7....	290*b*	+3.6	62....	261*b*	−1.7
60....	273*b*	+4.0	12....	290*a*	+2.3	63....	258*a*	−3.3
59....	276*a*	+1.0	12....	290*b*	(+6.9)	63....	258*b*	−3.6
58....	271*b*	+1.3	17....	290*a*	+2.6	65....	265*a*	−4.6
58....	272*a*	−0.7	17....	290*b*	(+4.3)	65....	265*b*	−1.7
50....	273*b*	+0.5	19....	283*a*	−1.0	66....	255*a*	−5.8
50....	274*a*	+1.5	21....	290*a*	+0.8	66....	255*b*	−1.8
49....	276*a*	(−4.1)	21....	290*b*	−1.0	66....	264*a*	−5.3
47....	271*b*	(+7.8)	22....	283*a*	−0.3	68....	264*b*	−2.0
47....	272*a*	+0.7	28....	283*a*	−1.0	72....	255*b*	−3.8
45....	274*a*	+0.3	31....	283*a*	−2.0	72....	265*a*	−2.6
44....	273*b*	+1.5	35....	283*a*	−0.2	72....	265*b*	−2.1
44....	276*a*	+1.7	37....	283*b*	−0.7	73....	255*a*	(−6.3)
42....	271*b*	+1.2	39....	261*b*	−0.3	73....	264*a*	+2.3
41....	272*a*	+5.0	41....	283*b*	−0.3	73....	264*b*	−1.7
40....	273*b*	+0.5	42....	258*a*	−2.0	79....	255*b*	−2.6
40....	274*a*	+2.6	42....	258*b*	0.0	79....	264*a*	−4.1
38....	276*a*	+1.8	42....	261*a*	(+5.3)	79....	264*b*	+0.3
37....	271*b*	−1.0	44....	261*a*	(+6.3)	82....	265*a*	(−6.9)
36....	272*a*	−0.7	44....	261*b*	−3.0	S. 82....	265*b*	−2.3
36....	273*b*	+3.3	46....	258*a*	−2.6			
36....	274*a*	+2.8	46....	258*b*	−4.1	λ 5856.312		
35....	271*b*	−0.3	47....	255*b*	+0.8	N. 69°...	273*b*	+4.8
30....	275*b*	+2.0	47....	264*b*	−3.1	69....	274*a*	+2.0
27....	275*a*	−2.0	47....	265*b*	+1.0	66....	272*a*	(+10.2)
26....	275*b*	+2.0	48....	265*a*	−4.0	61....	273*b*	+3.6
21....	288*a*	+0.8	48....	283*b*	−0.7	61....	274*a*	+4.0
21....	288*b*	+1.7	49....	255*a*	−2.0	59....	272*a*	+6.4
20....	275*a*	+2.3	51....	264*a*	(−7.3)	51....	274*a*	+6.9
20....	275*b*	+1.0	51....	264*b*	−1.8	50....	273*b*	+6.9
17....	288*a*	−3.0	51....	265*a*	−3.6	50....	276*a*	+4.3
17....	288*b*	0.0	51....	265*b*	−1.0	48....	272*a*	+7.4
16....	275*a*	+1.2	51....	283*b*	+2.6	44....	273*b*	+6.3
16....	275*b*	+1.3	52....	255*a*	−3.0	44....	274*a*	+5.8
14....	288*a*	+0.7	52....	255*b*	−5.6	44....	276*a*	+5.3
14....	288*b*	+3.6	52....	258*a*	−5.6	42....	272*a*	+6.6
12....	275*b*	+4.0	52....	258*b*	−5.9	39....	273*b*	+5.9
10....	275*a*	+1.3	52....	261*a*	−1.5	39....	274*a*	+2.1
10....	288*a*	−0.3	52....	261*b*	−3.1	39....	276*a*	+5.6
10....	288*b*	+1.5	56....	258*a*	+2.6	37....	272*a*	+3.3
9....	275*b*	(+6.3)	56....	258*b*	−1.2	36....	273*b*	+6.3
5....	288*a*	+1.3	57....	261*a*	−0.3	36....	276*a*	+5.9
5....	288*b*	+2.1	57....	261*b*	+0.5	35....	272*a*	+1.0
1....	288*a*	+1.3	59....	265*a*	(+5.0)	35....	274*a*	+4.5
N. 1....	288*b*	+1.7	S. 59....	265*b*	−4.6	N. 30....	275*b*	+4.5

TABLE V—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 5856.312—*Cont.*			λ 5928.013			λ 5928.013—*Cont.*		
N. 26°...	275*b*	+3.6	N. 69°...	276*a*	+3.6	S. 30°...	290*a*	−1.3
21....	288*a*	(−4.8)	68....	273*b*	+4.3	30....	290*b*	−2.8
20....	275*b*	+3.5	67....	271*b*	+5.3	31....	283*a*	−2.3
20....	288*b*	+4.3	66....	272*a*	+6.8	34....	290*a*	−6.3
18....	288*a*	(−3.0)	62....	273*b*	+4.0	34....	290*b*	−5.4
18....	288*b*	+3.6	61....	276*a*	+4.0	35....	283*a*	−4.8
16....	275*b*	+0.7	59....	272*a*	+5.6	35....	283*b*	−1.0
12....	275*b*	+4.0	57....	271*b*	+3.6	39....	283*b*	+2.3
11....	288*a*	−3.3	50....	273*b*	+3.6	42....	258*a*	−6.6
11....	288*b*	+3.3	50....	276*a*	+5.9	42....	258*b*	+1.5
10....	275*b*	+2.0	48....	272*a*	+3.6	42....	261*a*	−3.6
9....	288*a*	−2.3	44....	273*b*	+4.3	42....	261*b*	−2.3
9....	288*b*	+3.5	44....	276*a*	+4.0	44....	261*a*	−0.3
5....	288*b*	+1.5	42....	271*b*	+1.5	44....	261*b*	−1.7
4....	288*a*	+1.8	42....	272*a*	+4.6	46....	258*a*	+0.3
1....	288*a*	+1.5	39....	273*b*	+1.8	46....	258*b*	+2.5
N. 1....	288*b*	+0.3	39....	276*a*	+2.1	48....	264*a*	−2.6
S. 14....	290*b*	+0.2	37....	271*b*	+2.6	48....	264*b*	+0.7
15....	290*a*	−1.0	37....	272*a*	+1.0	48....	283*b*	(+4.0)
18....	290*a*	−1.2	36....	273*b*	+3.6	50....	264*a*	−1.7
18....	290*b*	−3.3	36....	276*a*	+3.5	50....	264*b*	−2.5
19....	283*a*	−2.6	35....	271*b*	+2.3	50....	283*b*	(+3.3)
21....	283*a*	−2.3	35....	272*a*	+1.3	52....	258*a*	−4.0
24....	290*a*	−4.3	30....	275*b*	+4.3	52....	258*b*	+1.7
24....	290*b*	−2.3	26....	275*b*	+1.3	52....	261*a*	−2.3
26....	290*a*	−4.1	21....	288*a*	+1.8	52....	261*b*	0.0
26....	290*b*	−4.3	21....	288*b*	(−5.6)	56....	283*b*	+2.8
29....	283*a*	(+3.6)	20....	275*b*	+0.8	57....	258*a*	(+5.1)
30....	290*a*	−5.0	17....	288*a*	+5.4	57....	258*b*	−1.5
30....	290*b*	−3.0	17....	288*b*	(−6.6)	57....	261*a*	−5.9
31....	283*a*	−4.3	16....	275*b*	0.0	57....	261*b*	(+3.3)
34....	290*a*	−3.3	12....	275*b*	+2.3	58....	264*a*	−5.4
34....	290*b*	−5.9	12....	288*a*	+4.3	58....	264*b*	−1.3
36....	283*b*	−6.4	12....	288*b*	(−5.0)	63....	258*b*	−2.3
39....	283*b*	−1.3	10....	275*b*	+1.7	63....	261*a*	−0.5
48....	264*a*	−1.3	10....	288*a*	−3.6	63....	261*b*	−4.3
48....	264*b*	−0.7	10....	288*b*	(−8.4)	64....	258*a*	−0.3
48....	265*a*	−5.6	5....	288*a*	+6.3	64....	264*a*	−4.3
48....	283*b*	(+2.1)	5....	288*b*	−4.3	64....	264*b*	−2.0
50....	264*a*	(+1.8)	1....	288*a*	+3.0	68....	261*b*	−1.0
50....	264*b*	−1.7	N. 1....	288*b*	−5.0	70....	264*a*	−0.7
50....	265*a*	−3.3	S. 15....	290*a*	−2.0	70....	264*b*	−3.0
51....	283*b*	−4.0	15....	290*b*	−1.7	S. 76....	264*a*	−4.3
58....	264*a*	0.0	18....	283*a*	−1.8			
58....	264*b*	−2.3	18....	290*a*	−4.6			
58....	265*a*	−3.5	18....	290*b*	−2.3			
64....	264*a*	−4.0	21....	283*a*	−1.3			
64....	264*b*	−3.3	24....	290*a*	−3.3			
64....	265*a*	−2.0	24....	290*b*	−3.0			
71....	265*a*	−1.8	26....	290*a*	−4.0			
75....	264*a*	−1.7	26....	290*b*	−3.3			
S. 75....	264*b*	−5.9	S. 29....	283*a*	−0.3			

TABLE VI

DISPLACEMENTS FOR LINES OF SERIES V

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
	λ 6007.540			λ 6007.540—*Cont.*			λ 6039.953—*Cont.*	
N. 64°	333*a*	+5.0	S. 41°	319*a*	−3.0	N. 39°	324*a*	+5.6
64	333*b*	+7.6	41	319*b*	−6.4	39	324*b*	+1.7
62	332*a*	−1.7	41	326*a*	+4.0	39	332*a*	+5.1
62	332*b*	+8.6	41	326*b*	−1.8	39	332*b*	+4.6
61	318*a*	+3.3	41	330*a*	−5.6	39	333*a*	+7.1
61	318*b*	+3.6	41	330*b*	−3.1	39	333*b*	+9.2
58	318*a*	+0.3	45	326*a*	(+5.6)	37	333*a*	+6.4
58	318*b*	+4.3	45	327*b*	−3.6	37	333*b*	+6.4
58	332*a*	−1.8	46	330*a*	−3.3	36	318*b*	+6.4
58	332*b*	−3.6	46	330*b*	−2.1	35	324*a*	(−5.3)
57	333*a*	+4.1	51	327*a*	−6.8	35	324*b*	+6.3
57	333*b*	+3.5	52	319*a*	−3.6	34	318*a*	+6.4
50	333*a*	+2.8	52	326*a*	+5.0	34	318*b*	+5.0
50	333*b*	+1.3	52	326*b*	−4.6	N. 34	332*a*	+4.1
49	318*a*	+4.6	52	330*a*	−2.8	S. 30	330*a*	−3.6
49	332*a*	+0.3	52	330*b*	−5.3	30	330*b*	−3.5
49	332*b*	−3.1	54	327*b*	−3.3	31	319*a*	−6.9
45	332*a*	−4.0	57	319*a*	0.0	31	319*b*	−6.6
45	332*b*	−1.0	57	326*a*	+3.3	32	327*b*	−6.1
45	333*a*	+5.0	57	326*b*	−3.6	35	319*a*	−6.9
45	333*b*	+4.8	S. 57	327*a*	−2.6	35	319*b*	−2.3
44	318*a*	+3.3				35	326*b*	−6.9
44	318*b*	+4.0		λ 6039.953		35	327*a*	−4.0
39	318*a*	+3.0	N. 65	333*b*	+7.4	35	327*b*	−5.9
39	318*b*	+5.6	62	318*a*	(+11.9)	35	330*a*	+0.5
39	332*a*	−2.6	62	332*a*	+4.0	35	330*b*	−7.3
39	332*b*	+4.5	62	332*b*	+2.8	41	319*a*	−5.8
39	333*a*	+2.6	60	324*a*	(−8.6)	41	319*b*	−9.9
39	333*b*	+5.0	60	324*b*	+3.0	41	326*a*	−4.5
35	318*a*	+4.6	59	318*a*	+7.3	41	326*b*	−4.6
35	318*b*	+3.5	59	318*b*	+3.3	41	327*a*	−5.3
35	332*a*	−4.6	59	333*a*	+4.8	41	327*b*	−7.9
35	332*b*	+4.6	59	333*b*	+9.2	41	330*a*	(+10.4)
35	333*a*	+2.3	58	332*a*	+6.8	41	330*b*	−1.3
N. 35	333*b*	+3.6	58	332*b*	+6.3	45	326*a*	−6.3
S. 30	330*a*	−5.9	57	324*a*	(−5.0)	45	326*b*	−5.6
30	330*b*	−3.3	57	324*b*	+4.0	45	327*a*	−4.3
31	319*a*	+2.1	50	318*a*	+7.3	45	327*b*	−1.3
31	319*b*	−4.0	50	318*b*	+6.4	45	330*a*	(+8.6)
31	327*a*	+5.0	50	324*a*	+6.6	45	330*b*	−5.9
31	327*b*	−4.6	50	324*b*	+5.3	46	319*a*	−7.6
34	330*a*	−3.0	49	332*a*	+7.9	46	319*b*	−2.0
34	330*b*	−5.0	49	332*b*	+4.3	51	327*a*	−6.9
35	319*a*	+5.4	49	333*a*	+4.0	51	327*b*	−6.8
35	319*b*	−1.3	49	333*b*	+3.8	52	319*a*	−7.6
35	327*a*	+4.0	45	324*a*	+9.2	52	319*b*	−6.9
35	327*b*	−3.1	45	324*b*	+3.6	52	326*a*	−7.6
37	326*a*	(+5.9)	45	332*b*	+7.9	52	326*b*	−5.6
37	326*b*	−4.6	45	333*a*	+10.1	52	330*a*	−6.6
40	327*a*	−3.6	45	333*b*	+7.1	52	330*b*	−7.6
S. 40	327*b*	−6.9	N. 39	318*b*	+3.5	S. 57	326*a*	−5.9

TABLE VI—*Continued*

Lat.	Plate	Δ
λ 6039.953—*Cont.*		
S. 57°	326*b*	−7.3
57	330*a*	−8.9
58	319*a*	−4.0
58	319*b*	−6.3
58	327*a*	−0.2
S. 58	327*b*	−6.8
λ 6079.227		
N. 64	333*a*	+5.6
64	333*b*	+5.4
63	318*a*	+4.3
63	318*b*	+5.0
62	332*a*	+4.0
62	332*b*	+2.6
61	324*b*	+2.0
59	332*a*	+3.5
59	332*b*	+1.7
57	318*a*	+4.3
57	324*a*	+4.6
57	324*b*	+4.3
57	333*a*	+6.8
57	333*b*	(+7.6)
50	318*a*	+6.1
50	318*b*	+5.6
50	324*a*	+5.6
50	324*b*	+5.6
50	333*a*	+7.4
50	333*b*	+5.9
49	332*a*	+4.3
49	332*b*	+3.5
45	318*a*	+4.0
45	318*b*	+5.6
45	324*a*	+5.3
45	324*b*	+4.6
45	332*a*	+4.1
45	332*b*	+2.3
45	333*a*	+5.6
45	333*b*	+5.8
40	324*a*	+2.6
40	324*b*	+5.3
39	318*b*	+4.0
39	332*b*	+4.1
39	333*a*	+4.0
39	333*b*	+5.9
36	318*a*	+5.1
36	318*b*	+5.9
35	324*a*	+4.6
35	324*b*	+3.0
35	332*a*	+4.6
35	332*b*	+2.6
35	333*a*	+3.3
N. 35	333*b*	+5.0
λ 6079.227—*Cont.*		
S. 30°	319*a*	−4.0
30	319*b*	−4.6
31	327*a*	−2.6
31	327*b*	−5.4
34	326*a*	−3.5
34	326*b*	−6.9
35	319*a*	−5.3
35	327*a*	(−0.2)
35	327*b*	−4.6
40	327*a*	−3.6
41	319*a*	−6.4
41	319*b*	−5.6
41	326*a*	−4.0
41	326*b*	−7.6
45	319*a*	−5.3
45	326*a*	−6.4
45	327*a*	−4.6
45	327*b*	−6.3
52	319*a*	(−7.9)
52	319*b*	−3.5
52	326*a*	−3.0
52	326*b*	−4.0
52	327*a*	−5.4
52	327*b*	−3.6
57	319*a*	−3.6
57	319*b*	−5.6
57	327*a*	−5.0
57	327*b*	−3.0
58	326*a*	−5.8
S. 58	326*b*	−5.9
λ 6111.290		
N. 64	332*a*	+0.7
64	333*a*	+3.0
64	333*b*	+4.3
62	332*b*	+2.3
59	332*a*	+3.3
59	332*b*	+0.2
57	324*a*	+4.3
57	324*b*	+0.5
57	333*a*	+2.1
57	333*b*	−1.3
50	324*a*	+1.0
50	324*b*	−1.7
50	333*a*	+2.3
50	333*b*	+3.5
49	332*a*	+4.0
49	332*b*	+3.0
45	324*a*	+0.7
45	324*b*	+4.0
45	332*a*	−1.0
N. 45	332*b*	+0.2
λ 6111.290—*Cont.*		
N. 45°	333*a*	+3.3
45	333*b*	+1.8
39	324*a*	+3.3
39	332*a*	+3.6
39	332*b*	+4.3
39	333*a*	+2.6
39	333*b*	+2.3
35	324*a*	−0.3
35	324*b*	+0.2
35	332*a*	+0.7
35	332*b*	+1.0
35	333*a*	(−3.1)
N. 35	333*b*	+2.3
S. 30	330*a*	(+3.6)
30	330*b*	−1.8
31	327*a*	−3.6
31	327*b*	−2.6
34	326*a*	−1.8
34	326*b*	−5.0
35	327*a*	−0.3
35	327*b*	−1.3
35	330*a*	+0.3
35	330*b*	−5.4
40	327*a*	−2.3
40	327*b*	−1.7
40	330*a*	(+4.0
40	330*b*	+0.3
41	326*a*	−0.3
41	326*b*	−2.8
45	326*a*	−1.3
45	327*a*	−1.3
45	327*b*	−4.0
45	330*a*	(+4.0)
45	330*b*	+0.7
51	330*a*	+2.3
51	330*b*	−5.9
52	326*a*	−1.7
52	326*b*	−3.0
52	327*a*	−3.0
52	327*b*	−1.7
57	327*a*	−2.5
57	327*b*	−4.5
57	330*a*	+0.2
57	330*b*	−3.6
58	326*a*	−1.3
S. 58	326*b*	−3.3
λ 6119.740		
N. 64	332*a*	+4.3
64	333*a*	(−4.0)
64	333*b*	+1.0
N. 63	318*b*	+6.4

TABLE VI—*Continued*

Lat.	Plate	Δ
λ 6119.740—*Cont.*		
N. 62°	332*b*	+4.6
60	324*b*	+5.6
59	332*a*	+6.9
59	332*b*	+3.8
57	318*b*	+2.6
57	324*a*	(−8.3)
57	324*b*	+2.6
57	333*a*	+5.3
57	333*b*	+4.3
50	318*b*	+4.0
50	324*a*	+6.4
50	324*b*	+3.3
50	333*a*	+2.1
50	333*b*	+5.3
49	332*a*	+4.5
49	332*b*	+5.8
45	318*b*	+2.3
45	324*a*	+4.3
45	324*b*	+2.0
45	332*a*	+4.0
45	332*b*	+5.8
45	333*a*	+4.0
45	333*b*	+6.9
39	318*a*	+8.3
39	318*b*	+2.8
39	324*b*	+5.4
39	332*a*	+6.3
39	332*b*	+5.4
39	333*a*	+7.3
39	333*b*	+4.0
36	318*a*	+6.9
36	318*b*	+6.4
35	324*a*	+5.3
35	324*b*	+5.6
35	332*a*	+3.3
35	332*b*	+2.3
35	333*a*	+2.3
N. 35	333*b*	+6.9
S. 30	319*a*	−3.3
30	319*b*	−5.3
30	330*b*	−3.0
31	327*a*	−1.0
31	327*b*	−5.4
34	326*a*	(+3.0)
34	326*b*	−3.0
35	319*a*	(+8.9)
35	319*b*	−5.0
35	327*a*	−4.6
35	327*b*	+1.5
35	330*a*	−3.0
35	330*b*	−3.1
S. 40	327*a*	−5.1

Lat.	Plate	Δ
λ 6119.740—*Cont.*		
S. 40°	327*b*	−6.3
40	330*a*	−5.9
40	330*b*	−3.3
41	319*a*	(+7.6)
41	319*b*	−3.6
41	326*a*	−3.3
41	326*b*	−5.0
45	319*a*	(+6.3)
45	319*b*	−3.0
45	326*a*	−5.1
45	327*b*	−3.3
45	330*a*	−3.6
45	330*b*	−6.9
51	330*a*	−6.1
51	330*b*	−5.9
52	327*b*	−3.6
52	326*b*	−4.6
52	326*a*	−6.3
52	319*b*	−1.2
52	319*a*	(+4.0)
57	319*a*	(+7.3)
57	319*b*	−6.3
57	327*a*	−5.9
57	327*b*	−6.3
57	330*a*	−5.0
58	326*a*	−4.1
S. 58	326*b*	−3.6
λ 6129.190		
N. 64	332*a*	+3.0
64	332*b*	+5.1
63	324*a*	+6.1
63	333*a*	+8.6
63	333*b*	+5.1
62	318*a*	(−6.3)
58	332*a*	+4.3
58	332*b*	+4.3
57	324*a*	+4.0
57	324*b*	+5.3
57	333*a*	−1.7
57	333*b*	+2.0
56	318*a*	(−4.6)
50	332*a*	+7.3
50	332*b*	+2.3
50	333*a*	−1.7
50	333*b*	+3.8
49	318*a*	(−3.3)
48	324*b*	+6.8
45	332*a*	+2.6
45	333*a*	+3.6
45	333*b*	+2.3
N. 44	318*a*	−1.8

Lat.	Plate	Δ
λ 6129.190—*Cont.*		
N. 44°	324*a*	+3.6
44	324*b*	+3.6
40	332*a*	+3.1
39	324*a*	+1.3
39	324*b*	+5.6
39	333*a*	+4.6
39	333*b*	+1.7
38	318*a*	−0.3
35	332*a*	+2.6
35	332*b*	+5.6
35	333*a*	+2.1
34	318*a*	(−4.8)
34	324*a*	+0.3
N. 34	324*b*	+5.4
S. 31	319*a*	+3.0
31	319*b*	−5.3
31	330*a*	−8.3
31	330*b*	−1.7
32	326*a*	−3.8
32	326*b*	−3.6
35	330*a*	−5.0
35	330*b*	−4.6
36	319*a*	−4.0
36	319*b*	−1.5
36	326*a*	−4.5
36	326*b*	−5.1
36	327*b*	−2.5
41	319*a*	−4.8
41	319*b*	−5.3
41	327*a*	+3.0
41	327*b*	−3.6
41	330*a*	+2.6
41	330*b*	−4.3
42	326*a*	−3.6
42	326*b*	−1.5
46	319*a*	−5.6
46	319*b*	−3.0
46	326*a*	−3.0
46	326*b*	−7.3
46	327*a*	−1.0
46	327*b*	−5.3
52	319*a*	−4.3
52	319*b*	(+3.5)
52	326*a*	−4.3
52	326*b*	−2.5
52	330*a*	−4.8
52	330*b*	−3.5
53	327*a*	−7.1
53	327*b*	+1.0
58	319*a*	−5.4
58	319*b*	−4.0
S. 58	326*a*	−2.1

TABLE VI—*Continued*

Lat.	Plate	Δ
λ 6129.190—*Cont.*		
S. 58°...	326*b*	−5.0
58....	330*a*	−4.6
58....	330*b*	−6.3
59....	327*a*	−5.3
S. 59....	327*b*	−5.6
λ 6149.458		
N. 64....	332*a*	+4.0
64....	332*b*	+5.1
63....	324*a*	+5.3
63....	333*a*	+5.1
63....	333*b*	+6.8
58....	332*a*	+2.6
58....	332*b*	+2.0
57....	324*a*	+4.1
57....	324*b*	+5.9
57....	333*a*	+1.3
57....	333*b*	+3.3
56....	318*a*	(−3.8)
50....	332*a*	+4.1
50....	332*b*	+5.3
50....	333*a*	−1.7
50....	333*b*	+5.1
49....	318*a*	(−4.0)
48....	324*a*	+2.6
48....	324*b*	+0.3
45....	332*b*	+3.5
45....	333*a*	−1.0
45....	333*b*	+5.3
44....	318*a*	(−5.0)
44....	324*a*	+3.6
44....	324*b*	+5.0
40....	332*a*	+5.3
40....	332*b*	+7.3
39....	324*a*	+3.6
39....	324*b*	+3.8
39....	333*a*	+3.3
39....	333*b*	+2.6
38....	318*a*	(−4.6)
35....	332*a*	+6.3
35....	332*b*	−0.3
35....	333*a*	+2.0
35....	333*b*	+5.0
34....	318*a*	(−3.3)
N. 34....	324*a*	+0.8
S. 31....	319*a*	−1.5
31....	319*b*	−2.6
31....	330*a*	−5.3
31....	330*b*	−1.5
32....	326*a*	−4.1
32....	326*b*	−2.3
S. 35....	330*a*	−6.9

Lat.	Plate	Δ
λ 6149.458—*Cont.*		
S. 35°...	330*b*	−2.8
36....	319*a*	−3.3
36....	319*b*	−4.6
36....	326*a*	−5.8
36....	326*b*	−6.3
36....	327*b*	+0.3
41....	319*a*	−4.3
41....	327*a*	−4.6
41....	327*b*	−5.0
41....	330*a*	−4.1
41....	330*b*	−3.3
42....	326*a*	−3.6
42....	326*b*	−4.6
45....	330*a*	−3.0
45....	330*b*	0.0
46....	319*a*	−6.6
46....	319*b*	−2.6
46....	326*a*	−3.0
52....	319*b*	−1.2
52....	326*a*	−4.3
52....	330*a*	−3.0
52....	330*b*	+2.3
53....	327*b*	−0.5
58....	319*a*	−4.3
58....	319*b*	−3.0
58....	326*a*	−2.3
58....	326*b*	−3.3
58....	330*a*	−4.0
58....	330*b*	−7.9
59....	327*a*	−4.5
S. 59....	327*b*	−4.3
λ 6173.553		
N. 64....	332*a*	+3.0
64....	332*b*	+5.3
63....	333*a*	+6.6
63....	333*b*	+8.2
62....	318*a*	+6.3
62....	318*b*	+7.3
58....	332*a*	+5.6
58....	332*b*	+4.3
57....	324*a*	+1.0
57....	324*b*	+2.1
57....	333*a*	+4.3
57....	333*b*	+3.3
56....	318*a*	+0.2
56....	318*b*	+5.8
50....	332*a*	+6.6
50....	332*b*	+3.8
50....	333*a*	+0.5
50....	333*b*	+5.3
N. 49....	318*a*	+4.0

Lat.	Plate	Δ
λ 6173.553—*Cont.*		
N. 49°...	318*b*	+9.2
48....	324*a*	+6.8
48....	324*b*	+6.9
45....	332*a*	+4.0
45....	332*b*	+6.3
45....	333*a*	+6.8
45....	333*b*	+5.8
44....	318*a*	+6.6
44....	318*b*	+5.1
44....	324*a*	(−0.7)
44....	324*b*	+5.0
40....	332*a*	+5.1
39....	324*a*	+4.6
39....	324*b*	+6.6
39....	333*a*	+0.2
39....	333*b*	+4.0
38....	318*b*	+6.4
35....	332*a*	+6.1
35....	332*b*	+5.0
35....	333*a*	+0.7
35....	333*b*	+4.8
34....	318*a*	+5.3
34....	318*b*	+4.6
34....	324*a*	+4.6
N. 34....	324*b*	+7.9
S. 31....	319*a*	−4.0
31....	319*b*	−4.3
31....	330*a*	(+1.6)
31....	330*b*	−3.3
32....	326*a*	−5.9
32....	326*b*	−5.4
35....	330*a*	(+0.6)
35....	330*b*	−2.8
36....	319*a*	−4.8
36....	319*b*	−6.4
36....	326*a*	−6.3
36....	326*b*	−5.3
36....	327*a*	−1.2
36....	327*b*	−5.6
41....	319*a*	−4.8
41....	319*b*	−4.3
41....	327*a*	−8.1
41....	327*b*	−3.8
41....	330*a*	−0.1
41....	330*b*	−4.3
42....	326*a*	−5.9
42....	326*b*	−6.1
45....	330*a*	(+5.1)
45....	330*b*	−2.6
46....	319*a*	−5.3
46....	319*b*	−5.3
S. 46....	326*a*	−5.9

TABLE VI—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 6173.553—*Cont.*			λ 6173.553—*Cont.*			λ 6173.553—*Cont.*		
S. 46°...	326*b*	−7.3	S. 52°...	330*a*	(+2.3)	S. 58°...	326*b*	−7.6
46....	327*a*	−6.9	52....	330*b*	−4.6	58....	330*a*	−2.2
46....	327*b*	−3.8	53....	327*a*	−5.3	58....	330*b*	−6.4
52....	319*a*	−1.7	53....	327*b*	−4.3	59....	327*a*	−3.6
52....	319*b*	−3.6	58....	319*a*	−6.3	S. 59....	327*b*	−6.9
52....	326*a*	−5.0	58....	319*b*	−5.0			
S. 52....	326*b*	−6.1	S. 58....	326*a*	−7.1			

Since the compound quarter-wave plate was sometimes used in the normal (+) and sometimes in the inverted (−) position, and since in nearly all cases the measurer was ignorant of its position and also of the hemisphere under observation, personal bias has been eliminated. The signs of displacements observed with the inverted quarter-wave plate have been changed, so that all results refer to the normal position of the plate.

Besides the lines given in Table III, others have also been measured. A list of these, which show no sensible displacement, is in Table IV. The close equality in the numbers of right and wrong signs is in striking contrast with the preponderance of correct signs in Table III.

4. DETERMINATION OF MAXIMUM DISPLACEMENT FOR DIFFERENT LINES

The first step in the calculation of the sun's general field is the determination of the displacement at $\phi=45°$ from the measures in Tables V–VIII. The displacements of a normal Zeeman triplet produced by the sun's field, assumed to be that of a uniformly magnetized sphere, may be represented by[1]

$$k\Delta=A\cos i+B\sin i\cos\lambda \qquad (1)$$

in which

Δ = displacement of spectral line,

k = a constant depending upon the strength of field, the magnetic separation of the line, and the units employed,

i = inclination of sun's magnetic axis to axis of rotation, and

λ = heliographic longitude of magnetic axis.

[1] Seares, *Mt. Wilson Contr.*, No. 72; *Astrophysical Journal*, 38, 99, 1913.

TABLE VII

Displacements for Lines of Series VII—λ5247.737

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
N. 70°...	424*a*	+4.0	N. 50°...	429*a*	+5.9	S. 25°...	440*b*	−5.3
70....	424*b*	+6.3	50....	429*b*	+3.6	29....	433*a*	−5.8
70....	426*a*	+5.1	50....	430*a*	+5.6	29....	433*b*	(+4.3)
70....	426*b*	+4.3	50....	430*b*	+6.3	29....	434*a*	−4.3
70....	427*a*	+4.1	50....	431*a*	+5.9	29....	434*b*	−2.8
70....	427*b*	+5.3	50....	431*b*	+3.6	29....	435*a*	−2.6
70....	428*a*	(−4.3)	50....	432*a*	+4.1	29....	435*b*	−5.6
70....	428*b*	+5.1	50....	432*b*	+2.6	29....	436*a*	−6.1
69....	429*a*	+5.6	46....	424*a*	+3.3	29....	436*b*	−4.0
69....	430*a*	+3.6	46....	424*b*	+5.0	29....	437*a*	−5.3
69....	430*b*	+2.0	46....	426*a*	+3.3	29....	437*b*	−5.6
69....	431*a*	+4.3	46....	427*a*	+0.3	29....	438*a*	−5.0
69....	431*b*	+4.8	46....	427*b*	+2.6	29....	438*b*	−7.3
69....	432*a*	(+8.3)	46....	428*a*	(−5.6)	29....	439*a*	−4.3
69....	432*b*	+5.9	46....	428*b*	+2.6	29....	439*b*	−4.0
63....	424*a*	+2.6	45....	429*a*	+5.3	30....	440*a*	−7.1
63....	424*b*	+5.1	45....	429*b*	+3.6	35....	433*a*	−6.6
63....	426*a*	+5.3	45....	430*a*	+5.9	35....	433*b*	(+6.3)
63....	426*b*	+3.6	45....	430*b*	+5.0	35....	434*a*	−5.0
63....	427*a*	+2.5	45....	431*a*	+4.3	35....	434*b*	−6.6
63....	427*b*	+7.6	45....	431*b*	+3.6	35....	435*a*	−5.6
63....	428*a*	(−3.6)	45....	432*a*	+4.8	35....	435*b*	−4.3
63....	428*b*	+3.6	45....	432*b*	+6.3	35....	436*a*	−4.0
62....	429*a*	+4.3	41....	424*a*	+4.0	35....	436*b*	−5.3
62....	430*a*	+3.3	41....	424*b*	+3.0	35....	437*a*	−4.0
62....	430*b*	+6.1	41....	426*a*	+1.7	35....	437*b*	−5.1
62....	431*a*	+6.6	41....	426*b*	+6.3	35....	438*a*	−4.6
62....	431*b*	+5.4	41....	427*a*	+4.6	35....	438*b*	−4.5
62....	432*a*	+4.5	41....	427*b*	+4.6	35....	439*a*	−3.8
62....	432*b*	+6.9	41....	428*b*	+4.3	35....	439*b*	−3.3
56....	424*a*	+4.3	40....	429*a*	+2.1	35....	440*a*	−5.0
56....	424*b*	+2.6	40....	429*b*	+4.0	35....	440*b*	−6.1
56....	426*a*	+3.3	40....	430*a*	+3.5	39....	433*a*	−6.3
56....	426*b*	+2.6	40....	430*b*	+4.8	39....	433*b*	(+4.1)
56....	427*a*	+4.6	40....	431*a*	+5.6	39....	434*a*	−4.6
56....	427*b*	+4.6	40....	431*b*	+4.0	39....	434*b*	−6.3
56....	428*a*	(−5.4)	40....	432*a*	+5.3	39....	435*a*	−3.6
55....	429*a*	+5.4	N. 40....	432*b*	+3.8	39....	435*b*	−5.6
55....	429*b*	+5.3	S. 25....	433*a*	−3.0	39....	436*a*	−5.6
55....	430*a*	+4.1	25....	433*b*	(+5.3)	39....	436*b*	−5.6
55....	430*b*	+2.6	25....	434*a*	−5.1	39....	437*a*	−1.0
55....	431*a*	+3.3	25....	434*b*	−1.8	39....	437*b*	−3.3
55....	431*b*	+6.4	25....	435*a*	−5.4	39....	438*a*	−4.3
55....	432*a*	+5.6	25....	435*b*	−3.3	39....	438*b*	−5.6
55....	432*b*	+5.4	25....	436*a*	−2.6	39....	439*a*	−4.0
51....	424*a*	+2.3	25....	436*b*	−6.3	39....	439*b*	−7.6
51....	424*b*	+4.5	25....	437*a*	−1.7	40....	440*a*	−4.6
51....	426*a*	+3.3	25....	437*b*	−5.3	40....	440*b*	−3.5
51....	426*b*	+3.5	25....	438*a*	−4.8	47....	433*a*	−2.3
51....	427*a*	+5.4	25....	438*b*	−4.1	47....	433*b*	(+2.8)
51....	427*b*	+5.9	25....	439*a*	−2.0	47....	434*a*	−4.8
51....	428*a*	(−3.0)	25....	439*b*	−4.8	47....	434*b*	−6.4
N. 51....	428*b*	+4.6	S. 25....	440*a*	−4.0	S. 47....	435*a*	−3.6

TABLE VII—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
S. 47° . . .	435*b*	−3.0	S. 48° . . .	440*a*	−6.3	S. 51° . . .	437*b*	−6.6
47	436*a*	−3.3	48	440*b*	−4.6	51	438*a*	−5.6
47	436*b*	−5.0	51	433*a*	−6.3	51	438*b*	−3.6
47	437*a*	−2.3	51	433*b*	(+4.0)	51	439*a*	−5.6
47	437*b*	−6.6	51	434*a*	−4.6	51	439*b*	−5.0
47	438*a*	−4.6	51	434*b*	−7.3	52	440*a*	−3.1
47	438*b*	−5.4	51	435*a*	−6.6	S. 52	440*b*	0.0
47	439*a*	−4.3	51	435*b*	−7.1			
S. 47	439*b*	−5.6	S. 51	437*a*	+0.3			

Lat.	Plate	Δ			Lat.	Plate	Δ		
		λ5250	λ5253	λ5263			λ5250	λ5253	λ5263
N. 69° . .	431*a*	+2.0	+5.1	+1.7	S. 25° . .	435*b*	−1.0	+0.3	−2.3
69 . . .	431*b*	+4.5	(−3.1)	+2.6	25 . . .	436*a*	−0.7	−0.3	−2.3
69 . . .	432*a*	+1.8	+2.3	+3.8	29 . . .	434*a*	−3.0	−3.0	+0.3
69 . . .	432*b*	−1.3	+1.3	+0.3	29 . . .	434*b*	(+3.6)	+1.3	−0.2
62 . . .	431*a*	+3.6	+2.0	(+5.0)	29 . . .	435*a*	+0.3	−0.3	−1.5
62 . . .	431*b*	+4.0	+4.3	+1.3	29 . . .	435*b*	−3.6	−3.6	−2.3
62 . . .	432*a*	+1.3	+1.7	(−1.3)	29 . . .	436*a*	+0.7	−3.5	−2.8
62 . . .	432*b*	+1.8	+4.1	+1.0	35 . . .	434*a*	−3.6	−3.8	−2.1
55 . . .	431*a*	+0.7	+0.7	0.0	35 . . .	434*b*	−2.3	−1.7	−3.3
55 . . .	431*b*	+2.0	+3.5	+1.7	35 . . .	435*a*	−3.6	−3.1	−1.3
55 . . .	432*a*	+3.3	+2.6	+3.1	35 . . .	435*b*	−2.6	−0.7	(+1.2)
55 . . .	432*b*	+1.3	+4.3	+2.5	35 . . .	436*a*	−2.6	−1.3	+0.7
50 . . .	431*a*	+1.7	+0.3	+2.3	39 . . .	434*a*	−3.0	−1.2	−3.6
50 . . .	431*b*	+4.3	+2.6	0.0	39 . . .	434*b*	−2.0	−3.5	−1.0
50 . . .	432*a*	+4.3	+3.6	+2.3	39 . . .	435*a*	−2.6	(+1.3)	−2.0
50 . . .	432*b*	+1.0	+3.1	+3.3	39 . . .	435*b*	−2.1	−5.3	−1.7
45 . . .	431*a*	+2.3	+2.0	+1.3	39 . . .	436*a*	−3.6	−3.3	−4.0
45 . . .	431*b*	+2.8	+3.3	+2.3	47 . . .	434*a*	−2.3	−4.0	−1.3
45 . . .	432*a*	+3.3	+2.3	+1.2	47 . . .	434*b*	−4.0	−0.7	−0.3
45 . . .	432*b*	+5.3	+2.6	+2.3	47 . . .	435*a*	+0.2	−3.6	−4.1
40 . . .	431*a*	+3.6	+2.1	+1.0	47 . . .	435*b*	−0.3	−3.6	−5.0
40 . . .	431*b*	+3.0	+1.0	+2.0	47 . . .	436*a*	−4.3	−2.3	−3.0
40 . . .	432*a*	+1.8	+2.6	+2.1	51 . . .	434*a*	−0.8	−2.6	−1.0
N. 40 . . .	432*b*	+0.2	+1.8	+3.6	51 . . .	434*b*	−5.6	(−6.3)	(−6.3)
S. 25 . . .	434*a*	(+2.6)	+0.3	+1.0	51 . . .	435*a*	−1.7	−2.3	−3.6
25 . . .	434*b*	−2.6	−1.0	−1.2	51 . . .	435*b*	−3.0	−2.6	−1.0
S. 25 . . .	435*a*	−2.3	−4.0	−1.8	S. 51 . . .	436*a*	−3.6	−1.0	−3.3

TABLE VII—*Continued*

Lat.	Plate	Δ						
		λ5300	λ5304	λ5328	λ5329.3	λ5329.9	λ5340	λ5348
N. 69°	430*a*	....	+8.3	+2.3	+1.3	+3.6	+4.8	+1.3
69	430*b*	+3.3	....	....	....	....	....	....
69	431*a*	+2.0	−0.2	+3.6	−1.0	+0.8	+4.6	−0.5
69	431*b*	+3.0	0.0	(+5.3)	+2.0	+5.8	+4.3	(+4.6)
69	432*a*	(+7.6)	(+14.2)	(+5.3)	+2.3	+5.9	+3.0	+1.5
69	432*b*	+4.6	+4.1	+1.8	+3.1	−2.3	+7.8	0.0
62	430*a*	+5.0	0.0	+2.6	−0.7	+2.1	+2.3	+0.7
62	430*b*	+0.7	....	....	....	....	....	....
62	431*a*	+4.8	+4.1	+3.0	+3.0	+5.0	+4.3	+3.8
62	431*b*	+5.3	+1.3	+2.8	(−2.3)	+1.3	+2.3	+1.7
62	432*a*	+2.6	+5.3	+1.3	+2.6	+3.6	+9.6	+2.0
62	432*b*	+5.0	+7.4	+2.6	−0.7	+6.9	+1.0	+3.6
55	430*a*	+1.7	(−4.6)	−0.7	+0.7	+2.6	−2.3	+0.7
55	430*b*	+0.8	....	....	....	....	....	....
55	431*a*	+3.6	+7.3	−0.5	−0.2	+2.6	−1.0	+1.7
55	431*b*	+3.8	+5.3	+2.0	+2.3	+1.2	−1.0	+1.5
55	432*a*	+4.3	+3.5	+4.0	+1.5	+4.0	+2.1	+3.0
55	432*b*	+1.3	+6.9	+3.6	+2.6	−2.6	+7.6	+1.3
50	430*a*	+3.3	+2.0	+2.6	+3.6	+5.3	+8.7	+2.8
50	430*b*	+3.6	....	....	....	....	....	....
50	431*a*	−0.3	−1.7	+1.8	+2.6	+4.3	+5.3	+0.8
50	431*b*	+2.8	+8.7	+2.3	+1.7	+0.8	+8.3	+2.3
50	432*a*	+2.1	+9.2	+0.7	+1.3	−0.3	+2.6	−0.3
50	432*b*	+5.0	+1.3	+1.2	+2.6	+8.6	+4.0	+2.3
45	430*a*	0.0	+6.9	+2.0	+1.7	+6.1	+4.6	+4.1
45	431*a*	+2.0	+4.6	+3.3	+2.6	−2.0	+2.5	+2.6
45	431*b*	+0.8	+2.0	+1.0	+2.0	+0.5	+8.4	+0.7
45	432*a*	+3.0	+5.6	+2.1	+1.3	+3.3	(−4.3)	+1.2
45	432*b*	+2.8	+7.8	+2.3	+2.3	+4.1	+7.9	+0.7
40	430*a*	+4.0	+5.9	+2.5	+5.3	+1.8	....	(+4.5)
40	431*a*	+3.0	0.0	+1.0	+3.0	+4.0	+5.0	+2.8
40	431*b*	−0.3	+6.3	+1.8	(−2.3)	+5.6	(−4.6)	+2.3
40	432*a*	+5.3	+3.0	+1.7	+1.8	−2.3	+8.7	+1.0
N. 40	432*b*	(−1.3)	+1.8	+2.0	0.0	−1.3	....	(−2.3)
S. 24	434*b*	....	(+10.9)	....	....	....	....	....
25	434*a*	−3.0	+2.5	0.0	−2.0	+0.7	−5.0	−1.7
25	434*b*	−3.6	....	....	0.0	−1.0	+0.7	−1.8
S. 25	435*a*	−2.0	....	−3.0	−3.6	−5.3	+1.3	−1.3

TABLE VII—*Continued*

Lat.	Plate	Δ						
		λ5300	λ5304	λ5328	λ5329.3	λ5329.9	λ5340	λ5348
S. 25	435*b*	....	....	−2.0	−3.8	....	....	(+1.7)
28	434*b*	....	+0.8	....	....	....	....	....
29	434*a*	....	+4.6	−3.3	−2.6	−4.8	−5.0	−1.3
29	434*b*	−0.7	....	−0.7	(+2.0)	−0.3	−5.9	−0.3
29	435*a*	−3.6	−6.6	−2.0	−2.3	−5.6	−10.2	−1.0
29	435*b*	....	....	−2.1	−2.3	....	....	−2.1
35	434*a*	−4.6	−4.5	(−5.3)	−3.6	−4.0	−4.1	−0.3
35	434*b*	−4.6	−7.4	−3.0	−0.5	−6.3	−0.3	−1.2
35	435*a*	−3.6	−5.0	−0.5	−2.1	+0.3	+0.3	+0.2
35	435*b*	....	....	−1.0	−1.6	....	....	+0.7
39	434*a*	−3.6	−11.6	+0.7	−2.3	−3.0	−0.2	−2.5
39	434*b*	−2.8	−3.6	−3.5	−5.0	−4.3	(−12.2)	−2.0
39	435*a*	−3.0	+0.3	−0.8	−0.3	−8.3	−2.6	−2.0
39	435*b*	....	....	−5.0	−4.5	....	....	−1.8
47	434*a*	+0.5	−8.6	−3.3	−4.3	−0.7	....	−2.0
47	434*b*	−5.0	−4.3	−0.5	−1.0	−6.4	−4.6	−0.2
47	435*a*	−0.2	−11.1	−2.6	−3.5	−3.1	−7.8	−0.3
51	434*a*	−2.0	−5.6	(+1.8)	−1.0	−2.1	−1.8	−0.7
51	434*b*	(−7.1)	−6.3	−2.6	−2.5	−9.2	−5.1	(−5.0)
51	435*a*	−3.1	−3.0	−1.0	+1.5	−3.8	−3.6	−1.2
S. 51	435*b*	....	....	....	....	....	....	−3.1

The coefficients A and B are defined by

$$\left.\begin{aligned} A &= 3 \sin (2\phi - D) + \sin D \\ B &= 3 \cos (2\phi - D) + \cos D \end{aligned}\right\} \qquad (2)$$

in which

ϕ = heliographic latitude of the point observed,

D = heliographic latitude of sun's center.

Were D and i both zero, $k\Delta$ would equal $3 \sin 2\phi$, which has zero values at $\phi = 90°$ N., 0°, and 90° S., a maximum at 45° N. and a minimum at 45° S. Since the observed displacement curves are approximately of this character, it follows that i must be a small angle (D is known to be small). This permits the calculation of the field-strength without further knowledge of i or λ, for, applying equation (1) to equal northern and southern values of ϕ,

$$k(\Delta_n - \Delta_s) = 6 \sin 2\phi \, (\cos D \cos i + \sin D \sin i \cos \lambda), \qquad (3)$$

whence[1]

$$\frac{4}{CH_p} (\Delta_n - \Delta_s) = 6 \sin 2\phi, \qquad (4)$$

[1] *Mt. Wilson Contr.*, No. 72, p. 11; *Astrophysical Journal*, **38**, 109, 1913.

in which

Δ is to be expressed in angstroms, and

H_p = field-strength in gaussian units at sun's magnetic pole,

C = separation of the n-components of the triplet in angstroms for a field of 1 gauss.

The omission of D, i, and λ has introduced an error of the order of i^2 (about 1 per cent), a precision that is ample, for the individual values of Δ are subject to uncertainties of 25 per cent or more.

For the determination of H_p it is convenient to apply equation (4) to the displacements observed at $\phi=45^\circ$, thus giving

$$H_p=\frac{4\Delta_{45}}{3C} \tag{5}$$

in which Δ_{45} is the mean of the absolute values for $\phi=45^\circ$ N. and S.

To determine Δ_{45}, equation (1) may be written

$$k\Delta=A \tag{6}$$

whence

$$\Delta_{45}=\Delta\frac{A_{45}}{A} \tag{7}$$

This neglects quantities of the order of i, but a combination of results for the observations in the northern and southern hemispheres, which with few exceptions are symmetrically distributed, reduces the error to one of the second order. In fact, the application of (6) to equal values of ϕ, N. and S., leads directly to (4), which is of this precision. The use of (7) is facilitated by the tabulation of A with the arguments ϕ and D.

To shorten the calculation, means were found for groups of displacements observed for each line in neighboring latitudes, the limits in general being 20°–29°, 30°–39°, 40°–49°, 50°–59°, 60°–69°, N. and S. The occasional values of Δ below 20° and above 69° were disregarded, since their contribution to the weight of Δ_{45} would have been insignificant.

The value of Δ_{45} having been found by a least-squares solution (the two hemispheres were treated separately), k was calculated by (6), which was then used for the detection of discordant

TABLE VIII

Displacements for Lines of Series VIII

Lat.	Plate	Δ
λ 4406.810		
N. 60°...	470*a*	+4.0
60....	470*b*	+3.0
52....	470*a*	+3.5
52....	470*b*	−0.3
52....	480*a*	+0.2
48....	470*b*	+1.0
47....	480*a*	+2.5
41....	470*a*	+3.5
40....	480*a*	+5.6
36....	470*a*	+3.3
36....	470*b*	+3.5
35....	480*a*	+2.8
31....	480*a*	(+7.4)
N. 27....	480*a*	+2.3
S. 33....	463*a*	−4.8
33....	463*b*	−4.5
33....	464*a*	−1.3
33....	464*b*	(+1.5)
38....	463*a*	−4.8
38....	463*b*	−4.0
38....	464*a*	−3.0
38....	464*b*	−1.8
43....	463*a*	−3.1
43....	463*b*	−2.5
43....	464*a*	−5.0
43....	464*b*	−5.9
51....	463*a*	−2.6
51....	463*b*	+0.2
51....	464*a*	−2.3
51....	464*b*	−2.0
57....	463*a*	−1.7
57....	463*b*	0.0
57....	464*a*	−2.0
S. 57....	464*b*	−5.1
λ 4418.499		
N. 60....	470*a*	+2.6
60....	470*b*	+4.6
52....	470*a*	+3.3
52....	470*b*	+1.0
52....	480*a*	+1.3
48....	470*a*	+3.3
48....	470*b*	+2.3
47....	480*a*	+2.5
41....	470*a*	+4.8
41....	470*b*	+1.2
40....	480*a*	+5.8
36....	470*a*	+2.6
36....	470*b*	+3.1
35....	480*a*	+3.3
31....	480*a*	+2.0
N. 27....	480*a*	+4.3
λ 4418.499—*Cont.*		
S. 33°...	463*a*	−3.0
33....	463*b*	−4.3
33....	464*a*	−3.8
33....	464*b*	−1.7
38....	463*a*	−6.3
38....	463*b*	−5.1
38....	464*a*	(−6.6)
38....	464*b*	−0.3
43....	463*a*	−3.8
43....	463*b*	−3.3
43....	464*a*	−1.3
43....	464*b*	−2.3
51....	463*a*	−1.3
51....	463*b*	−3.3
51....	464*a*	−4.1
51....	464*b*	−0.2
57....	463*a*	−2.3
57....	463*b*	−4.5
57....	464*a*	−1.3
S. 57....	464*b*	−2.6
λ 4421.733		
N. 60....	470*a*	+2.0
60....	470*b*	+1.3
52....	470*a*	+1.7
52....	470*b*	+3.0
52....	480*b*	+5.0
48....	470*a*	+3.3
48....	470*b*	+2.3
46....	480*b*	+5.6
41....	470*b*	+0.3
40....	480*b*	+4.3
36....	470*a*	+4.0
36....	470*b*	+1.5
35....	480*b*	+5.3
31....	480*b*	+1.3
N. 27....	480*b*	+6.3
S. 32....	478*a*	−8.9
32....	478*b*	+1.3
32....	479*a*	−5.3
32....	479*b*	−8.9
33....	463*a*	−6.8
33....	463*b*	−7.9
33....	464*a*	−2.1
36....	478*a*	−7.6
36....	478*b*	(+5.3)
37....	479*a*	−5.3
37....	479*b*	−5.9
38....	463*a*	−6.3
38....	463*b*	−9.9
38....	464*a*	−7.9
S. 42....	478*a*	(+5.0)
λ 4421.733—*Cont.*		
S. 42°...	479*b*	−5.3
43....	463*a*	−9.6
43....	463*b*	−7.3
43....	464*a*	−7.6
47....	478*a*	−3.0
47....	478*b*	−4.6
47....	479*a*	−4.0
47....	479*b*	−5.0
51....	463*a*	−5.9
51....	463*b*	−3.6
51....	464*a*	(−11.4)
53....	478*a*	−5.6
53....	478*b*	−2.8
53....	479*a*	−1.2
53....	479*b*	−6.9
57....	463*a*	+0.5
57....	463*b*	−8.9
57....	464*a*	−2.0
58....	478*a*	−8.9
58....	478*b*	−2.8
58....	479*a*	−5.8
S. 58....	479*b*	−2.6
λ 4430.785		
N. 52....	480*a*	(−0.8)
47....	480*a*	+2.3
40....	480*a*	+3.3
35....	480*a*	+2.0
31....	480*a*	+3.0
N. 27....	480*a*	+3.6
S. 33....	463*a*	−2.5
33....	463*b*	−3.0
33....	464*a*	−1.0
33....	464*b*	−2.8
38....	463*a*	−5.1
38....	463*b*	−5.3
38....	464*a*	−4.0
38....	464*b*	−2.6
43....	463*a*	−5.1
43....	463*b*	−3.3
43....	464*a*	−3.6
43....	464*b*	−1.8
51....	463*a*	−0.7
51....	463*b*	(+2.3)
51....	464*a*	−3.1
51....	464*b*	−2.5
57....	463*a*	−1.5
57....	463*b*	−3.6
57....	464*a*	0.0
S. 57....	464*b*	−4.0

TABLE VIII—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 4438.006			λ 4438.006—*Cont.*			λ 4438.006—*Cont.*		
S. 33°...	463*a*	+0.3	S. 43°...	463*a*	−0.5	S. 51°...	464*b*	−3.1
33....	463*b*	−3.6	43....	463*b*	−2.3	57....	463*a*	−3.3
33....	464*a*	−2.6	43....	464*a*	−0.7	57....	463*b*	−2.3
33....	464*b*	0.0	43....	464*b*	−2.0	57....	464*a*	+0.7
38....	463*a*	−3.5	51....	463*a*	−3.3	S. 57....	464*b*	−4.0
38....	463*b*	−3.3	51....	463*b*	+0.3			
S. 38....	464*b*	(+3.8)	S. 51....	464*a*	(−9.2)			

observations. Values giving residuals greater than three times the probable error were rejected, and the calculation was then revised for the determination of final values of Δ_{45} and k. Rejected observations appear in parentheses in Tables V–VIII.

The results are given in Table IX, which contains the values of Δ_{45} for each hemisphere without distinction as to sign, their differences and their means, the value of k, the original number of observations within the chosen limits for ϕ, the number rejected, and the probable error of a single observed value of Δ derived from a comparison with the theoretical displacement curve written in the form (6). The unit for Δ_{45} and the probable error is 0.001 mm. The table also indicates, in the last column, the value of one angstrom in millimeters, which is necessary for the transformation of Δ_{45} into angstroms.

In order to utilize all the available data, the results by van Maanen on λ 5929.898, Series I and III, and on λ 5812.139 and λ 5828.097, Series IV,[1] are also given in Table IX. To secure homogeneity these have been rediscussed by the foregoing method, which accounts for small differences between the values in Table IX and those previously published. The only results not included are the fragmentary measures of λ 5812 and λ 5828 in Series I and the results of Series II, which were from second-order spectra.

The consistency of the data is illustrated by the differences in the fifth column of Table IX, which by equation (1) are of the form

$$k(\Delta_{+45}+\Delta_{-45}) = 2 \sin D \cos i + 2 \cos D \sin i \cos \lambda \qquad (8)$$

[1] *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, **38**, 87, 1913.

and have the limiting values $2\sin(D+i)$ and $2\sin(D-i)$. The numerical results are such that i, as already assumed, cannot exceed a few degrees. Considering the difficulty of measurement

TABLE IX

VALUES OF Δ_{45}

(Unit for Δ_{45} and P.E. is 0.001 mm)

SERIES	λ	Δ_{45}				k	NO. OBSER.		P.E. 1Δ	FACTOR
		N.	S.	Diff.	Mean		All	Rej.		
I....	5929.898	3.69	5.02	−1.33	4.36	0.70	71	7	±2.96	4.91
III...	5929.898	4.77	5.50	−0.73	5.14	0.59	125	9	3.38	4.91
	5812.139	4.96	4.16	+0.80	4.56	0.66	96	10	1.61	4.87
	5828.097	4.44	3.74	+0.70	4.09	0.74	98	11	1.44	4.88
IV...	5831.821	1.48	2.14	−0.66	1.81	1.69	86	6	1.66	4.88
	5856.312	5.36	3.55	+1.81	4.46	0.70	54	5	1.72	4.89
	5928.013	3.70	2.10	+1.60	2.90	1.12	74	5	2.15	4.91
	6007.540	2.44	2.31	+0.13	2.38	1.26	72	2	2.51	4.93
	6039.953	6.15	5.79	+0.36	5.97	0.50	85	6	2.17	4.94
	6079.227	4.81	5.07	−0.26	4.94	0.60	74	3	0.99	4.95
V....	6111.290	1.99	2.18	−0.19	2.08	1.44	66	4	1.49	4.96
	6119.740	4.90	4.42	+0.48	4.66	0.64	83	8	2.10	4.96
	6129.190	3.41	3.83	−0.42	3.62	0.83	80	5	2.08	4.96
	6149.458	3.64	3.73	−0.09	3.68	0.81	76	5	1.79	4.96
	6173.553	5.14	5.10	+0.04	5.12	0.58	90	5	1.58	4.97
	5247.737	4.61	4.89	−0.28	4.75	0.62	184	12	1.50	4.72
	5250.817	2.60	2.50	+0.10	2.55	1.16	54	2	1.14	4.72
	5253.633	2.69	2.39	+0.30	2.54	1.18	54	3	1.14	4.72
	5263.486	2.00	2.06	−0.06	2.03	1.47	54	4	1.03	4.72
	5300.929	2.96	2.96	0.00	2.96	1.01	50	3	1.31	4.74
VII..	5304.355	4.46	6.15	−1.69	5.30	0.58	47	3	2.94	4.74
	5328.515	2.12	2.02	+0.10	2.07	1.44	51	4	1.01	4.75
	5329.329	1.94	2.36	−0.42	2.15	1.40	52	3	1.17	4.75
	5329.975	2.75	3.98	−1.23	3.36	0.92	48	0	2.07	4.75
	5340.639	4.73	3.58	+1.15	4.16	0.73	45	3	2.60	4.75
	5348.511	1.79	1.29	+0.50	1.54	1.98	53	5	0.95	4.75
	4406.810	2.88	3.13	−0.25	3.00	1.00	34	2	1.28	4.50
	4418.499	3.14	3.04	+0.10	3.09	0.97	36	1	1.10	4.51
VIII.	4421.733	3.33	5.74	−2.41	4.54	0.72	52	3	2.12	4.51
	4430.785	3.09	3.04	+0.05	3.06	0.98	26	2	1.01	4.51
	4438.006	......	2.09	....	2.09	1.41	19	2	±1.44	4.51

and the smallness of the unit in which the differences are expressed ($1\ \mu$), the internal agreement is very good, especially for Series V and VII, which include later measures and plates of somewhat better quality.

5. CONFIRMATION OF THE EXISTENCE OF THE SUN'S GENERAL FIELD

It is appropriate at this point to remark upon the character of the evidence now presented as bearing upon the existence of the sun's general magnetic field. It is scarcely necessary to state that it confirms the results of the preliminary investigation. For example, a moment's inspection of Tables V–VIII shows that the displacements of the twenty-six additional lines agree with those found for the four lines discussed in *Mount Wilson Contribution*, No. 71. The algebraic signs are opposite in the northern and southern hemispheres and give the same magnetic polarity as was deduced from the earlier results. Such differences as occur relate mainly to the amplitude of the displacement-curves, and this is determined by the field-strength at the points in which the different lines originate and the magnitude of their respective Zeeman separations, modified to some extent perhaps by systematic influences depending upon line-intensity.

Comparing the variation of the displacements as a function of latitude with the theoretical behavior of a uniformly magnetized sphere, we find as before an agreement that is within the limits of the uncertainty affecting the measured displacements. Moreover, the small differences in the values of Δ_{45} for the northern and southern hemispheres, which have been collected in Table IX, leave no doubt as to the correctness of the original estimate that the inclination of the magnetic axis to the solar axis of rotation cannot exceed a few degrees.

Added weight is given to these conclusions by the precautions taken to avoid physiological error and by the independent confirmation of the displacements by measurers who were entirely ignorant of the data of observation. But by drawing upon the results of another investigation, which will be published in detail later,[1] we can make the case even stronger.

In the present paper we have been content to indicate that the inclination of the sun's magnetic axis does not exceed a few degrees and may be disregarded without appreciably affecting the results

[1] See, however, a preliminary account in *Mount Wilson Communication*, No. 50; *Proceedings National Academy of Sciences*, 4, 4, 1918.

of the discussion. It is easy, however, to rearrange the fundamental equation (1) in such a way as to derive from suitably arranged observations made on a single day a value of the quantity

$$Y = \tan i \cos \lambda \tag{9}$$

The longitude λ varies from day to day, and a comparison of the values of Y for different days extending over a sufficient interval will enable us to determine the inclination i and the period of revolution P. From an extended series of observations on three chromium lines, λ 5247, λ 5300, and λ 5329, it has been found that the calculated values of Y are actually in close agreement with a periodic function of the form of (9).[1]

It is of interest to consider the implications of this result. The curve of displacements defined by equation (1) is very nearly a sine curve. Disregarding D, which is always small, the curve will pass through the origin if i is zero. In general, for $\phi = 0$,

$$k\Delta = -2 \sin D \cos i + 4 \cos D \sin i \cos \lambda, \tag{10}$$

and since i as well as D is small, the curve always passes near the origin, Δ_0 having values that are sometimes positive and sometimes negative.

Thus the changing position of the magnetic axis caused by its revolution around the sun's axis of rotation produces a small shift of the displacement-curve in its own plane, together with some change of form; and from these second-order effects the inclination and period have been derived. Since the three lines selected for the investigation were observed on sixty-three days (the observations extend over an interval of one hundred and ten days), nearly two hundred separate curves enter into the calculation. Not only is the characteristic form revealed in every case, but, of far more importance as evidence of the reality of the field, the curves for the separate days are so related in form and in position with respect to the origin that the second-order quantities Y satisfy equation (9) throughout the entire series, including more than

[1] It may be added that the resulting value of i is $6\overset{\circ}{.}2 \pm 0\overset{\circ}{.}4$, while the magnetic axis revolves about the sun's axis of rotation in a period of 31.79 ± 0.31 days.

three complete revolutions of the magnetic axis about the axis of rotation.

Various details are illustrated by Figs. 3 and 4. The first gives a series of displacement curves for two dates, September 2 and 14, 1914—both the original observations and the theoretical curves—based upon the unknowns

$$x=\frac{\cos i}{k}, \qquad y=\frac{\sin i \cos \lambda}{k}$$

calculated from the data for the respective days. Aside from the close accordance of the plotted points, the shift in the position of the curves with respect to the origin is to be noted. Since D was sensibly constant and equal to $+7^\circ\!.2$, the change in the algebraic sign of Δ for $\phi=0$ indicates a similar change in the sign of $Y=\sin i \cos \lambda$; see equation (10). The northern end of the magnetic axis was accordingly directed toward the observer on September 2 (λ near 0°) and away from him on September 14 (λ near 180°). Fig. 4 shows the close agreement of the values of $Y=y/x$ with equation (9). These results, particularly those illustrated in Fig. 4, exhibit a degree of internal consistency which is a searching test of the validity of the conclusions as to the existence of the sun's general field.

On the other hand we must consider the significance of the negative results from the lines listed in Table IV, which are not affected to any measurable amount by the sun's general field, although all of them are susceptible to the action of the magnetic fields in sun-spots. Certain aspects of the question thus raised are discussed in a later section. At present it is sufficient to state that the probabilities established by the evidence favorable to the existence of the field are so great that the negative results are presumably to be attributed to some class distinction separating the two lists of lines.

Our experience indicates that a very considerable number of lines will ultimately be found to show the influence of the sun's general field. The list of elements represented by displaced lines now includes iron, chromium, nickel, vanadium, and titanium, but it will doubtless be possible to add other elements.

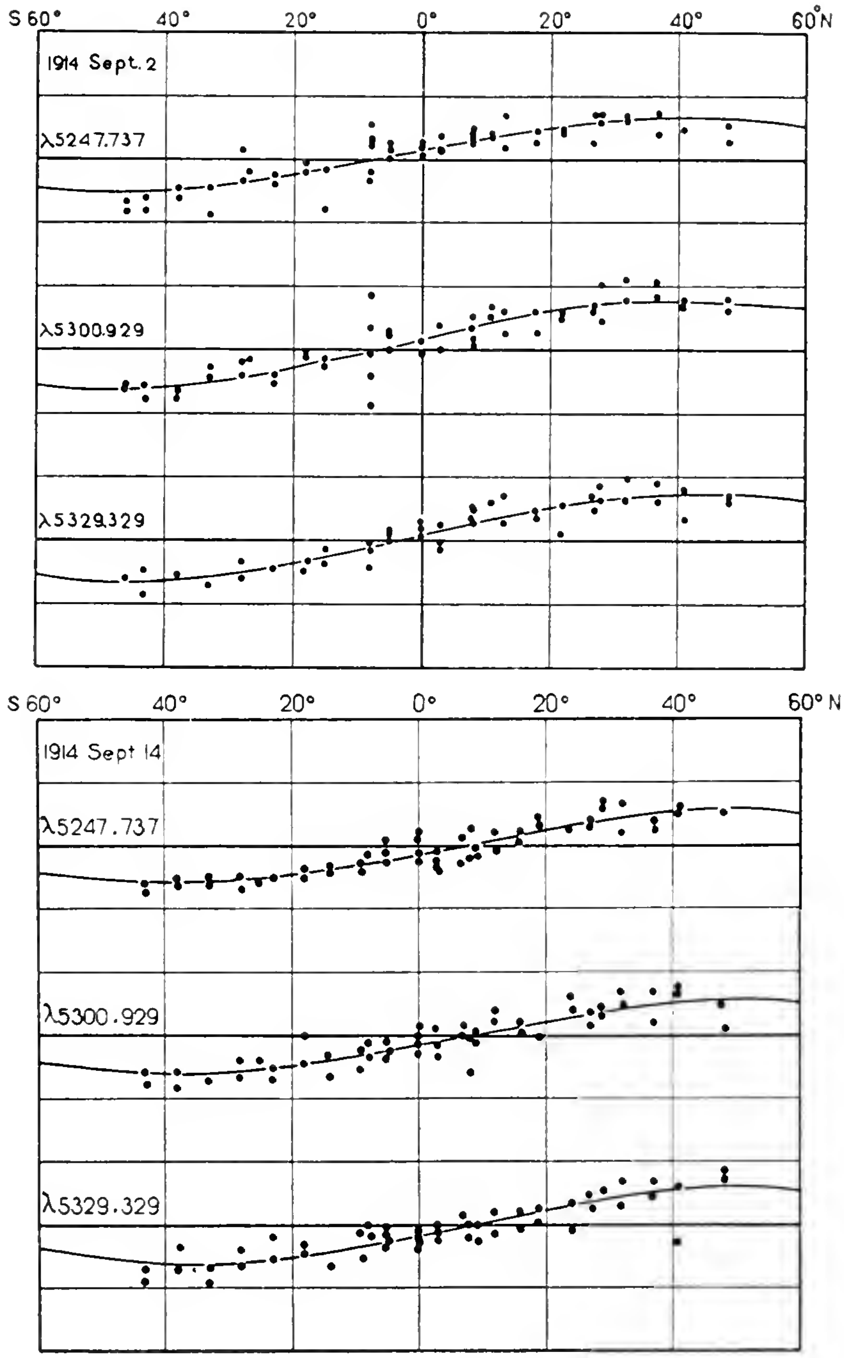

FIG. 3.—Displacement-curves for September 2 and 14, 1914. Abscissae are heliographic latitudes. Ordinates are displacements, the scale being 1 division of diagram $=0.005$ mm. The curves, which correspond to equation (1), have been derived from the observed values of Δ. Their ordinates for $\phi=0$ represent the combined influence of D, i, and λ. The data for the three lines give, as mean values of $V=\tan i \cos \lambda$, $+0.213$ for September 2 and -0.159 for September 14. These are plotted as single points in Fig. 4, together with similar values of V for each of the other dates.

6. CALCULATION OF THE FIELD-STRENGTH

The results in Table IX are now to be combined with the laboratory data in the fourth, fifth, and sixth columns of Table X, where C, the magnetic separation for a field of one gauss, is expressed in 0.0001 A as a unit. The values of Δ_{45} in the seventh column, expressed in the same unit, depend on the sixth and last columns of Table IX. The field-strength at the magnetic pole of the sun, which, it should be repeated, is here considered to be a uniformly magnetized sphere, is then found from C and Δ_{45} by equation (5).

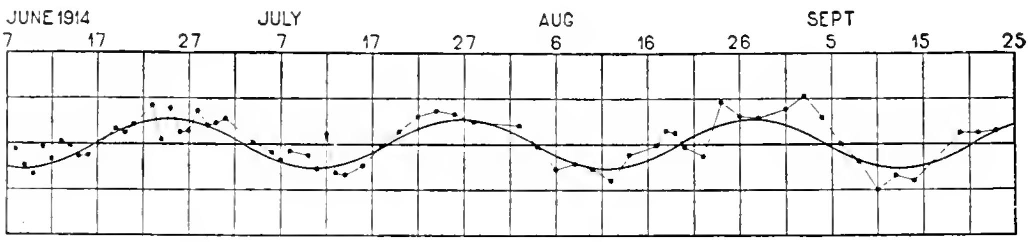

FIG. 4.—The curve $V = \tan i \cos \lambda$. Each plotted point is derived from data of a single day similar to those illustrated in Fig. 3. Approximate values of i, P, and t_0 were read from a provisional curve. Differential corrections derived by a least-squares solution gave the final values, which correspond to the curve shown in the figure. The close agreement of the values of V derived from the observed displacements with the theoretical curve is a most exacting test of the existence of the sun's general field.

Several of the laboratory data are from Mr. King's table[1] for iron, but most of the lines had to be specially observed. In spite of the great difficulties involved in the laboratory investigation of faint lines, Mr. Babcock has succeeded in determining C for all but four of the lines. One of these is unidentified, a second is a chromium line of intensity 0, and the others are the iron lines λ 5928.013 and λ 5929.898, both of solar intensity 2. In the previous paper doubt was expressed as to the reliability of the identification of λ 5929.898 with iron. The line has since been observed by Mr. Babcock in the spectrum of the core of the iron arc, but it does not appear in the spark and its separation by the magnet has not yet been determined. Hence we must still depend upon the approximate value of C derived from its separation in spots.

[1] *Papers Mt. Wilson Observatory*, 2, Pt. 1; *Carnegie Institution Publication*, No. 153, 1912.

TABLE X

LABORATORY DATA AND FIELD-STRENGTH

λ	El.	Int.	No. Components	$\frac{\Delta\lambda}{a\lambda^2}$	C	Δ_{45}	H_p	Weight
4406.810	V	2	....	1.5	0.274	6.7	32.5	1
4418.499	Ti	1	6?	1.3	0.230	6.9	40.1	1
4421.733	V	0	12	2.4	0.45	10.1	29.9	1
4430.785	Fe	3	3	2.5	0.460	6.8	19.7	2
4438.006	V	0	4?	1.6	0.297	4.6	20.6	0.5
5247.737	Cr	2	3	2.54	0.653	10.1	20.6	3
5250.817	Fe	3	3?	1.5	0.381	5.4	18.9	2
5253.633	Fe	2	3?	1.5	0.377	5.4	19.1	2
5263.486	Fe	4	3	1.5	0.390	4.3	14.7	2
5300.929	Cr	2	3	1.9	0.498	6.2	16.6	2
5304.355	Cr	0	4	1.6	0.424	11.2	35.2	1
5328.515	Cr	2	9?	1.2	0.307	4.4	19.1	1
5329.329	Cr	3	....	1.7	0.440	4.5	13.6	1
5329.975	Cr	0	....	....	....	7.1	......	....
5340.639	Cr	0	9	1.6	0.438	8.8	26.8	2
5348.511	Cr	4	10?	1.6	0.429	3.2	9.9	2
5812.139	Fe	0	....	0.7	0.230	9.4	54.7	1
5828.097	......	0	....	....	....	8.4	......	....
5831.821	Ni	1	....	0.85	0.272	3.7	18.1	1
5856.312	Fe	2	....	1.0	0.306	9.1	39.6	1
5928.013	Fe	2	....	....	....	5.9	......	....
5929.898	Fe	2	....	....	0.72	9.9	18.3	1
6007.540	Ni	1	....	0.8	0.290	4.8	22.0	1
6039.953	V	0	4 or 6	1.5	0.502	12.1	32.2	2
6079.227	Fe	2	....	2.0	0.700	10.0	19.1	1
6111.290	Ni	2	3 or 4	0.9	0.315	4.2	17.8	1
6119.740	V	1	12	1.0	0.40	9.4	31.3	1
6129.190	Ni	1	....	1.1	0.332	7.3	29.3	1
6149.458	Fe	2	4?	1.3	0.446	7.4	22.2	1
6173.553	Fe	5	3	2.5	0.888	10.3	15.5	2

REMARKS RELATING TO LABORATORY DATA

λ 4406 Two groups each of p- and n-components, much widened.
4421 Weighted mean separation, based on measures of unresolved components.
4438 Components wide, but separation well determined; two sharp p-components.
5329.3 Only one p-component. Diffuse.
5329.9 Very weak and diffuse. Perhaps two p-components.
5340 Weak on laboratory plates.
5348 Two narrow groups of n-components; probably three or four in each, unresolved.
5831 p-components not observed; n-components not sharp.
5856 p-components not observed.
5928 Completely covered by an air line in spark. Inductance sufficient to cut out air line obliterates the iron line.
5929 Not observed in spark. C from observations of sun-spots.
6007 Difficult; p-components not observed.
6039 Two narrow groups of n-components, unresolved.
6079 Two n-components; violet n-component blended with adjacent line.
6111 Very difficult; p-components not observed; two n-components.
6119 Three groups of four components each. Measures of high weight, but C is approximate because of unequal intensities of n-components.
6129 Difficult; p-components not observed.

The results for the structure of the lines are far from complete. In several cases the number of components is uncertain or unknown. The number of normal triplets is relatively small; nevertheless the theory developed for lines of this class or, more generally, for those with three groups of components of definite intensity relations,[1] has been applied to all of the lines observed. The error thus involved in cases of unusual structure is probably small, for the field-strength is based upon the displacement at $\phi=45°$, where the inclination of the line of sight to the lines of force is so small that the p-components and one group of the n-components are not transmitted to any appreciable extent by the analyzing apparatus of the spectrograph. Consequently the displacements must be nearly independent of the structure of the lines.[2] But Δ_{45} is not alone the result of observations at $\phi=45°$, for it includes displacements observed throughout the interval $\phi=20°$ to $\phi=69°$. The substantial agreement of values of Δ_{45} found from different latitudes leads, however, to the conclusion that the influence of complex structure is unimportant.

The weights of H_p in the last column of Table X have been assigned after a consideration of various circumstances that affect the precision. Both faint and strong lines are difficult to measure. Thus far it has not been found possible to use lines fainter than 0, while the large scale of the photographs gives to lines of intensity 4 or 5 a width that greatly increases the difficulty of the settings. Lines of intensity 2 or 3 are perhaps most easily measured, but, independently of intensity, contrast and sharpness enter to such a degree that general statements must be cautiously made. Further, the number of observations naturally influences the precision of Δ_{45}, the reliability in this respect being indicated by the data in Table IX. On the laboratory side there is also a wide range in precision, according to the number, character, and the more or less perfect resolution of the components of the individual lines. The adopted weights represent an attempt to strike a balance between these different factors.

It will be noted that the value of H_p for λ 5929.898 (18.3 gausses) differs widely from that of 28 gausses previously published.

[1] *Mt. Wilson Contr.*, No. 72, p. 11; *Astrophysical Journal*, **38**, 109, 1913.

[2] *Mt. Wilson Contr.*, No. 72, p. 12; *Astrophysical Journal*, **38**, 110, 1913.

The discrepancy arises from the large systematic difference between the measures of Miss Lasby and van Maanen. The original value was based upon the mean of the two series of measures, while, to secure homogeneity with the other results, that of Table X depends on the measures of van Maanen alone.

We desire at this point to express our obligations to Miss Richmond, Miss Wolfe, and Miss Felker of the Computing Division for their assistance with the least-squares reductions and the extensive calculations necessary for the determination of the field-strength from the individual lines.

7. VARIATION OF FIELD-STRENGTH WITH INTENSITY OF THE LINES

From an examination of Table X it is evident that the values of H_p vary with the intensity of the lines observed. To exhibit the nature of this relation, the data have been collected in Table XI, which also gives the weighted mean field-strength (the weights are in parentheses) corresponding to lines of each intensity for each of the elements thus far observed. It will be noted that, in general, the values of H_p corresponding to a given intensity, at least for any single element, are approximately equal. In the case of iron, intensity 2, for example, the agreement is surprisingly good, with the exception of the result for λ 5856. The mean results are shown graphically in Fig. 5.

In forming the means, the iron lines λ 6079 and λ 6149, which are slightly enhanced, perhaps should have been omitted, since lines of this character probably correspond to a higher level than that indicated by unenhanced lines of the same intensity. The mean field-strength would not, however, have been appreciably modified. The single titanium line λ 4418 is also enhanced, though not to an important degree.

For iron and chromium we find a rapid decrease in field-strength with increasing line-intensity. For vanadium, nickel, and titanium the data are too slender to establish independently the nature of the relation; but, including lines of all intensities, the mean value of H_p for each of these elements is in general agreement with the results for iron and chromium. The field-strength for iron,

TABLE XI

FIELD-STRENGTH, INTENSITY, AND LEVEL

Element	Intensity 0	1	2	3	4	5
Fe	5812 54.7(1)	…	5253 19.1(2)	4430 19.7(2)	5263 14.7(2)	6173 15.5(2)
	…	…	5856 39.6(1)	5250 18.9(2)	…	…
	…	…	5929 18.3(1)	…	…	…
	…	…	6079 19.1(1)	…	…	…
	…	…	6149 22.2(1)	…	…	…
Means	5812 54.7(1)	…	5753 22.9(6)	4840 19.3(4)	5263 14.7(2)	6173 15.5(2)
Level	270	…	335	370	410	450
Adopted	253	…	342	387	410	405
Cr	5304 35.2(1)	…	5247 20.6(3)	5329 13.6(1)	5348 9.9(2)	…
	5340 26.8(2)	…	5300 16.6(2)	…	…	…
	…	…	5328 19.1(1)	…	…	…
Means	5328 29.6(3)	…	5278 19.0(6)	5329 13.6(1)	5348 9.9(2)	…
Level	335	…	368	390	418	…
Adopted	335	…	368	390	418	…
Ni	…	5831 18.1(1)	6111 17.8(1)	…	…	…
	…	6007 22.0(1)	…	…	…	…
	…	6129 29.3(1)	…	…	…	…
Means	…	5989 23.1(3)	6111 17.8(1)	…	…	…
Level	…	361	375	…	…	…
Adopted	…	331	338	…	…	…
V	4421 29.9(1)	6119 31.3(1)	4406 32.5(1)	…	…	…
	4438 20.6(0.5)	…	…	…	…	…
	6039 32.2(2)	…	…	…	…	…
Means	5350 29.9(3.5)	6119 31.3(1)	4406 32.5(1)	…	…	…
Level	365	390	418	…	…	…
Adopted	315	317	387	…	…	…
Ti	…	4418 40.1(1)	…	…	…	…
Level	…	360	…	…	…	…
Adopted	…	361	…	…	…	…

line-intensity o, is large in comparison with the corresponding value for chromium and may be considerably in error, for it depends upon a single line, difficult of measurement on the solar plates and troublesome to investigate in the laboratory because of its faintness.

In a series of important papers[1] Mr. St. John has shown in a convincing way that lines of increasing intensity represent successively higher levels in the solar atmosphere. In accordance with

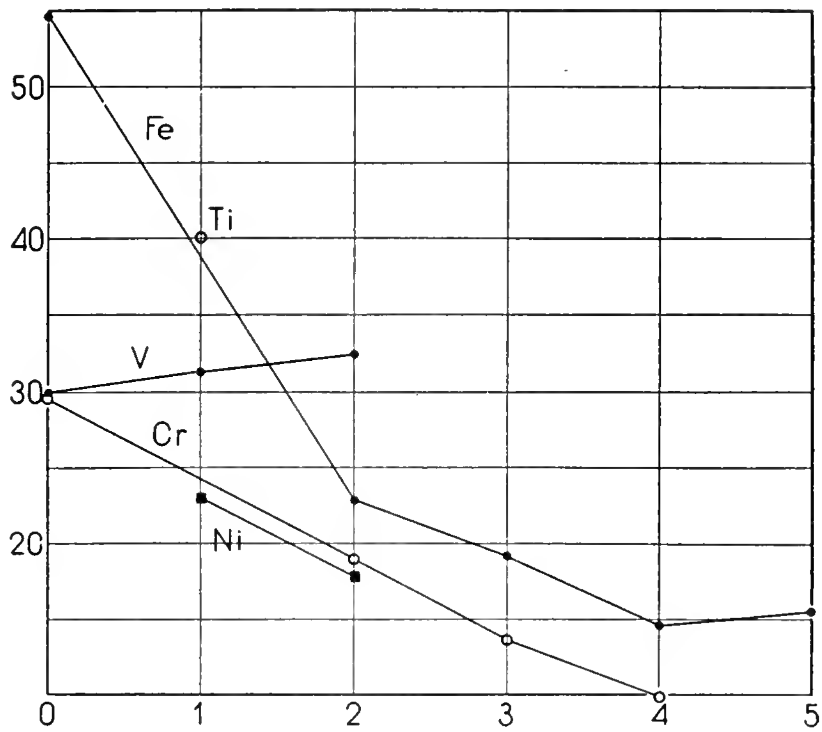

FIG. 5.—Variation of field-strength (ordinates, in gausses) with the Rowland intensity (abscissae) of lines in the solar spectrum.

the suggestion on an earlier page, the data for field-strength and intensity in Table XI and Fig. 5 are therefore open to interpretation as the result of a variation of field-strength with level, and would indicate that the intensity of the sun's general field decreases rapidly in passing upward from the level represented by lines of intensity o.

In the main, the curves of Fig. 5 are in agreement with Mr. St. John's measures of radial velocities in sun-spot vortices. Thus

[1] *Mt. Wilson Contr.*, Nos. 69. 74, 88; *Astrophysical Journal*, **37**, 322, **38**, 341, 1913; **40**, 356, 1914.

we find from his chart[1] that lines of iron, chromium, vanadium, and nickel having the same intensity occur at approximately the same level, while titanium lines are relatively higher by one unit of intensity. In other words, we should expect the curves to lie close together, as they actually do. The systematic difference for iron and chromium is probably real. The lines observed may actually occur at slightly different elevations, as the two curves would indicate, for we are dealing with a limited number of lines observed over the general solar disk, while St. John's results are averages for large numbers observed at the outer edge of the penumbrae of sun-spots.

But, as already stated, the correlation of the observed changes in field-strength with variations in level depends upon the assumption that the measured displacements are free from systematic errors which vary with the intensity of the lines observed. It is therefore important that the question of systematic errors be given careful attention.

8. SYSTEMATIC ERRORS

Comparing the results of different observers, the displacements determined by Miss Lasby[2] appear to be 50 per cent larger than those by van Maanen, and seem also to have been in excess of those found by Mr. Adams in his trial measures.[3] On the other hand the mean values for Miss Richmond and Miss Felker in Table I are appreciably smaller than those by van Maanen. Further, disregarding the inconclusive discordance for λ 5928, the measures by van Maanen are in satisfactory agreement with those from the Koch registering photometer, although the data are insufficient for final conclusions.

Personal errors, however, we may suppose to be approximately constant, and should not therefore affect the relative values of the field-strength derived from lines of different intensities. Depending upon the measures of a single observer, the data discussed in this paper should be homogeneous, and it is highly improbable that the

[1] *Mt. Wilson Contr.*, No. 74, p. 5; *Astrophysical Journal*, **38**, 345, 1913.

[2] *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, **38**, 87, 1913.

[3] *Mt. Wilson Contr.*, No. 71, p. 32; *Astrophysical Journal*, **38**, 56, 1913.

important relation between field-strength and line-intensity should be a consequence of personal error.

Approaching the question of systematic errors from a different direction, we are confronted by the following apparently contradictory results:

1. The three lines λ 5329.329 (Cr, 3), λ 5812.139 (Fe, 0), and λ 5828.097 (—, 0) show displacements in the second order which are sensibly equal, when reduced to the same scale, to those observed in the third order (van Maanen estimates the intensity of λ 5329 on Mount Wilson plates to be 2).

2. In third-order spectra the lines λ 5247.737 (Cr, 2) and λ 5929.898 (Fe, 2) both show displacements attributable to the sun's general field, although neither is appreciably displaced in second-order spectra (van Maanen's estimate of the intensity of λ 5247 on Mount Wilson plates is 3).

3. The lines λ 6173.553 (Fe, 5) and λ 6302.709 (Fe, 5) have wide separations of nearly equal magnitudes in the spectra of sun-spots. The former shows one of the largest displacements thus far observed in the case of the sun's general field (the measures are of third-order spectra); but the latter, λ 6302, has no measurable displacement in the first, second, or third orders.[1] (On Mount Wilson photographs λ 6302 is decidedly stronger than λ 6173 and much more difficult of measurement.)

It is difficult to find in these results any influence certainly due to intensity. The fact that the intensities of the lines under (1) are less than those under (2) might be suspected of having something to do with the failure of λ 5247 and λ 5929 to show displacements in the second order. But we should then be at a loss to know the bearing of this inference upon the third-order results for lines of high intensity, such as λ 6173 and λ 6302, referred to under (3); and the difficulty would only be increased when we

[1] This statement refers to the mean result from a number of photographs extending over a wide range of heliographic latitude. Small displacements are shown by individual groups of spectrum strips which are probably real in part, although perhaps not the result of a magnetic field. Small irregularities of photographic density may produce appreciable shifts, but these will be accidentally distributed, and have nothing to do with the systematic displacements produced by the general field. Note in this connection the results for the registering photometer given on p. 211.

attempted to account for the behavior of the faint line λ 6012.450 (Ni, 1), which has a very large displacement in the spectra of sunspots and none at all in the general-field photographs. Clearly something besides line-intensity is involved in these phenomena.

There are reasons for believing that the amount and distribution of the photographic density across a line are factors of importance in the measurement of minute shifts.[1] Both the lines which are undisplaced in the second order have sharp edges and show strong contrast on the photographs taken in this order. The third-order images of these lines, and both second- and third-order images of the lines mentioned under (1), are much flatter. Their density-curves have neither the steepness nor the height that characterize the second-order photographs of λ 5247 and λ 5929; and, in general, lines that show displacements do not possess intensity-curves of this type. The minimum density within a line (the reference is to the negative) and the contrast vary with exposure and development. Very faint lines naturally give difficulty in measurement, but low density and excessive contrast also produce unsatisfactory results. The ease of setting is of course greater in the case of sharp narrow lines, but for such lines displacements have not been detected.

These facts may perhaps be explained by supposing that it is the point of minimum density—the highest point of the intensity-curve—rather than the edges of the line that determines the value of a setting. In cases of excessive contrast the middle of the line is likely to be underexposed and it will then be impossible to locate the minimum with precision, for throughout the central section there will be no appreciable deposit of silver.

The explanation also applies to the peculiar behavior of λ 6302 described under (3). On the Mount Wilson photographs its intensity is at least two units greater than that of λ 6173. In the third order it is so broad and diffuse that satisfactory measurement is impossible, while in the first and second orders the contrast and density are such that the point of minimum density cannot be located; λ 6173, on the other hand, is well suited to measurement.

[1] In addition to the accompanying discussion, see also Hale, *Mt. Wilson Contr.*, No. 71, p. 50; *Astrophysical Journal*, 38, 74, 1913.

It seems likely, therefore, that the form of the intensity-curve rather than intensity in the usual sense is a controlling factor in the detection and measurement of the displacements. This conclusion is not directly an answer to the inquiry as to the existence of systematic errors depending upon intensity, but it seems to account for the irregularities that might have been attributed to such an influence, and we are thus left without any evidence which would indicate that such errors have entered into the results.[1] Tentatively, we are therefore disposed to accept the relation between field-strength and intensity illustrated by the curves of Fig. 5 as an indication of a decrease in the field with increasing level in the solar atmosphere.

9. DISCUSSION OF THE RESULTS IN RELATION TO LEVEL IN THE SOLAR ATMOSPHERE

To examine further the results in relation to elevation in the solar atmosphere, they have been compared with the flash-spectrum measures of Mitchell.[2] A number of the lines employed were observed by him in the chromosphere; but in the nature of the case the precision of the calculated levels is less than that required to make the comparison effective, and we are therefore obliged to use mean levels. To this end the results calculated from the length of the chromospheric arcs were taken from Mitchell's tables for individual lines and grouped into means for each intensity unit of Rowland's scale. All blends and bracketed wave-lengths were omitted, and to avoid photographic influences only those lines between λ 4000 and λ 5000 were included.

This limitation is necessary in order that the calculated levels for the different elements may be comparable. The distribution of the lines throughout the spectrum is different for different elements, and, since the sensitiveness of the photographic equipment changes with the wave-length, systematic differences between the different intensity-and-level curves are likely to enter unless the

[1] A method of testing this question by measurements in the second and third orders of lines slightly displaced by the weak fields surrounding sun-spots will be tried in the future.

[2] *Publications McCormick Observatory*, **2**, Pt. 2; *Astrophysical Journal*, **38**, 407, 1913.

data relate to the same limited region of the spectrum. An exception, however, was made in the case of nickel, for which the region $\lambda\,5000$–$\lambda\,6000$ was also used in order to compensate for irregularities in the interval $\lambda\,4000$–$\lambda\,5000$.

The mean levels thus derived (in kilometers) have been entered in Table XI. Those for iron, chromium, and nickel show a regular increase in elevation with decreasing field-strength. For vanadium and titanium the results are discordant, but the evidence is inconclusive. Although the levels for these are systematically higher than for the corresponding intensities of other elements,

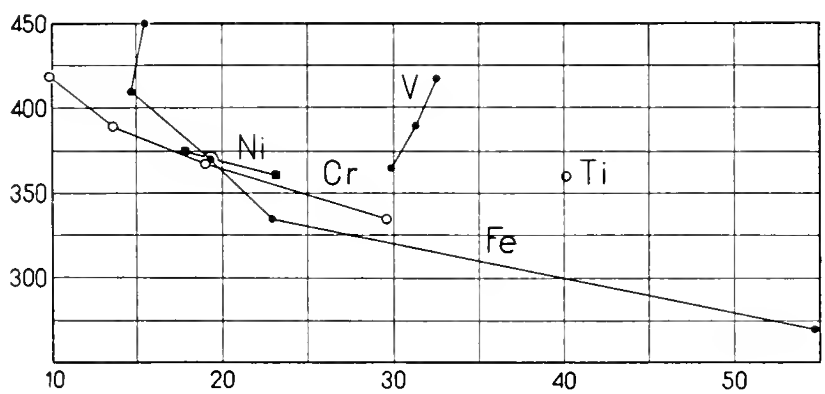

Fig. 6.—Variation of field-strength (abscissae, in gausses) with elevation in the solar atmosphere (ordinates, in kilometers). The levels require correction as described in sec. 9.

the extreme range, including all elements, is small, and apparently that portion of the sun's general field effective in producing the observed displacements is limited to a very thin shell in the solar atmosphere. The results are illustrated in Fig. 6.

Various reservations and exceptions to these conclusions immediately suggest themselves and have now to be considered.

First, we have tacitly assumed that lines of a given element and intensity correspond to the same level in the sun's atmosphere independently of wave-length. But we have reason to believe that the levels determined from line-intensity are affected by scattering,[1] which certainly varies with the wave-length; in addition there may be influences depending on wave-length which are peculiar to the different elements. We can remove the effect of scattering by the

[1] St. John, *Mt. Wilson Contr.*, No. 69, p. 17; *Astrophysical Journal*, **37**, 338, 1913.

application of an appropriate correction; but possible disturbances arising from other causes will be so involved with various uncertainties inherent in the observation of flash spectra that their consequences are necessarily disregarded.

Second, elevations derived from flash spectra must be accepted with caution, owing to the fact that for different elements lines having the same solar intensity may behave quite differently in the chromosphere, some showing much greater intensities in the flash spectrum than others. Since lines of high intensity are represented in such spectra by relatively long arcs, the calculated levels may exhibit differences that are wholly fictitious. Such, for example, is the case with vanadium as compared with the other elements observed.

Third, since the evidence for relatively greater elevation in the case of the enhanced lines seems unquestioned, it is not permissible to disregard this fact in the combination of the data now under discussion.

For a change in wave-length of 1000 A the influence of scattering is equivalent to about one unit of intensity,[1] lines to the red corresponding to a relatively lower level than their observed solar intensities would indicate. Levels assigned on the basis of the observed intensity will therefore be too high in the red and too low in the blue. The corrections apparently should vary directly as the fourth power of the wave-length, and may be derived with the aid of the levels in Table XI, care being taken to use for each element the rate of change in elevation per unit of intensity corresponding to the element and intensity in question.

As a matter of convenience we reduce the elevations to λ 5300, the wave-length of the chromium lines. The corrections in kilometers are given in Table XII.

To study the consequences of abnormal behavior of any element in the chromosphere, we may compare the intensities of its lines in the flash spectrum with those of the flash-spectrum lines of other elements. Flash intensities do not ordinarily correspond to those of the general solar spectrum. Lines of faint solar intensity commonly have flash intensities which are one or more units higher than

[1] St. John, *loc. cit.*

the Rowland values, while the stronger lines of the general solar spectrum have relatively low intensities in the chromosphere.

TABLE XII

CORRECTIONS TO LEVEL DEPENDING ON WAVE-LENGTH

Element	Intensity					
	0	1	2	3	4	5
Fe....	5812, −17	....	5753, −18	4840, +17	5263, 0	6173, −45
Ni....	....	5989, −17	6111, −21	....	....	....
V.....	5350, 0	6119, −28	4406, +17	....	....	....
Ti....	....	4418, +21	....	....	....	....

For the intensities involved in this investigation the behavior of iron and chromium in the chromosphere is substantially the same, and the levels calculated for their respective lines should be comparable, in so far as the point now under discussion is concerned. But with nickel, vanadium, and titanium there are deviations to be considered, which are indicated by Table XIII. For example,

TABLE XIII

FLASH INTENSITIES

SOLAR INTENSITY	FLASH INTENSITY				DEVIATIONS FROM FE AND CR		
	Fe, Cr	Ni	V	Ti	Ni	V	Ti
0..........	0.4	0.6	0.9	0.5	−0.2	−0.5	−0.1
1..........	0.7	1.1	1.4	1.0	−0.4	−0.7	−0.3
2..........	1.0	1.5	2.1	1.5	−0.5	−1.1	−0.5
3..........	1.4	1.4	3.0	1.8	0.0	−1.6	−0.4
4..........	2.0	1.3	4.0	2.2	+0.7	−2.0	−0.2

the relatively high flash intensities of vanadium as compared with iron and chromium lines of the same solar intensity will lead to calculated levels that are too high unless suitable corrections are applied. To derive such corrections, the flash-spectrum intensities may be plotted against the levels given in Table XI under the corresponding solar intensities. The changes in level corresponding to the differences of flash intensity in the last three columns of

Table XIII can at once be read from the curves thus defined. The results thus found are given in Table XIV.

TABLE XIV

CORRECTIONS DEPENDING ON ABNORMAL FLASH INTENSITY

Element	Solar Intensity		
	0	1	2
Ni	....	−13	−16
V	−50	−45	−48
Ti	....	−20	....

Finally, to arrive at a rough correction of the levels of the enhanced lines λ 6079 and λ 6149 which will reduce them to the system of the other lines, we may make use of the results of Mr. Adams on the displacement of lines at the sun's limb.[1] These show that the enhanced lines are shifted by larger amounts than unenhanced lines of the same intensity. For λ 6079 and λ 6149 he finds shifts of +0.009 and +0.010 mm, respectively. These are equal to the displacements of unenhanced lines whose intensities are at least two units greater than those of the lines in question. We accordingly find corrections of +75 km. The result is not very reliable, but the scanty data available suggest that the corrections should be larger rather than smaller. The corresponding effect upon the level for iron lines of intensity 2 given in Table XI is +25 km.

The application of this and the corrections in Tables XII and XIV to the levels in Table XI gives the values which are provisionally adopted as corresponding to the mean field-strengths. These results also appear in Table XI opposite the word "Adopted."

With this revision of the levels, the curves of Fig. 6 assume the form shown in Fig. 7. Notwithstanding the tentative character of several of the corrections, an improved agreement is noticeable in various directions. The iron and chromium curves are now nearly coincident, and four of the five vanadium lines, which before were seriously discordant, are now in excellent agreement with iron and chromium. The fifth vanadium line, λ 4406 (2), and the single

[1] *Mt. Wilson Contr.*, No. 43; *Astrophysical Journal*, **31**, 30, 1910.

titanium line, λ 4418 (1), remain discordant, and the agreement for nickel is less satisfactory than before. The titanium line is slightly enhanced and presumably something should be added to the adopted level, but the data necessary for the calculation of the correction are lacking. It is not surprising, however, that individual lines should have levels which differ from the mean level of all lines of the same intensity; and the uncertainty of the corrections which reduce the results for nickel to λ 5300 is more than sufficient to account for the systematic deviation shown by that element in Fig. 7.

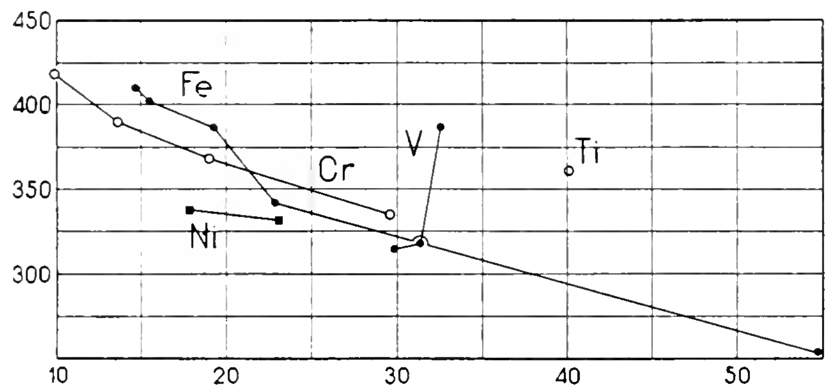

FIG. 7.—Adopted variation of field-strength with level in the sun's atmosphere. The elevations (ordinates) are in kilometers and the field-strengths (abscissae) in gausses.

The main point of this detailed discussion of corrections is that none of the disturbing influences mentioned is capable of modifying essentially what seems to be a fundamental relation between field-strength and level in the sun's atmosphere. Qualitatively the results of Fig. 7 are the same as those of Fig. 6, and quantitatively, even, the differences are confined within narrow limits.

10. INTERNAL EVIDENCE

It is now possible to point out certain internal agreements in the results that add much to the weight of the conclusions.

First, there is a wide range in the Zeeman separation of the components of the various lines under the influence of a constant field. Nevertheless, the measured displacements lead to closely accordant values for the field. As an example, the iron lines λ 5253 and λ 6149, of intensity 2, coefficient of separation about 0.4, give for H_p the same value as that derived from λ 5929 and λ 6079,

also of intensity 2, whose coefficient is 0.7. In fact the ten lines of intensity 2 may be divided into two groups according to large and small values of C, respectively, with mean results as follows:

C	H_p	Weight	No. of Lines
0.676	19.8	5	3
0.378	22.5	9	7

The two groups of lines give nearly the same value for H_p, although for a given field-strength the separations of one group as observed in the laboratory are nearly twice those of the other. Their solar displacements must therefore bear a similar relation to each other, and the precision of the measures has accordingly been such as to reflect these differences in the behavior of individual lines.

Second, reference has been made to the systematic difference in field-strength for iron and chromium lines of the same intensity illustrated in Fig. 5. The difference does not appear in Fig. 7, and upon examination we find that this is due to the circumstance that the calculated levels for chromium are higher than those for iron (see Table XI), the differences in field and intensity compensating each other in such a way that for a given level the calculated strength is the same whether we use the lines of iron or chromium. In other words the hypothesis of a change of field with level reconciles the differences of Fig. 5, and the observed displacements for these two elements differ by just the amount required for them to yield the same field-strength for a given elevation.

Third, the application of the corrections derived in the preceding section led in general to an improved agreement. Since all of the corrections are based upon the hypothesis of levels, the accordance of the final results is in favor of the applicability of this hypothesis.

11. FAILURE OF CERTAIN LINES TO REVEAL THE GENERAL FIELD

The influence of the density and contrast of a spectral line upon the detection of displacements due to the sun's general field has already been discussed. The apparently great importance of these factors raises a question as to the part they may have played in producing the negative results summarized in Table IV. It is certain, however, that the character of these lines is not wholly responsible for the absence of measurable displacements, and it is unlikely

that it has had any considerable influence, for many of the lines are of a satisfactory quality.

It is evident, therefore, that in the case of Table IV we are dealing with an anomaly of another kind. All of the lines there listed show displacements in the spectra of sun-spots. For some of these lines the separation of the components in spots is less than the average spot-separation of the lines in Table I, but the behavior of others (for example, λ 6005, λ 6012, and λ 6081) in spots is entirely comparable with that of the lines in Table III. The line λ 6012 (Ni, 1) is especially noteworthy. Its components are more widely separated by the magnetic fields in sun-spots than those of any of the lines in Table III excepting λ 6173 (Fe, 5); but on the general-field plates it shows no appreciable displacement, although other lines of similar intensity on the same photographs, such as λ 6007 and λ 6039, are certainly displaced.

There is obviously a lack of parallelism between the results for spot fields and for the sun's general field. At present no complete explanation of these differences can be given; we can only offer various suggestions that later experience may prove to be of more or less significance.

In general it is to be noted that sun-spot fields extend over a much greater range of level than can at present be explored in the case of the sun's general field. Moreover, the conditions of pressure, temperature, and ionization are more or less different in the two cases; and it is not surprising that complete parallelism in results should not be found for fields existing under such diverse conditions. It seems not improbable that, in the sun's general atmosphere, some of the lines listed in Table IV originate outside the thin shell of a hundred or more kilometers' thickness which seems to include that part of the field now accessible to observation.

A careful examination of the displacements in sun-spot vortices by Mr. St. John and Miss Ware reveals no essential difference between the two classes of lines. On the other hand, Mr. King, from a consideration of temperature conditions, finds for chromium and vanadium, at least, some indication of a class distinction. His conclusions are as follows:

The iron and nickel lines of Table III are faint in the arc and for the most part have not been observed in the furnace. The chromium and vanadium lines are usually strong at all furnace temperatures. The vanadium lines λ 4438 and λ 6120 are relatively strong at the lowest temperature at which the spectrum appears.

Table IV contains no lines which are in any respect low-temperature lines. The iron lines are of the type seeming to require not only high temperature but high-vapor density. They are shown best by the core of the arc and by the tube-arc, and with great difficulty by the furnace and the spark. Two titanium lines and one calcium line are produced by the furnace, but are faint or absent at low temperature.

12. THE LOCAL-WHIRL HYPOTHESIS

In *Mount Wilson Contribution*, No. 71. it was suggested that the general magnetic field of the sun might result from the combined effect of a great number of local whirls. Evidence opposing this view was offered, but the material available for discussion was insufficient, and we may now return to a consideration of the suggestion, which is regarded favorably by both Birkeland[1] and Brunt.[2]

The hypothesis assumes the existence of a large number of minute spots or pores, too small to be distinguished as such, but having magnetic and other properties similar to those of visible spots. Recent investigations at Mount Wilson show these properties to include:

1. An almost universal tendency of spots to occur in pairs, the chief members of which are of opposite magnetic polarity. This tendency to form bipolar groups was shared by the few extremely small spots which appeared near the last sun-spot minimum, when the general magnetic field was under observation.

2. The preceding members of bipolar spot-groups, in the great majority of cases, are of opposite polarity in the northern and southern hemispheres.

3. Since the last sun-spot minimum the preceding members of bipolar groups have been opposite in polarity to those observed in the same hemisphere before the minimum.

[1] *Comptes Rendus*, **157**, 394, 1913. [2] *Astronomische Nachrichten*, **196**, 169, 1913.

4. The magnetic axes of sun-spots are in approximate coincidence with solar radii.

5. The strength of the magnetic field of a spot is roughly proportional to its area.

Minute bipolar spots, even if distributed uniformly over the sun, could not account for the general field, for the following reasons:

1. If the members of each bipolar group were of equal field-strength, they would exactly neutralize each other in their combined effect.

2. If, as is usually the case, the preceding members of bipolar groups were larger, on the average, than the following members, the polarity of a general field due to their predominating magnetic influence should have reversed at the sun-spot minimum. Our observations show, on the contrary, that the polarity of the general field did not reverse at the minimum.

Suppose we assume, however, that the minute dark spaces between the brighter granules of the solar surface represent small single spots, opposite in polarity in the northern and southern hemispheres. It is not clear why they should fail to show the bipolar characteristics of larger spots, but, as single spots with almost no indications of bipolar structure are not very uncommon, we may make this assumption. It is more difficult to admit the possibility that their polarity did not reverse at the sun-spot minimum, because single spots, as well as bipolar groups, shared in this remarkable change. In any case, however, the hypothesis must face the fact that the magnetic axes of visible spots are very nearly radial, and that the granulated structure is best seen at the center of the sun, as though we were looking down through the bright granules into the darker regions between them. Under such conditions it is hard to see how we could fail to observe the Zeeman effect to the best advantage (along the lines of force) at the center of the sun, where the general magnetic field shows no displacements. If this objection were waived, it would still be necessary to assume a distribution of the granules such as to account for the observed inclination of the magnetic axis to the axis of rotation (6°2). Finally, there would remain the serious difficulty, pointed out by Brunt, of supposing that the total charge

of electricity in a pore, or in a minute vortex between the granules, is almost as great as that required to account for the intense magnetic field in a large sun-spot.

While for the foregoing reasons we are not inclined to give favorable consideration to the hypothesis of local whirls, further attempts will be made to overcome the observational difficulties of photographing the spectra of the dark spaces between the granules on a sufficient scale to detect possible local Zeeman effects.

In considering the bearing of our results on theories of the solar magnetism it should be remembered, quite apart from the question of systematic errors, that our highest values do not necessarily represent the full intensity of the sun's field. On the contrary, the apparent variation of the field-strength with level in the solar atmosphere renders it probable that more intense fields may ultimately be detected at lower levels. This change of field-strength, if actually as rapid as the results seem to indicate, will also afford a useful criterion of a satisfactory theory, which must furthermore account for the observed inclination of the magnetic axis to the sun's axis of rotation.

13. SUMMARY

The present investigation is a continuation of that in *Contribution*, No. 71. Measures of displacements are given for twenty-six additional lines in the solar spectrum belonging to the elements iron, chromium, nickel, vanadium, and titanium (Tables V–VIII). Eighteen other lines (Table IV), all of them susceptible to the influence of magnetic fields in sun-spots, show no measurable shift. The twenty-six lines, which through changes in position indicate the presence of a magnetic field (Table IX), confirm the results detailed in *Contribution*, No. 71, and seem to place beyond reasonable doubt the conclusion that the sun behaves approximately as a uniformly magnetized sphere (secs. 1, 2, and 5), with the magnetic axis only slightly inclined to the solar axis of rotation and a polarity corresponding to that of the earth.

Laboratory data are available for twenty-seven of the thirty lines (indirectly, for one line) known to be influenced by the sun's general field. Combined with displacements observed in the solar

spectrum, these yield for each line a value of the field-strength at the magnetic pole (sec. 6 and Table X). An intercomparison of results (Table XI) shows that the field decreases with increasing values of the Rowland intensity of the respective lines (sec. 7). Since line-intensity increases with the level at which the various lines originate, it would appear that the strength of the sun's general field falls off rapidly with increasing elevation in the solar atmosphere.

Using Mitchell's observations of the flash spectrum, we find that the part of the field now accessible to observation lies within the bounding surfaces of a thin shell in the solar atmosphere, whose thickness seems to be of the order of 150 km (Table XI, Fig. 6). The application of certain necessary corrections (Tables XII and XIV) improves the internal agreement and leaves outstanding as discordant only two of the twenty-seven lines (Fig. 7). With proper allowance for peculiarities in the behavior of different elements and in the Zeeman effect for individual lines, the results indicate that definite values of the calculated field-strength always correspond to definite levels in the solar atmosphere.

Systematic errors in the measured displacements varying in an appropriate manner with the intensity of the spectral lines would explain the observed dependence of field-strength upon line intensity; but there is no indication of the existence of such errors (sec. 8), and the internal evidence, on the other hand, is strongly in favor of the hypothesis of changing field-strength with changing level (sec. 10).

The anomalous behavior of the lines which show no displacement is not satisfactorily explained. Because of possible peculiarities of pressure, temperature, and electrical conditions necessary for the emission of these lines, they may originate outside the very limited region within which it has been possible thus far to observe the sun's general field (sec. 11).

The underlying causes of the sun's general magnetic field remain obscure. The evidence bearing on the local-whirl hypothesis is unfavorable to its acceptance as an adequate explanation of the existence of the field (sec. 12).

Mount Wilson Solar Observatory
February 1918

ON PARALLAXES AND MOTION OF THE BRIGHTER GALACTIC HELIUM STARS BETWEEN GALACTIC LONGITUDES 150° AND 216°[1]—*Concluded*

By J. C. KAPTEYN[2]

21. LUMINOSITY-CURVE OF THE B0–B5 STARS

In accordance with earlier results, I will assume that the luminosity-curves are Gaussian error-curves, with the equation

$$y=\frac{h}{\sqrt{\pi}}e^{-h^2(M-k)^2}\,. \qquad (100)$$

Care will, of course, be taken to note any indicated deviations from this form, particularly for the fainter half of the curve, for which we have scarcely any data from direct observations.

First solution: from the Cannon stars in the Nebula-group.—Since it was assumed that the number of B0–B5 stars fainter than 9.05 necessary to complete the quantities in the third column of Table XXX probably lies between 0 and 6, I will assume for this number the value 3. Any error thus committed will be of little importance. The arithmetic mean of the magnitudes thus becomes 6.58, and the median magnitude 6.48. If the luminosity-curve is an error-curve, or even a symmetrical curve, both the arithmetic mean and the median will coincide with the maximum. I thus adopt for the maximum of the frequency-curve of the apparent magnitudes $m_{max}=6.53$. As one-twelfth of the stars in this region belong to the outside stars (see Section 18) whose maximum (arithmetic mean) lies at $m=6.06$, we have

$$\text{Corrected } m_{max}=6.57. \qquad (101)$$

For the determination of the probable amount of deviation (probable error) we have from the seventh column of Table XXX

$\frac{1}{4}$ of the stars are fainter than 7.90. Therefore $r=7.90-6.57=1.33$

$\frac{1}{4}$ of the stars are brighter than 5.78. Therefore $r=6.57-5.78=0.79$,

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 147.

[2] Research Associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

so that in the mean we have $r=1.06$. Further, the parallax of the group being $0''.0054$,

$$M=m-6.34. \qquad (102)$$

We thus have for the constants of the luminosity-curve (100)

$$\left.\begin{aligned} K&=6.57-6.34=+0.23 \\ h&=\frac{0.4769\ldots}{r}=0.450 \end{aligned}\right\} \qquad (103)$$

Second solution: from the outside Cannon stars.—Treating the numbers of the eighth column of Table XXX in the same way, we find, as remarked in Section 18, a value of K which must be substantially correct and a value of h which must be too low. Noting that for the mean parallax $0''.0081$ of the outside stars

$$M=m-5.46 \qquad (104)$$

we find (27 stars)

$$K=+0.60 \qquad h>0.40 \qquad r<1.19. \qquad (105)$$

Third solution: from the bright stars in Table XXIX—Arranging the stars for which $\lambda<165°$ according to absolute magnitude, M, and parallax, we obtain the summary[1] given in Table XXXI.

Treating the numbers in Table XXXI exactly as was done in the similar case of *Mount Wilson Contribution* No. 82, pp. 41 and 42, I find for the luminosity-curve the results shown in Table XXXII.

The third column of Table XXXII gives the numbers corresponding to the observed luminosity-curve, the number of stars on which each value rests being added in parentheses. The extreme values are thus seen to be very uncertain. It seems impossible to determine from these data the values of both K and h in (100). I therefore adopted $K=+0.36$, which is the weighted mean of the values obtained in the first and second solutions, and found

$$h=0.50. \qquad (106)$$

[1] The absolute magnitudes were obtained by using the values of $\pi_{2.5}$ instead of the finally adopted π. For the luminosity-curve the difference is immaterial.

The theoretical curve computed with these values of h and K is in the fourth column of Table XXXII. The enormous residual for $M=+0.5$ is not to be taken too seriously. It corresponds to an irregularity in the actually observed numbers of less than three stars.

TABLE XXXI

NUMBER OF B0–B5 STARS

M	$\bar{M}$	π 0.″0194	0.″0154	0.″0122	0.″0097	0.″0077	0.″0061	0.″00485	0.″00385	0.″00305	
+1.75 to +2.25	+2.0				Domain of the stars apparently fainter than 5.80 (Harvard) and part of the stars 5.31 to 5.80						
+1.25 " +1.75	+1.5	.....	.....	1							
+0.75 " +1.25	+1.0	.....	.....	0.5	3.5						
+0.25 " +0.75	+0.5	.....	.....	1.0	4.5	4.5	1				
−0.25 " +0.25	0.0	.....	.....	2.0	4.0	4.0	3				
−0.75 " −0.25	−0.5	.....	.....	.....	1.0	0.5	5.5	1.5	0.5		
−1.25 " −0.75	−1.0	.....	.....	.....	1.0	4.0	3.0	1.0	1.0		
−1.75 " −1.25	−1.5	.....	.....	.....	.....	.....	5.0	2.0	0.5	0.5	
−2.25 " −1.75	−2.0	.....	.....	.....	1 0	.....	1.5	2.5	1.0	.....	.....
−2.75 " −2.25	−2.5	.....	.....	.....	.....	.....	.....	2.0	.....	.....	.....
−3.25 " −2.75	−3.0	.....	.....	.....	.....	.....	3.0	.....	.....	.....	.....
−3.75 " −3.25	−3.5	.....	.....	.....	.....	1.0	.....	.....	.....	.....	.....
−4.25 " −3.75	−4.0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
−4.75 " −4.25	−4.5	.....	.....	.....	.....	.....	0.5	0.5	.....	.....	.....
−5.25 " −4.75	−5.0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Total below crosslines..	.....	.....	.....	3.5	11.5	9.5	18.5	8.0	1.5	.....	.....

TABLE XXXII

LUMINOSITY-CURVE B0–B5 STARS

M	$\bar{M}$	Number	Comp.	O−C
+1.25 to +0.75	+1.0	17.0 (0.5)	17.3	− 0.3
+0.75 " +0.25	+0.5	38.6 (5 5)	18.8	+19.8
+0.25 " −0.25	0.0	23.9 (10)	18.2	+ 5.7
−0.25 " −0.75	−0.5	10.2 (7)	15.9	− 5.7
−0.75 " −1.25	−1.0	9.0 (9)	11.9	− 2.9
−1.25 " −1.75	−1.5	7.0 (7)	8.1	− 1.1
−1 75 " −2.25	−2.0	6.0 (6)	4.9	+ 1.1
−2.25 " −2.75	−2.5	2.0 (2)	2.4	− 0 4
−2.75 " −3.25	−3.0	3.0 (3)	1.2	+ 1.8
−3.25 " −3.75	−3.5	1.0 (1)		
−3.75 " −4.25	−4.0	0.0 (0)	0.7	+ 1.3
−4.25 " −4.75	−4.5	1.0 (1)		

Collecting these results, including the solution of *Mount Wilson Contribution*, No. 82, p. 43, with weights according to a rough estimate of the reliability, we have:

CONSTANTS LUMINOSITY-CURVE B0–B5 STARS

	No. of Stars	K	Weight	h	Weight
First solution	48.0	+0.23	1.8	0.450	1
Second "	27.0	+0.60	1.0	(>0.400)	0
Third "	69.0	....	....	0.500	1
Mt. Wilson Contr. No. 82	142.5	+0.885	1.0	0.409	2
Adopted means	....	+0.500	....	0.442	......

(107)

To this value of h corresponds the value $r=\pm 1.01$ mag., that is, the B0–B5 stars are distributed according to an error-curve about the central value $M=+0.50$, with a probable error of about one magnitude.

22. LUMINOSITY-CURVE OF THE B8–B9 STARS

First solution: from the Cannon stars in the Nebula-group.—In Section 18 reasons were given for the assumption that, for the nebula region, the number of stars from 9.05 to 9.55 in Table XXX, in order to become complete, must be multiplied by a factor not exceeding 2. This being admitted, it is evident that the maximum of the frequency-curve of the apparent magnitudes must lie at about

$$m=8.80. \tag{108}$$

Fitting an error-curve having this central value to the observed numbers, I find

$$h=0.90. \tag{109}$$

The correction required for the position of the maximum on account of the admixture of stars not belonging to the group is +0.10 mag. Consequently, by (102), the elements of the luminosity-curve are

$$\left.\begin{aligned} K &= 8.80+0.10-6.34=+2.56 \\ h &= 0.90 \end{aligned}\right\} \quad (109 \text{ stars}) \tag{110}$$

Second solution: from the outside stars.—Neglecting the stars fainter than 9.05, the maximum is located somewhere near $m=8.30$.

The value of h found by fitting an error-curve to all data is 0.78, but, for reasons already given, this value must be too small. Hence, by (104),

$$\left.\begin{aligned} K &= 8.30-5.46=+2.84 \\ h &> 0.78 \end{aligned}\right\} \qquad \text{(143 stars)} \qquad (111)$$

The bright B8–B9 stars are too few in number to yield any reliable result. The same holds for the stars in *Mount Wilson Contribution*, No. 82. Finally, therefore, we have from (110) and (111), giving equal weight to the two values of K,

$$K=+2.70 \qquad h=+0.90 \qquad \text{(B8–B9 stars)} \qquad (112)$$

23. LUMINOSITY-CURVE OF THE B3 STARS

In passing from the B0–B5 stars to the B8–B9 stars, the change in the value of K is so considerable that it becomes highly important to attempt a still further subdivision. For the B3 and B5 stars the data are still fairly adequate to such a determination. For the former I use the Cannon stars, both for the nebula and the outside region and further the data contained in *Mount Wilson Contribution*, No. 82, discussed in the same way as the bright B0–B5 stars in the third solution of Section 21.

In the case of the outside stars there is no objection to the value for K. The result for h is really (see Section 18) a lower limit. However, in the cases already treated, this value differs so little from that for the nebula region that in the present instance—for which only a rough result is all that can be hoped—I have no hesitation in using the h furnished by outside stars on the assumption that they are all at the same distance. The reduction to absolute magnitude is of course made with the mean parallax (33). Table XXXIII summarizes all the data. The apparent magnitudes corresponding to the values of M in the table were obtained by (102) and (104). The numbers from *Mount Wilson Contribution*, No. 82, were multiplied by a factor such that the total brighter than $M=+1.75$ becomes equal to that for the stars in the nebula region increased by those outside. They thus become comparable with the sums of the two preceding columns. This reduction is necessary because only that part of the curve which corresponds

to the stars brighter than $M=+2.25$ is given. The sudden breaking off of the curve at this point by no means implies the non-existence of fainter stars. In parentheses are added the

TABLE XXXIII

LUMINOSITY-CURVE B3 STARS

M	Number of Stars					
	Neb.	Outside	*Contr.* 82	Adopted	Comp.	O−C
>+ 2.75......	0	0.0	....	0.0	1.7	−1.7
+2.25 to +2.75..	0	0.0	....	0.0	1.7	−1.7
+1.75 " +2.25..	3	0.0	9.0 (1)	3.0	2.7	+0.3
+1.25 " +1.75..	3	4.0	5.0 (2)	6.0	3.7	+2.3
+0.75 " +1.25..	3	0.0	5.0 (5)	4.0	4.5	−0.5
+0.25 " +0.75..	1	0.0	4.5 (9)	3.0	4.8	−1.8
−0.25 " +0.25..	4	2.5	3.1 (11)	4.8	4.4	+0.4
−0.75 " −0.25..	4	0.5	2.8 (13)	3.7	3.5	+0.2
−1.25 " −0.75..	0	2.0	2.7 (12.5)	2.4	2.7	−0.3
−1.75 " −1.25..	2	2.0	1.2 (5.5)	2.1	1.6	+0.5
−2.25 " −1.75..	....	1.0	1.3 (6)	1.2	0.9	+0.3
−2.75 " −2.25..	....	....	0.4 (2)	0.4	0.4	0.0
<− 2.75......	....	....	....	....	0.3	−0.3

observed numbers of stars on which the results of *Mount Wilson Contribution*, No. 82, rest. They determine the weight with which these values are to be combined with the totals of the two preceding columns. The results of the combination are given in the column headed "adopted," and are fairly well represented by an error-curve for which

$$K=+0.53 \qquad h=+0.52 \qquad \text{(B3 stars)} \qquad (113)$$

24. LUMINOSITY-CURVE OF THE B5 STARS

It is seen that among the outside B5 stars in Table XXX there is none fainter than 8.5. We may conclude, I think, that we have before us the complete frequency-curve. According to the last line but one we have for the maximum, $m_{max}=7.73$ (outside stars). In the nebula region two of the observed stars are fainter than 8.5, but none fainter than 9.0. It seems hardly safe to suppose that here, too, we have the whole of the frequency-curve before us. If we assume (*a*) that the frequency-curve is complete, (*b*) that there are two stars of magnitude 9.1 really existing but not observed, the

probability is that the truth lies between the two. Corresponding to supposition (*a*) we find $m_{max}=7.50$, and to (*b*) $m_{max}=7.69$. The difference is not very material and I adopt

$$m_{max}=7.60 \quad \text{(neb. region)} \tag{114}$$

Transferring to absolute magnitudes by (104) and (102) we have

$$\left.\begin{array}{lll} \text{for the outside stars} & K=2.27 & (7 \text{ stars}) \\ \text{for the Nebula-group} & K=1.26 & (15 \text{ stars}) \\ \hline \text{Weighted mean} & K=1.58 & (22 \text{ stars}) \end{array}\right\} \tag{115}$$

The agreement is poor. For the derivation of h I added the data of *Mount Wilson Contribution* No. 82 to those of Table XXXIII, just as was done for the B3 stars, and found

$$h=0.50. \tag{116}$$

25. FURTHER LUMINOSITY-CURVES AND SUMMARY OF RESULTS

By the combination of the results (112) with (115) and (116) we obtain for the B5–B9 stars

$$K=+2.61 \qquad h=0.87 \qquad (274 \text{ stars}) \tag{117}$$

There is further the determination of *Mount Wilson Contribution*, No. 82, p. 45,

$$K=+2.00 \qquad h=+0.508 \qquad (69 \text{ stars}) \tag{118}$$

which is confessedly poor; the solution (117), too, is not of high weight, but doubtless much better. Combining with the respective weights 4 and 1, I find

$$K=+2.49 \qquad h=+0.80 \qquad (\text{B5–B9 stars}) \tag{119}$$

For the B0, B0–B2, and B1–B2 stars, the value of K was obtained by taking the arithmetic means of the apparent magnitudes of the Cannon stars in the nebula region and reducing to absolute magnitudes by subtracting 6.34 according to (102). Further, in the case of the B0–B2 stars, I derived a very crude value of h. For the B0–B9 stars a good result could not be obtained. All the results are summarized in Table XXXIV, in which, for convenience, I also insert the constants for the A0–A9 stars, which will be obtained presently.

In the fifth column have been inserted the numbers of stars on which the several determinations rest. These give a very imperfect idea of the reliability of the corresponding luminosity-curve, not only because the absolute magnitudes are wholly dependent on the accuracy of a few parallaxes, but mainly because the observations embrace very different fractions of the whole curve. For this reason the part of the curve covered by observations has been

TABLE XXXIV

CONSTANTS OF LUMINOSITY-CURVES

	K	h	r	No. of Stars	Fraction of Curve	Quality
B0	−2.5	?	?	6	1.00	Poor
B3	+0.5	0.52	0.92	99	0.95	Good
B5	+1.6	0.50	0.95	22, 49*	0.81	Fair
B0–B2	−1.6	0.43:	1.11:	13	1.00	Poor
B0–B3	−0.4	0.52	0.92	33	1.00	Fair
B1–B2	−0.9	?	?	7	1.00	Poor
B0–B5	+0.5	0.442	1.01	286	0.96	Good
B5–B9	+2.5	0.80	0.60	343	0.50	Fair
B8–B9	+2.7	0.90	0.53	252	0.50	Fair
A0–A9	+3.4	0.80	0.60	474, 601†	0.50	Fair

* Twenty-two stars used for the derivation of K; 49 for h.

† 474 stars used for the derivation of K; 601 for h.

roughly indicated in the sixth column. When the fraction is less than 0.50, the maximum is not included, and its determination becomes more or less precarious. As a consequence, the elements for the stars of type B3, resting on 99 stars, must be far better than those for the B5–B9 stars, for which 343 stars were available. Even the values of K for the B0, B0–B2, B1–B2 stars, though poor, deserve more confidence than we might at first sight be willing to concede on account of the extremely small numbers of stars on which they rest, simply because practically the whole of the curve is covered by observation. A crude estimate of the reliability of the elements has been given in the last column.

I have computed these numerous luminosity-curves in the hope that they might help materially in obtaining good parallax estimates when other means are defective or wanting.

Remark.—There are a few exceptionally bright stars in our lists, such as ϵ and ζ Orionis and ϵ Canis Majoris.[1] For all three the absolute magnitudes are brighter than -4.0. Since the spectrum of the last is B1, and the others are probably B1 or B0, the magnitudes do not seem irreconcilable with the luminosity-curves of Table XXXIV. The case of β Orionis (Boss 1250) is different. It is the brightest star in our lists, $M=-5.5$, whereas the spectrum is B8; hence it is more than 8 mags. brighter than the mean for stars of the same spectrum, and seems to deserve particular attention.

26. LUMINOSITY-CURVE OF THE A STARS

The value of K was determined from the outside stars. The main difficulty was the probable incompleteness of the numbers of stars between magnitude limits 9.05 and 9.55. We are not altogether without means for overcoming the difficulty, however.

We can make a rough determination of ϵ, the factor by which the observed number must be multiplied to make it complete, with the aid of the B5–B9 stars, by comparing the observed and the theoretical numbers obtained with the constants of Table XXXIV. We find

	No. Obs.	No. Theor.	ϵ
Nebula region	12	30.5	2.5
Outside stars	7	11.2	1.6
Total	19	41.7	2.2

As already remarked, the A stars in the nebula region scarcely admit of a value of ϵ as high as 2.0. The theoretical number for the nebula region, therefore, and hence also for the outside stars, must be too high. I assume

$$1.0<\epsilon<2.4. \tag{120}$$

If we try to represent the numbers of the last column of Table XXX by an error-curve on the two hypotheses $\epsilon=1$ and

[1] The Boss numbers are respectively 1370, 1398, 1804.

$\epsilon=2.4$, we find maxima at $m=8.80$ and 9.05, respectively, so that we cannot greatly err if we write $m=8.93$. We thus get, by (104),

$$K=3.47. \tag{121}$$

Adopting this value, the maximum in the Nebula-group, according to (102), must lie at $m=9.81$, and, corrected for the large admixture of external stars projected on the group, for all the A stars within the limits (78) $m=9.29$. From both regions together I then obtain the best representation by assuming

$$h=0.80. \tag{122}$$

As there may still remain some doubt as to the correctness of K, particularly because the preceding determination tacitly assumes that for the region considered the parallax of the bright A stars agrees with that of the bright B0–B9 stars, I have tried to find some verification by entirely independent data. The most suitable for the purpose is that furnished by the Pleiades and the Hyades.

If the stars in these groups be arranged in order of magnitude, they will also be arranged nearly in the order of spectrum. This remark is not new, but Table XXXV, which brings the fact clearly into view, may not be unwelcome.

a) *The Pleiades.*—The brighter stars are all B5–B9. For stars of magnitude 6.59 and fainter, with relatively few exceptions down to 9.05, the spectrum is A. Stars still fainter show an F spectrum. Outside the limits 6.5 and 9.1 no A stars occur, though we should expect a few to appear were the number of stars a hundred times greater. It is evident that we have before us the entire luminosity-curve and that the absolute magnitudes of the bulk of the A stars lie within a range of 2.6 mags. If we assume that the luminosity-curve is an error-curve, its maximum must coincide with the arithmetic mean. From the photographic magnitudes determined at Potsdam I find from 23 A stars

$$m_{max}=7.70 \qquad h=0.82 \tag{123}$$

and for 8 B5 stars

$$m_{max}=4.49. \tag{124}$$

It thus appears that the maximum of the frequency-curve for the A stars is 3.21 magnitudes fainter than that for the B5 stars.

TABLE XXXV

DISTRIBUTION OF SPECTRA

Mag.	Hyades*	Pleiades†
3.41	....	B5
3.62	A5	....
3.63 to 4.10	K, K, K, K	B8, B5
4.10 " 4.55	A, A3, A3, A5	B5, B5
4.55 " 5.05	A5, A5, A2, A5, A3	B5
5.05 " 5.55	A5, A8, A5, A, A2, A2, A, A5, A5	B8
5.55 " 6.05	A, A5, A5, G, A2, A9, F, A, F, A4, A9, F, A5, A9	B5, B8, B8, B8
6.05 " 6.55	A9, A8, A8	B8
6.55 " 7.05	F2, F5, F2, G0	A, B8, A, A, A
7.05 " 7.55	G0	A, A, A, A, A
7.55 " 8.05	G0, G0, G, G0, F8	A, A, A, A, A5
8.05 " 8.55	G0, G0, G0, G5	A2, A3, A5, A2, A2, A5, A5
8.55 " 9.05	G5, F8, K0, K0, K2, G0, K0, K0, G5	F, F, A5, A
9.05 " 9.55	....	F2, F, F2, F
10.45	....	F5

* The stars belonging to the Hyades were taken from the preface to *Groningen Publications*, No. 23, the data for spectrum being largely supplemented by private letters from the Harvard Observatory and by plates taken at Potsdam by Dr. Zernike. The list contains all the stars of determined spectrum known to belong to the group.

† The photographic magnitudes for the Pleiades are by Hertzsprung, *Pub. Astr. Obs. Potsdam*, **22**, No. 63, p. 21. It would have been preferable to use the visual magnitudes by Müller and Kempf (*Astronomische Nachrichten*, **150**, 193, 1899), but I have not deemed it important to make the change. The brighter stars have been reduced to the Harvard scale by subtracting 0.24 mag. The spectra for the brighter stars are from Miss Maury (*Harvard Annals*, **28**, Part I); for the fainter stars they are from Tikhoff (*Mitteilungen Pulkowo*, No. 40, Tableau II). The proper motions necessary for determining whether a star belongs to the physical group are from the following sources: Lagrula or Elkin (weight 1), Boss (weight 1), and Stratton (weight $\frac{1}{3}$).

The same must also be true for the luminosity-curves, and since by (115) the maximum of the luminosity-curve for the B5 stars lies at absolute magnitude $K=+1.58$, we find

$$\text{A stars} \qquad K=4.79, \tag{125}$$

a determination which is independent of the parallax of the Pleiades.

b) *The Hyades.*—Table XXXV contains all the stars of determined spectrum which are known to belong to the group.

We here meet with the curious fact that four of the five brightest stars have K spectra, a type not again appearing until we reach

stars fainter than 8.5. If I am not mistaken this fact first gave rise to the general[1] theory of giant and dwarf stars.

For the determination of the luminosity-curve we cannot use all the objects in Table XXXV, for it is evidently necessary to know *all* the stars belonging to the group, together with their spectra, down to a specified and rather faint limit, a condition satisfied only for that part of the group for which proper motion plates have been measured at Groningen.

As the spectra are still rather uncertain, I give in full all the A stars which, according to the data at my disposal, occur in the restricted area. It will thus be easy to improve the results as soon as better data are available.

No. *Gron.* 23	Harv. Mag.	Spectrum	No. *Gron.* 23	Harv. Mag.	Spectrum
227	3.62	A5	87	5.68	A2
143	4.24	A	245	5.70	A9
170	4.60	A5	64	5.76	A4
315	4.75	A5	371	5.80	A6
107	4.84	A2	222	5.97	A5
251	4.84	A5	282	6.04	A9
385	4.85	A3	44	6.14	A9
38	5.27	A5	1	6.35	A8
255	5.49	A5	150	6.39	A8
34	5.59	A5			

Among the stars fainter than 6.39 there is not a single A star.

The mean magnitude

$$m = 5.36 \quad (19 \text{ stars}) \tag{126}$$

is the apparent magnitude corresponding to the maximum of the frequency-curve. For the best-fitting modulus of precision I find

$$h = 0.88. \tag{127}$$

For the conversion into absolute magnitudes we require the parallax of the group, which according to *Groningen Publications*, No. 23, p. 4, is

$$\pi = 0\overset{''}{.}024, \text{ therefore } M = m - 3.10. \tag{128}$$

[1] General in the sense that the theory applies, not only to certain clusters, but to the whole of the stellar system.

Finally, for the elements of the luminosity-curve

$$K = 5.36 - 3.10 = 2.26 \qquad h = 0.88 \tag{129}$$

The agreement of the values for h in (129) and (123) *inter se* and with (122) is good. The agreement of (125), (129), and (121) for K is surprisingly bad.

I think that one of the causes must be an error in the value (129) furnished by the Hyades. For, adopting $K = +2.26$, we find by (102) and (104) that the maximum in the Nebula-group is at $m = 8.60$ and in the outside group at $m = 7.72$. A glance at Table XXX shows that such values are not to be thought of. After mature consideration I believe that the error must in great part be attributed to the parallax. We have two excellently agreeing values, one by Boss from the convergent of the proper motions and some measures of radial velocity; the other a direct determination of parallax made at Groningen. The following criticisms may, however, be made.

In the Groningen determination the magnitude error offered considerable difficulty and it does not seem impossible that, notwithstanding all our care, an error of 0″.01, though four times the probable error, may have crept into the result.

In Boss's determination,[1] the small extent of the group makes the evaluation of the convergent precarious. In order to estimate the possible error of his parallax, I made the following redetermination, first condensing his data into the four normals in Table XXXVI, chosen to give the most favorable determination of the convergent.

TABLE XXXVI

NORMAL PLACES

	α	δ	$100\,\mu$	p	No.	p comp.		$(O-C)\sin\lambda$	
						Boss	Kapt.	Boss	Kapt.
$\delta > +18°$	4^h19^m	$+20°.5$	$11''.6$	$113°.6$	9	$113°.6$	$116°.1$	0.0	−1.1
$\delta < +13$	4 27	+ 9.5	11.0	92.8	7	94.0	90.8	−0.5	+0.7
Rest, α small........	4 11	+15.9	12.2	104.7	13	104.1	104.5	+0.3	+0.1
Rest, α great.......	4 27	+15.3	10.5	104.3	12	106.1	106.6	−0.8	−0.8
Weighted mean..	4 20	+15.6	11.35	......	....	......	......	......	......

[1] *Astronomical Journal*, **26**, 31, 1911.

According to Boss the convergent lies at

$$6^h 7^m.2 \qquad +6°56' \qquad (130)$$

which indeed satisfies the observations excellently (last column but one); but

$$5^h 46^m \qquad +8°40' \qquad (131)$$

satisfies them almost equally well (last column). For the stream-velocity Boss used only three stars, while we now have the nine objects in Table XXXVII. The values $\rho_{corr.}$ were obtained by

TABLE XXXVII

RADIAL VELOCITIES, HYADES

α	δ	ρ	Sp	ρ corr.	λ	Authority
$3^h.8$	+17°	+25.7	F	+25.7	149°	Küstner, *A.N.*, **198**, 409
[4.0	+19	+21.6	K	+19.1	151]	" *A.N.*, **198**, 409
4.2	+15	+39.6	K	+37.1	155	" *Ap.J.*, **27**, 301
4.3	+17	+40.8	K	+38.3	156	" *Ap.J.*, **27**, 301
4.3	+22	+40.0	A3	+39.0	155	*Lick Bull.*, **7**, 20
4.3	+18	+36.4	A	+35.4	156	" " **7**, 20
4.4	+19	+39.4	K	+36.9	157	Küstner, *Ap.J.*, **27**, 301
4.4	+16	+39.4	K	+36.9	157	" *A.N.*, **198**, 409
4.9	+21	+42.0	A5	+41.0	161	*Lick Bull.*, **7**, 20
Mean 4.3	+18	....	......	+36.3	155.8	

reducing Küstner's results to those of the Lick observers by applying the systematic corrections found by Küstner himself and then adding the constant corrections found by Campbell (*Lick Bulletin*, **6**, 127); λ is the angular distance from the point (131).

The second star was excluded on account of its great divergence. Forming for the others the equations of condition

$$V \cos \lambda = \rho_{corr.}$$

and solving by least squares,

$$V = 40.0 \text{ km} \quad (8 \text{ stars}). \qquad (132)$$

This and $100v = 100\mu = 11''.35$ (Table XXXVI) give by (51)

$$\pi = 0''.033; \text{ therefore } M = m - 2.41. \qquad (133)$$

The maximum at apparent magnitude 5.36 (126) thus entails

$$K = +2.95. \qquad (134)$$

The value (125) furnished by the Pleiades is open to the criticism that it rests on (124), determined from only eight B5 stars. We may obtain a result based on a greater number of stars if we adopt the parallax[1] derived on the hypothesis that the motion of the group in space, referred to the center of gravity of the whole system, is parallel to the Milky Way. The parallax thus found is

$$\pi = 0\overset{''}{.}018, \text{ therefore } M = m - 3.72. \tag{135}$$

The value (123), first reduced to the Harvard scale by subtracting 0.24 (see footnote to Table XXXV), thus leads to

$$K = 3.74 \quad (23 \text{ A stars}) \tag{136}$$

Collecting results, we thus have, the adopted weights being in parentheses:

	K		h
	First Solution	Second Solution	
Pleiades (125), (136) . .	4.79 (0.5)	3.74 (1)	(123) 0.82
Hyades (129), (134) . . .	2.26 (1)	2.95 (1)	(129) 0.88
Mean	3.10	3.34	0.85

The values of K, which in the first solution were extremely divergent, have come very much nearer together. The mean values for both solutions agree well with (121). The value of h is surprisingly close to that of (122). In conclusion, therefore, the Pleiades and the Hyades furnish values of K which leave much to be desired; still, as far as they go, they decidedly confirm the value (121). I finally adopt

$$K = +3.4. \tag{137}$$

27. REMARKS ON THE GEOMETRICAL FORM OF THE LUMINOSITY-CURVES

a) In what precedes we have assumed that the luminosity-curves are error-curves. In how far is this assumption justified? For stars of all spectra together the form was very decisively

[1] *Proceedings Amsterdam Academy of Sciences*, **14**, Part 2, 909, 1912.

indicated by the results in *Groningen Publications*, No. 11, though the representation by an error-curve was not there given. This was first done in *Astronomical Journal*, **24,** 115, 1904. The representation over a range of 16 magnitudes leaves little to be desired. For spectra of the first and second types separately, luminosity-curves were also given in *Publication*, No. 11. These, too, are well represented by error-curves. But the data available for these investigations established the curve only as far as the maximum, or nearly so; the fainter branch is altogether wanting. What will be the form for the fainter magnitudes?

The question was considered in *Mount Wilson Contribution*, No. 82, p. 43, but could be answered only very imperfectly. Even now the data are scanty, though somewhat improved, especially for types B and A. They lead to the conclusion that for these spectra the whole curve, both ascending and descending branches, can be represented—with some rough approximation at least—by an error-curve.

Take, for instance, the stars of spectrum B3 for which the whole curve is pretty well covered by observations. In Table XXXIII a comparison is made between the observed numbers (adopted) and the best-fitting error-curve (computed). The divergences O−C for the fainter part of the curve are admittedly large, but they can hardly be qualified as systematic or as greater than can be explained by the scarcity of stars.

In order to obtain more material, I proceeded as follows. The *whole* curve is covered by observation for:

B5–B9	13	stars	in the	Pleiades
A	26	"	"	"
A	19	"	"	Hyades
B3	20	"	"	Nebula-group
	78			

Since, by Table XXXIV, the best-fitting error-curves do not differ excessively in their values of h, I formed the deviations from the separate means for each of these four groups. The question is whether the positive and negative deviations for all these 78 stars

are symmetrical and in accordance with an error-curve. The result is as follows:

Deviations	Observed	Smoothed	Computed	O−C
<− 2.25	0.0	1.0	1.0	0.0
−2.25 to −1.75	3.0	1.3	2.2	−0.9
−1.75 " −1.25	1.0	6.0	5.2	+0.8
−1.25 " −0.75	14.0	10.0	9.5	+0.5
−0.75 " −0.25	15.0	12.8	13.5	−0.7
−0.25 " +0.25	9.5	14.5	15.2	−0.7
+0.25 " +0.75	19.0	13.3	13.5	−0.2
+0.75 " +1.25	11.5	11.8	9.5	+2.3
+1.25 " +1.75	5.0	5.5	5.2	+0.3
+1.75 " +2.25	0.0	1.7	2.2	−0.5
>+ 2.25	0.0	0.0	1.0	−1.0

The theoretical curve has been computed with $h=0.70$. The last column shows the divergence of the observations (smoothed by taking means of three consecutive values) from the theoretical curve. The conclusion is as before: the representation by an error-curve is fairly satisfactory for the entire curve.

b) But this resemblance of the luminosity-curve to an error-curve must not be taken too literally. With our present data it is impossible to investigate the matter very closely. Still it may be well to call attention to the fact that even now there are signs indicating that the agreement probably is by no means absolute. Thus the representation for the A stars in Table XXX is defective. As compared with an error-curve, there is a decided excess of very large and very small deviations, and the same thing is indicated in other ways. Hertzsprung[1] has derived the absolute magnitudes of 15 stars belonging to the Ursa Major group. Twelve of these have A spectra, and their absolute magnitudes (increased by 5 mags. to reduce to our scale) range from −1.03 to +2.20;[2] they are accordingly, by Table XXXIV, from −4.4 to −1.2 mags. brighter than the average A star.

It is not at all surprising that the deviations are all on the bright side. Hertzsprung confined himself to Bradley stars and has, by this choice, given strong preference to very luminous stars. In

[1] *Astrophysical Journal*, **30**, 139, 1909.

[2] There seems to be a mistake in the parallax computed for β Aurigae. I have used the correct value. Without the correction the range would be −0.53 to +2.20.

fact, we may say that such catalogues as those of Bradley and Boss are really catalogues of exceptionally luminous stars. It is just this absence of stars of mean and faint absolute magnitude from our catalogues which makes the determination of the luminosity-curves so difficult. Nevertheless, in the case of the Ursa Major stars, the deviations seem to be somewhat excessive for a curve whose probable error is ± 0.60 mag.[1] There is accordingly a strong indication of an excess of very luminous stars.

Similarly there seems to be an excess of stars which are absolutely very faint. The companion to *o* Eridani, which shows an A spectrum, has absolute magnitude $+10.3$, which is 6.9 mags. fainter than the average A star. Other examples might also be given. As already remarked at the end of Section 25 we have, among the B8 stars, such an exceptional object as β Orionis, which is 8.2 mags. brighter than the average B8–B9 stars.

With more extensive data it may become necessary to give up the error-curve as the best representation of the frequencies of the absolute magnitudes. The most convenient form for trial will then probably be the sum of two error-curves having the same maximum but different moduli.

c) I have taken much trouble in deriving what in many cases can be considered only as first approximations to the luminosity-curves in the conviction that such curves form one of the most important elements in attempting to learn the structure of the universe. Scarcity of data is responsible for the fact that in former endeavors stars of all spectra have been grouped together or that, at most, a separation into only two types has been attempted. The results of the present paper will illustrate how crude is such a procedure, for it involves the combination of stars having -2.5 as a

[1] More important than the size of the deviations is the conclusion that the Ursa Major group must contain many more members than those we know, the bulk of which are among the fainter stars. Even now I think we can specify a great number of stars that belong to the group. There is, however, a great difficulty in the way of deciding with certainty, owing to the similarity of the elements of this group (vertex 20^h31^m, $-40°2$; velocity, -18.4 km) with those of the second star-stream, especially the A stars of this stream (vertex 19^h12^m, $-47°$; velocity, -18.5 km). For instance, the brightest of Hertzsprung's stars, β Aurigae, fits practically as well in the second stream as in the Ursa Major group. If we assign it to the former, its absolute magnitude changes from -1.03 to $+0.49$.

mean absolute magnitude (Bo stars) with others for which the mean magnitude is +10.3 (second-type stars). The corresponding ratio of intensities is more than 100,000 to 1.

To secure entirely reliable results we must attack the spectral classes separately. This will be an immense task, for it involves obtaining the numbers, magnitudes, spectra, and proper motions of a suitable fraction of the faint and very faint stars. In fact the best of the results given in the present paper are due to the invaluable data for stars to magnitudes 9.0 or 9.5 placed at my disposal by Professor Pickering and Miss Cannon.

The requirements for a solid foundation in deriving the luminosity-curves are perhaps best seen from Table XXXVIII, which shows the mean absolute magnitudes for stars of given apparent magnitude and proper motion. It was computed with

TABLE XXXVIII

ABSOLUTE MAGNITUDES M

μ	m		
	6.0	9.0	12.0
0″.05	+1.2	+3.6	+ 5.9
0.10	+2.3	+4.6	+ 7.0
0.20	+3.4	+5.7	+ 8.1
0.40	+4.4	+6.8	+ 9.2
0.80	+5.5	+7.9	+10.2

the data for all stars in *Groningen Publications*, No. 8. To all appearances the parallax for any specified magnitude and proper motion does not differ very markedly for stars of different spectra, so that for a rough estimate a single table can be used. The table shows, what of course is evident, that for the absolutely brightest stars we are dependent on the apparently bright stars of small proper motion. As these stars have been observed more completely and accurately than any others, the conditions are favorable for the brighter end of the luminosity-curve. The faint absolute magnitudes, on the contrary, are to be found among the apparently faint stars of large proper motion. The conditions here are nearly as unfavorable as possible. The table indicates, roughly, how far

we may hope to go with the available data and what will be required for further progress.

Since we have found the mean absolute magnitude of the B5 stars to be +1.6, and of the A stars to be +3.4, whereas for the second-type stars it is +10.3,[1] it is rather more than a guess if we assign the following series of values:

B5	stars at	+ 1.6
A5	" "	+ 3.4
F5	" "	+ 7
G5	" "	+10
K5	" "	+13
M	" "	+15

Assuming that the luminosity-curves are error-curves and that for a satisfactory determination of their constants we must possess reliable observational data, at least to a magnitude well past the maximum, we see from Table XXXVIII that Boss's *Catalogue* and the *Revised Harvard Photometry* (*Harvard Annals*, **50**), which are complete to magnitudes 5.8 and 6.5, respectively, may be considered as perhaps just sufficient for the derivation of the curve for the A5 stars. For the F stars we cannot expect altogether reliable results until the stars to 9.0 or somewhat beyond can be included. The *Revised Draper Catalogue* will furnish the required data for the spectra. For the large proper motions (which are the more important) we shall probably be able to manage with what is known, especially when the much-longed-for determinations of proper motion, now in preparation at Albany, have been placed in the hands of astronomers. For the G stars even these data will be inadequate, and we shall have for them no thoroughly sound determination until the data for the Selected Areas become available.

For the K and M stars the outlook might seem to be nearly hopeless. But there is one circumstance which probably will bring success along as soon as the definitive luminosity-curve for the G stars is found. It has long been known that the stars of very large proper motion, almost without exception, are of the second type.

[1] This value was obtained by representing the numbers of *Groningen Publications*, No. 11, p. 31, Type II, Solution B, by an error-curve.

We further find that the larger the proper motion and the fainter the apparent magnitude, the more do the K5–M stars begin to predominate. From van Maanen's table of large proper motions (*Mount Wilson Contribution*, No. 96), I find that among the stars fainter than magnitude 6.0, with proper motion ≧ 1″.0, 60 per cent are of type K5–M. For still fainter stars we must expect that this percentage will increase largely, so that finally a limit of faintness will be reached for which practically all the stars of large proper motion will be K5–M. Not only must this be so theoretically, but there is a strong indication of its confirmation by observed color indices such as those of Seares and Hertzsprung (*Mount Wilson Contributions*, Nos. 81, 100, and 102).

We therefore expect that beyond the magnitude for which we can determine spectra we shall be justified in including with the K5–M stars all swiftly moving objects, instead of limiting ourselves to those for which the spectra have been ascertained. Perhaps we shall have to multiply our numbers by a certain fraction, slightly less than unity, but it seems probable that this fraction will be capable of a sufficiently accurate determination with the aid of the somewhat brighter stars. Consequently, as soon as the data now under observation for the Selected Areas have become available, we shall probably be able to complete the investigation down to the faintest stars observed for the *Durchmusterung* of the Selected Areas, that is, approximately to:

Mag.	16.0	in the Northern	Hemisphere	(Harvard plates)
"	18.2	" "	"	(Mount Wilson plates)
"	16.8	" Southern	"	(Harvard Arequipa plates)

Of course there will remain the necessity of finding all the stars having large proper motions. This, however, will present no real difficulty. A repetition of the plates, even if made at the present moment, would be sufficient for the purpose. If photographed through the glass and superposed on the originals for differential measurement, or if taken as usual and carefully measured with the stereocomparator, the labor of finding all the proper motions exceeding 0″.10 would not be very trying. We may thus hope to secure the data necessary for a thorough study of the arrangement

in space of all the stars, including even those, thousands of times fainter than the sun, which are barely visible in our largest telescopes when near, and escape all observation[1] when situated in the remoter regions of space.

28. SUMMARY

1. The bulk of the B stars within the limits (2) form a local group. The great Orion nebula lies within these limits, and I have therefore called this group the *Nebula-group*.

2. Boss's proper motions in declination, for the stars in the region of the sky considered in the present paper, require the corrections (13) of Section 4.

3. Of the stars outside the Nebula-group, those within the limits $l = 180°$ to $216°$, $b = -30°$ to $+4°$ *may* form a local group. It seems more probable, however, that they all have the same systematic motion. I have assumed this to be the case. If later investigation should confirm the existence of the local group, relatively little will be changed in the conclusions of the present paper (Section 7).

4. The definitive elements of the motion of the helium stars outside the nebula region are given in (33), Section 8. The vertex nearly coincides with the vertex of the first stream of the non-helium stars.

5. The mean parallaxes for three separate groups of these stars are given in Table IX (Section 7), the average parallax for all being $0''.0081 \pm 0''.0007$.

6. The error in the position of the vertex due to remaining systematic errors in the proper motions cannot exceed $3°$ or $4°$, and the remaining uncertainties in V and π from this source must be negligible (Section 8).

7. The components u and t of the peculiar motion are distributed approximately in accordance with error-curves. The probable amount of these components is:

for region of present paper (outside stars) $r_u = \pm 1.67$ km
for region of *Mount Wilson Contribution*, No. 82 $r_u = \pm 1.0$ km

[1] I do not mean that we must be idle until the Selected Areas are completed. On the contrary, I believe that even now we can find very useful representations of the most urgently needed portions of the luminosity-curves.

The smallness of these values is probably the most important fact brought to light by the present investigation.

8. The distribution of u being found, we can derive the distribution of υ as soon as we know the fraction of the stars $\phi(\pi)$ having any given parallax π; $\phi(\pi)$ is determined by the condition that the theoretical distribution of υ shall agree with the observed distribution (Section 13).

9. $\phi(\pi)$ being known, it becomes easy to determine the mean parallax of all the stars having given values of υ and λ. These mean parallaxes have been adopted as the individual parallaxes of stars having the same υ and λ. They have been tabulated for $r_u = \pm 2.5$ in Table XXI, for $r_u = 0.0$ in Table XXIV, and the definitive values, corresponding to $r_u = \pm 1.67$, have finally been obtained by interpolation between these tables.

10. The same method applied to τ does not lead to valuable results (Section 16).

11. The parallax of any star more than 24° from the antivertex, derived in the manner described, is, roughly speaking, $\pm 0''.0021$. This includes the effect of the observational errors in the proper motions (Section 15).

12. The parallax of the Nebula-group, $\pi = 0''.0054 \pm 0''.0009$, has been found by a consideration of the luminosity-curves (Section 18).

13. To the parallaxes various tests have been applied. In general they are strongly confirmatory, both for the outside stars and for the Nebula-group (Sections 17 and 19).

14. The systematic motion of the Nebula-group can differ but little from the motion of the outside stars (Section 20).

15. Luminosity-curves have been determined for many subdivisions of the B stars. Further, a curve was also derived for the A stars. All the results are summarized in Table XXXIV.

16. The frequency-curve of the absolute magnitudes (luminosity-curve) has in former publications been found to coincide nearly with a normal error-curve; in consequence of a lack of data, in particular for the faint stars, the agreement could not be established beyond a point near the maximum. In the present paper occur several cases for which the descending branch could also be included more or less completely. From all available material it appears that the

TABLE XXXIX

B STARS, GAL. LONG. 150° TO 216°

Boss No.	Sp.	Harv. Mag.	1900 α	1900 δ	Gal. Long	Gal. Lat.	Correction (13) Applied 100μ	Correction (13) Applied ρ	Vert. 17^h48^m, +9° ρ comp.	Vert. 17^h48^m, +9° λ	100τ	100υ	100r	ρ−4.3	ρ comp.	Unit 0″.0001 $\pi_{2.5}$	Unit 0″.0001 $\pi_{0.0}$	Unit 0″.0001 Adopted π	Unit 0″.0001 Adopted M
a) Lat. 0° to +30°; 100μ ≧ 1″.7																			
														km	km				
1635....	B5	4.06	$6^h 23^m$	+20°.3	160°	+ 5°	2″.2	201°	198°	140°	−0″.1	+2″.2	0″.23	....	....	88	90	89	−1.19
1550....	B3	4.92	6 6	+16.2	161	0	2.4	165	190	154	+1.0	+2.2	0.44	....	....	94	101	96	−0.17
1568....	B9	5.28	6 10	+16.2	161	+ 1	2.1	163	192	153	+1.0	+1.8	0.36	+25.2W	+17.8	85	84	85	−0.07
1751....	B8	5.69	6 44	+16.3	166	+ 8	2.3	236	210	151	−1.0	+2.1	0.52	+ 8.5W	+17.5	88	91	89	+0.44
1788....	B8	5.88	6 51	+10.1	172	+ 7	2.7	246	220	155	−1.2	+2.4	0.69	+28.7W	+18.1	101	113	105	+0.99
1944....	B8	3.09	7 22	+ 8.5	177	+13	6.6	229	233	150	+0.5	+6.6	0.18	....	....	163	240	189	−0.53
2071....	B8	5.11	7 47	+ 2.0	186	+15	2.5	256	249	148	−0.3	+2.5	0.36	....	....	94	99	96	+0.02
2330....	B3	4.32	8 38	+ 3.8	191	+28	2.2	265	253	136	−0.5	+2.2	0.40	....	....	79	70	76	−1.28
2365....	B9	5 19	8 44	− 3.1	198	+25	2 9	226	260	136	+1.6	+2.4	0.40	....	....	82	77	80	−0.29
1935....	B8	5.20	7 20	−16 0	199	+ 1	3.4	260	285	156	+1.4	+3.1	0.74	−11.9W	+18.3	118	149	126	+0.70
2451....	B8	5.50	9 4	− 8 2	206	+27	3.0	245	265	132	+1.0	+2.8	0.40	....	....	87	82	85	+0.15
2492	B9	5.54	9 12	− 8 3	207	+28	4.0	274	265	130	−0.6	+4.0	0.46	+ 6.7W	+12.8	115	114	115	+0.84
2042....	B3	5.53	7 40	−24.4	208	+ 1	3.9	301	294	148	−0.5	+3.9	0.70	....	....	121	147	130	+1.10
b) Lat. 0° to −30°; 100μ ≧ 1″.7																			
975....	B8	5.15	4 9	+ 9 8	151	−27	4 2	170	128	149	−2.8	+3.1	0.72	....	....	105	122	111	+0.38
1315....	B3	4.83	5 22	+21 9	151	− 6	1.7	140	168	148	+0.8	+1.5	0.32	+12.2	+17.0	72	66	70	−0.94
1354 ...	B3	5.28	5 29	+24.0	151	− 4	3.2	167	171	146	+0.2	+3.2	0.43	+18.9W	+16.6	106	119	110	+0.49
981....	B3	4.32	4 10	+ 8.6	152	−28	3.4	120	127	149	+0.1	+3.4	0.34	+13.7	+17.1	112	132	119	−0.30
1097....	B9	5.37	4 34	+12 0	153	−21	2.7	105	138	152	−1.2	+2.4	0.44	+13.8W	+17.7	95	103	98	+0.33
1165....	B8	5.74	4 50	+14 9	153	−16	2.4	145	147	152	+0.1	+2.4	0.74	+ 4.7W	+17.7	95	103	98	+0.70
1375....	B3	3.00	5 32	+21.1	153	− 5	2.8	174	172	149	−0.1	+2.8	0.18	+12.1	+17.1	100	112	104	−1.02
1203....	B9	4 65	4 59	+15.3	154	−14	4.1	152	152	153	0.0	+4.1	0.28	....	....	128	168	141	+0.39
1464....	B2	5.89	5 49	+19 7	157	− 2	2.2	177	180	151	+0.1	+2 2	0.34	....	....	90	94	91	+0.69
1504....	B8	5.17	5 58	+19.7	158	0	2 8	172	185	150	+0.6	+2 7	0.44	....	....	99	110	103	+0.23
1396....	B3	4.87	5 36	+16.5	159	− 6	3.1	155	173	154	+1.0	+2.9	0.40	+27.7	+18.0	108	129	115	+0.17
1048....	B8	5.50	4 23	+ 1.2	161	−30	3.5	141	117	157	−1.4	+3.2	0.48	....	....	124	158	135	+1.15
1448....	B9	6.61	5 45	+14.4	161	− 6	2.8	172	178	157	+0.3	+2.8	0.38	....	....	116	141	124	+2.08
1455....	B9	5.57	5 47	+14.1	161	− 5	1.7	230	179	157	−1.3	+1.1	0.38	....	....	72	64	69	−0.24
1084....	B5	5.32	4 32	+ 0.8	162	−28	1.8	220	119	158	−1.8	−0.3	0.45	+18.3W	+18.5	51:	29:	44:	−1.46:
1438....	B9	4.92	5 44	+12 6	162	− 6	2.9	207	178	158	−1.4	+2 5	0 44	+16 8	+18.5	110	133	118	+0.28

TABLE XXXIX—*Continued*

1525....	B3	4.40	6 2	+14.8	162	−2	3.7	166	188	155	+1.4	+3.4	0.20	....	......	121	153	132	0.00
1548....	B3	4.35	6 6	+14.2	161	−1	3.5	165	191	156	+1.5	+3.2	0.38	....	......	121	152	131	−0.06
1567....	B2	5.81	6 9	+13.9	161	−1	4.1	74	193	155	+3.6	−2.0	0.66	....	......	44::	28::	39::	−1.23::
1303....	B2	1.70	5 20	+6.3	165	−15	2.0	200	158	162	−1.3	+1.5	0.16	+13.7:	+19.0	85	90	87	−3.60
1572..	B0	5.36	6 10	+12.6	165	−1	1.7	135	193	156	+1.4	+0.9	0.36	+8.3W	+18.3	68	57	64	−0.61
1331..	B3	4.32	5 25	+5.9	166	−14	3.8	162	162	163	0.0	+3.8	0.36	....	......	141	187	156	+0.28
1123	B5	4.18	4 41	−3.4	168	−28	1.8	112	110	162	−0.1	+1.8	0.22	....	......	92	104	96	−0.91
1302.	B3	4.73	5 20	+1.8	168	−17	2.1	209	154	166	−1.7	+1.2	0.62	....	......	83:	83:	83:	−0.67:
1343	B2	5.30	5 28	−1.7	174	−17	2.4	182	134	170	−1.8	+1.6	0.74	....	......	90:	89:	90:	+0.07:
1262...	B5	3.68	5 13	−7.0	170	−23	1.6	250	80	169	0.0	−1.6	0.22	+13.7:	+19.6	57n	57n	57n	−2.54
1376 ...	B2	5.02	5 32	−6.1	178	−18	2.0	192	80	172	−1.9	−0.7	0.65	....	......	71:	60:	67:	−0.25:
1382....	B1	5.75	5 33	−6.0	178	−18	2.4	272	80	174	+0.5	−2.4	1.06	....	......	93::	65::	84::	+0.37::
1666....	B3	5.02	6 29	−1.1	180	−3	4.7	220	229	166	+0.2	+4.7	0.66	....	......	158:	200:	172:	+1.20:
1239 .	B8	4.54	5 8	−12.0	181	−20	3.2	108	55	168	−2.6	+1.9	0.39	....	......	95:	105:	98:	−0.50:
1625...	B5	6.88	6 22	−4.3	181	−6	2.0	84	252	168	+0.4	−2.0	0.57	....	......	71::	40::	61::	+0.81::
1242....	B8	4.46	5 9	−13.1	182	−26	2.0	246	52	169	+0.5	−1.9	0.45	....	......	73::	40::	62::	−1.58::
1639... 1640...	B3p B3	4.73 5.22	6 24	−7.0	184	−7	3.7	278	279	169	+0.1	+3.7	0.39 0.84	+16.3	+19.6	143:	167:	151:	+0.62: +1.11:
1603....	B2	5.49	6 17	−11.7	188	−11	2.7	251	319	170	+2.5	+1.0	0.82	....	......	79:	78:	79:	−0.02
1831....	B3	5.28	7 2	−11.1	192	0	2.2	236	274	161	+1.4	+1.7	0.52	....	......	89	97	92	+0.10
1754....	B5	5.29	6 44	−15.0	194	−6	1.6	195	290	164	+1.6	−0.1	0.80	....	......	65:	47:	59:	−0.86:
1792....	B3	5.26	6 52	−22.8	202	−8	2.7	306	308	150	+0.1	+2.7	1.00	....	......	115	144	125	+0.74
1802.	B3	5.66	6 55	−25.3	203	−8	2.8	321	311	157	−0.5	+2.7	0.58	....	......	113	138	121	+1.08
1807. ..	B3	5.80	6 57	−25.1	204	−8	3.6	318	310	156	−0.5	+3.6	0.78	....	......	129	167	142	+1.56
1838.	B3	5.75	7 3	−23.7	204	−6	2.3	310	306	157	−0.2	+2.3	0.96	....	......	103	118	108	+0.92
1847	B3	5.76	7 6	−25.1	205	−6	2.3	280	307	156	+1.0	+2.0	0.59	....	......	93	100	95	+0.65
1958	B8	5.69	7 23	−22.7	206	−2	2.2	43	298	153	−2.1	−0.6	1.14	....	......	50:	28:	43:	−1.14:
1517.	B	5.64	6 1	−32.2	206	−22	13.1	359	353	156	−1.3	+13.0	1.66	....	......	300:*	400:*	333:	+3.25:
1401	B5	2.75	5 36	−34.1	207	−28	2.7	172	7	154	−0.7	−2.6	0.24	....	......	excl.	......	......	......
1467	B5	4.89	5 49	−33.8	207	−25	3.6	2	0	155	−0.1	+3.6	0.74	+20.7	+18.1	125	160	137	+0.57
1642	B3	5.80	6 25	−32.3	208	−17	2.2	321	339	155	+0.7	+2.2	0.70	....	......	94	100	96	+0.71
1641 .	B5	4.48	6 24	−32.5	209	−18	3.7	311	339	155	+1.7	+3.3	0.44	+20.7(1)	+18.1	119	150	129	+0.03
2011 .	B8	4.50	7 35	−26.6	210	−1	3.4	341	300	148	−2.2	+2.6	0.61	+18.7	+17.0	96	103	98	−0.54
1961 .	B9	5.52	7 24	−29.0	210	−5	2.7	304	307	149	+0.1	+2.7	0.54	....	......	98	108	101	+0.54
2012 .	B3	4.62	7 35	−26.6	210	−1	3.6	307	300	148	−0.4	+3.6	0.60	....	......	115	137	122	+0.05
1605....	B2	5.60	6 17	−34.1	210	−19	3.3	358	344	153	−0.8	+3.2	0.74	....	......	112	137	120	+1.00
1882 ...	B5	5.31	7 11	−30.5	211	−8	3.2	300	314	151	+0.8	+3.1	0.85	....	......	107	127	114	+0.60
1904 .	B8	4.55	7 31	−28.1	211	−3	7.1	255	303	149	+5.3	+4.8	0.60	....	......	excl.	......	......	......
1805...	B5	5.07	6 55	−34.0	213	−12	3.7	327	325	150	−0.1	+3.7	0.66	....	......	118	145	127	+0.59
1907.	B3	5.80	7 25	−31.2	213	−5	2.8	278	310	148	+1.5	+2.4	0.68	....	......	92	95	93	+0.64
1955	B	5.98	7 23	−33.9	215	−7	4.1	270	313	146	+2.8	+3.0	0.76	....	......	102	113	106	+1.11
2004 ..	B8	4.62	7 34	−34.7	216	−6	4.2	302	310	144	+0.6	+4.2	0.71	....	......	124	149	132	+0.22

*By extrapolation of the value for $\frac{\pi}{\nu}$, guiding ourselves by the way in which $\frac{\pi}{\mu}$ changes in *Gron. Publ.* No. 8, p. 31.

TABLE XXXIX—*Continued*

Boss No.	Sp.	Harv. Mag.	1900 α	1900 δ	Gal. Long.	Gal. Lat.	Correction (13) Applied 100 μ	Correction (13) Applied ρ	Vert. 17^h48^m, $+9°$ ρ comp.	Vert. 17^h48^m, $+9°$ λ	100 τ	100 υ	100 r	ρ−4.3	ρ comp.	Unit 0″.0001 $\pi_{2.5}$	Unit 0″.0001 $\pi_{0.0}$	Unit 0″.0001 Adopted π	Unit 0″.0001 Adopted M
									c) Lat. −30° to +30°; 100 μ≦1″.6										
1475....	B2	4.90	5 52	+25.9	151	+ 2	0.4	194	182	145	−0.1	+0.4	0.40	....	......	50	36	45	−1.83
1313....	B3	5.31	5 21	+17.9	154	− 8	1.6	150	166	152	+0.4	+1.5	0.54	....	......	77	73	76	−0.29
1307....	B3	6.18	5 20	+16.6	155	− 9	0.9	207	165	152	−0.6	+0.7	0.37	....	......	63	50	59	+0.03
1578....	B5	6.26	6 11	+23.8	155	+ 5	1.3	108	190	146	+1.3	+0.2	0.36	+ 7.3W	+16.6	48	32	43	−0.57
1345....	B3	5.50	5 28	+18.5	156	− 7	1.2	121	169	152	+0.9	+0.8	0.40	....	......	65	52	61	−0.57
1336...	B9	6.07	5 26	+17.0	156	− 8	0.9	198	168	152	−0.4	+0.8	0.46	....	......	65	52	61	0.00
1337...		6.44	5 26	+17.0	156	− 8	0.9	320	168	152	−0.4	−0.8	0.80	....	......	46:	26:	39:	−0.60:
1507....	B2	4.71 c	5 58	+20.1	158	0	1.5	148	185	150	+0.9	+1.2	0.39	+16.7	+17.3	69	60	66	−1.19
1346....	B3	5.58	5 28	+14.2	159	− 9	0.6	189	168	156	−0.2	+0.6	0.43	....	......	63	49	58	−0.60
1147....	B3	3.78	4 46	+ 5.4	161	−23	0.7	214	130	160	−0.7	+0.1	0.28	+19.0 orb.	+18.8	59	40	53	−2.60
1357....	Oe5	3.66	5 30	+ 9.9	163	−11	1.1	175	168	160	−0.1	+1.1	0.27	....	......	74	69	72	−2.05
1353....	B	4.53	5 29	+ 9.4	164	−12	0.8	187	167	160	−0.3	+0.8	0.32	....	......	69	59	66	−1.37
1159....	B3	3.87	4 49	+ 2.3	164	−24	0.4	225	126	162	−0.4	−0.1	0.25	+13.7	+19.0	61	42	55	−2.43
1314....	B2	4.66	5 22	+ 3.0	168	−16	1.4	197	157	165	−0.9	+1.1	0.32	+ 7.7 orb.	+19.3	57n	57n	57n	−1.56
1312....	B9	(7.7)	5 21	+ 2.8	168	−16	0.2	297	156	165	−0.1	−0.2	0.40	....	......	51n	51n	51n	(+1.2)
1391....	B3	4.54	5 34	+ 4.1	168	−13	0.6	72	168	166	+0.6	−0.1	0.36	....	......	51n	51n	51n	−1.92
1289....	B3	4.99	5 18	+ 3.4	168	−17	0.7	180	153	164	−0.3	+0.6	0.36	....	......	51n	51n	51n	−1.47
1332....	B3	5.52	5 26	+ 3.2	169	−15	0.9	140	160	165	+0.3	+0.8	0.31	+17.3W	+19.3	57n	57n	57n	−0.70
1295....	B3	5.64	5 19	− 0.3	170	−18	1.5	341	151	167	+0.3	−1.5	1.02	+18.1W	+19.5	57n	57n	57n	−0.58
1284....	B3	4.65	5 17	− 0.5	170	−19	0.5	281	149	167	−0.4	−0.3	0.33	+23.7 est.	+19.5	51n	51n	51n	−1.81
1339....	B	2.48	5 27	− 0.4	171	−16	0.1	45	159	166	+0.1	0.0	0.14	+18.8 orb.	+19.4	51n	51n	51n	−3.98
1706....	Oe5	4.68	6 35	+10.0	171	+ 3	0.8	157	211	157	+0.6	+0.5	0.26	....	......	62	46	57	−1.54
1283....	B3	5.65	5 16	− 0.5	171	−19	1.2	306	148	167	−0.4	−1.1	0.39	....	......	57n	57n	57n	−0.57
1301....	B1	3.44	5 19	− 2.5	172	−19	0.8	50	130	168	+0.8	+0.1	0.28	+31.2 orb.	+19.6	57n	57n	57n	−2.78
1370....	B	1.75	5 31	− 1.3	173	−16	0.2	0	149	169	+0.1	−0.2	0.15	+20.1 est.	+19.6	51n	51n	51n	−4.71
1349....	B2	5.37	5 28	− 1.2	173	−17	0.3	180	147	169	−0.3	+0.2	0.64	S.B.	......	51n	51n	51n	−1.09
1399....	B3	5.00	5 36	− 1.2	174	−15	1.3	252	153	170	−1.3	−0.2	0.64	+21.8 orb.	+19.7	57n	57n	57n	−1.22
1192....	B8	4.81	4 57	− 7.3	174	−26	1.4	0	92	166	+1.4	0.0	0.32	....	......	67:	55:	63:	−1.19:
1398....	B	2.05	5 36	− 2.0	175	−15	0.8	114	138	171	+0.3	+0.7	0.16	+13.2	+19.8	57n	57n	57n	−4.17
1389 ...	B	3.78	5 34	− 2.7	175	−16	0.5	0	123	171	+0.4	−0.3	0.23	....	......	51n	51n	51n	−2.68
1363....	Oe5	5.36	5 30	− 5.5	177	−18	0.9	20	81	172	+0.8	+0.4	0.31	....	......	57n	57n	57n	−0.86
1364....	B2	4.65	5 30	− 4.9	177	−18	0.2	26	94	172	+0.2	+0.1	0.36	....	......	51n	51n	51n	−1.81
1250....	B8pc	0.34	5 10	− 8.3	177	−24	0.3	19	74	170	+0.2	+0.2	0.12	+18.3 orb.	+19.7	71:	64:	69:	−5.47:
1231....	B2	4.34	5 4	− 8.9	177	−26	0.5	143	73	169	−0.5	+0.2	0.28	....	......	71:	63:	68:	−1.50:
1223....	B8	5.67	5 4	− 8.8	177	−26	1.0	180	73	169	−1.0	−0.3	0.56	....	......	67:	58:	64:	−0.30:
1366....	Oe5	2.87	5 31	− 6.0	178	−18	0.3	90	80	172	−0.1	+0.3	0.26	+17.0 orb.	+19.8	51n	51n	51n	−3.59
1361....	B1	5.58	5 30	− 6.1	178	−19	0.8	7	81	172	+0.8	+0.2	0.64	....	......	57n	57n	57n	−0.64
1362....	B1	4.67	5 30	− 6.1	178	−19	1.0	16	81	172	+0.9	+0.4	0.54	+24.7	+19.8	57n	57n	57n	−1.55
1340....	B3	4.64	5 27	− 7.4	178	−20	0.9	159	70	172	−0.9	0.0	0.40	+12.7	+19.8	57n	57n	57n	−1.58
1523....	B3	5.37	6 2	− 4.2	179	−11	0.5	116	252	172	+0.3	−0.4	0.72	+11.8W	+19.8	69:	62:	67:	−0.50:
1454....	B3	5.32	5 47	− 7.5	181	−16	1.2	90	37	177	−1.0	+0.7	0.41	....	......	67::	68::	67::	−0.55::
1277....	B1	4.29	5 15	−13.3	182	−25	0.7	0	45	170	+0.5	+0.5	0.32	+16.7	+19.7	62:	60:	61:	−1.78:
1435....	B	2.20	5 43	− 9.7	182	−17	0.3	108	8	178	−0.3	−0.1	0.20	+16.7	+20.0	62::	65::	63::	−3.80:
1512....	B3	5.12	5 59	− 6.7	182	−13	1.0	241	297	174	+0.8	+0.6	0.71	....	......	66:	67:	66:	−0.78.
1634....	B3	4.98	6 23	− 4.7	183	− 6	1.8	350	261	168	−1.8	0.0	0.41	+23.7	+19.6	68:	57:	64:	−0.99:
1598....	B3	5.13	6 15	− 7.8	183	− 9	0.7	286	297	171	+0.1	+0.7	0.44	....	......	76:	73:	75:	−0.49:

TABLE XXXIX—*Continued*

1554....	B3	5.09	6 7	− 6.5	183	−11	1.1	236	278	172	+0.7	+0.8	0.60	....	......	76:	75:	76:	−0.51:
1686....	B9	5.48	6 32	− 5.1	184	− 4	0.8	157	259	167	+0.8	−0.2	0.74	....	......	66:	53:	62:	−0.56:
1297....	B3	5.17	5 19	−14.0	184	−24	0.8	0	37	170	+0.5	+0.6	0.52	....	......	66:	64:	65:	−0.77:
1503....	B8	4.97	5 57	−10.6	185	−15	1.0	5	347	175	−0.3	+1.0	0.48	....	......	80::	79::	80::	−0.51::
1812....	B3	4.89	6 58	− 4.1	186	+ 2	1.2	355	254	161	−1.2	−0.2	0.37	....	......	58:	38:	51:	−1.57:
1580....	B9	4.99	6 11	−13.7	189	−13	0.8	66	330	171	−0.8	−0.1	0.95	....	......	69:	64:	67:	−0.88:
1739....	B8	5.19	6 42	−14.3	193	− 6	1.5	42	292	165	−1.4	−0.5	0.50	+13.0W	+19.3	65:	47:	59:	−0.96:
1609....	B1	1.99	6 18	−17.9	194	−13	0.8	300	329	167	+0.4	+0.7	0.22	+29.4 est.	+19.5	74:	70:	73:	−3.69:
1594....	B3	5.31	6 14	−19.9	195	−15	1.7	287	336	166	+1.3	+1.1	1.12	....	......	81:	80:	81:	−0.15:
1819....	B5	4.07	6 59	−15.5	196	− 3	1.0	185	289	160	+1.0	−0.2	0.24	+24.2	+18.8	56:	36:	49:	−2.48:
1793....	B5	4.39	6 52	−19.9	196	− 5	1.5	356	294	162	−1.3	+0.7	0.36	+35.7	+19.0	70	61	67	−1.48
1597....	B5	5.66	6 15	−20.9	196	−15	1.5	318	336	165	+0.5	+1.4	1.08	....	......	86:	90:	87:	+0.36:
1781....	B1	4.66	6 49	−20.1	199	− 7	1.5	12	305	162	−1.4	+0.6	0.46	....	......	69	58	65	−1.28
1660	B1	4.35	6 28	−23.3	200	−13	1.2	42	327	162	−1.2	+0.3	0.56	+25.7 est.	+19.0	64	50	59	−1.80
1924....	B8	4.87	7 18	−18.8	201	− 1	0.7	270	292	156	+0.3	+0.7	1.03	+12.7	+18.3	65	52	61	−1.20
1998....	B3	5.66	7 32	−19.5	203	+ 2	1.8	315	291	153	−0.7	+1.6	1.06	....	......	81	77	80	+0.18
1652....	B8	5.81	6 27	−27.7	204	−15	1.1	26	334	158	−0.9	+0.7	0.63	....	......	64	51	60	−0.30
1817....	B5pe	3.12	6 59	−23.7	204	− 7	0.6	288	308	157	+0.2	+0.6	0.32	+44.7 est.	+18.4	63	48	58	−3.06
2150....	B3	5.54	8 5	−16.0	204	+10	1.1	138	277	145	+0.7	−0.8	0.78	+28.4W	+16.4	38:	20:	32:	−1.93:
1899....	Oe	4.90	7 15	−24.4	205	− 4	1.4	270	302	154	+0.7	+1.2	0.44	....	......	74	66	71	−0.84
2158....	B3	4.34	8 5	−19.0	206	+ 9	1.5	254	281	145	+0.7	+1.3	0.40	....	......	67	56	63	−1.66
1954....	B2	5.48	7 23	−22.9	206	− 2	1.5	11	298	152	−1.4	+0.4	0.99	....	......	59	43	54	−0.86
1901....	Oe5	4.40	7 15	−24.8	206	− 4	1.6	330	304	154	−0.7	+1.4	0.50	....	......	78	73	76	−1.20
1872....	B5p	4.66	7 10	−26.2	206	− 6	1.6	334	307	154	−0.7	+1.4	0.35	....	......	78	73	76	−0.94
1858....	B5	5.86	7 8	−25.8	206	− 6	1.3	321	308	154	−0.3	+1.3	0.50	....	......	76	70	74	+0.21
1601....	B3	3.10	6 16	−30.0	206	−18	1.2	42	342	157	−1.0	+0.6	0.29	+19.7 est.	+18.4	63	48	58	−3.08
1877....	B3	3.83	7 11	−26.6	207	− 6	0.4	270	309	153	+.03	+0.3	0.51	+24.7 est.	+17.8	59	42	52	−2.59
2009....	B8	4.64	7 34	−25.1	208	− 1	1.4	270	298	149	+0.7	+1.2	0.62	....	......	68	58	65	−1.30
1804....	B1ac	1.63	6 55	−28.8	208	−10	0.7	8	318	155	−0.5	+0.4	0.22	+24.9	+18.1	61	45	56	−4.63
1736....	B	5.91	6 42	−30.8	208	−14	1.5	53	328	154	−1.5	+0.1	0.69	....	......	58:	40:	52:	−0.51:
1731....	B	5.16	6 41	−31.0	208	−14	2.3	347	328	154	−0.8	+2.2	0.66	....	......	94	101	96	+0.07
1669....	B3	5.70	6 29	−32.0	208	−16	0.9	18	336	154	−0.6	+0.7	0.70	....	......	66	53	62	−0.34
1532....	B5	5.93	6 3	−34.3	208	−22	0.9	333	353	154	+0.3	+0.8	0.72	....	......	67	55	63	−0.07
1490....	B3	4.36	5 54	−35.3	208	−24	1.1	338	357	154	+0.4	+1.0	0.38	+18.7	+18.0	70	60	67	−1.51
2061....	B2	4.59	7 44	−25.7	210	+ 1	1.2	275	295	148	+0.4	+1.1	0.56	....	......	66	54	62	−1.45
1934....	B5pe	2.43	7 20	−29.1	210	− 5	1.5	323	308	150	−0.4	+1.5	0.26	+36.0	+17.3	74	69	72	−3.28
1761....	B2	3.78	6 46	−32.4	210	−13	1.3	312	325	153	+0.3	+1.3	0.38	+ 3.7:*	+17.8	75	68	73	−1.90
1600....	B8	5.83	6 16	−34.4	210	−20	0.7	63	344	153	−0.7	+0.1	0.74	....	......	56	38	50	−0.68
1870....	B	6.53	7 10	−30.9	211	− 8	1.3	293	315	150	+0.5	+1.2	0.67	....	......	69	60	66	+0.63
1946....	B	6.20	7 22	−31.5	213	− 6	1.3	4	311	148	−1.0	+0.8	0.68	....	......	60	47	56	−0.06
1932....	B	5.47	7 20	−32.0	213	− 7	1.6	340	314	148	−0.8	+1.4	0.59	....	......	70	63	68	−0.37
1929....	B	5.43	7 19	−31.7	213	− 7	0.5	12	314	148	−0.4	+0.3	0.66	....	......	52	36	47	−1.21
1966....	B	6.51	7 25	−31.6	214	− 6	0.3	252	311	148	+0.3	+0.2	0.70	....	......	51	34	45	−0.22
1891....	B3	5.01	7 13	−36.4	215	−10	0.8	220	320	146	+0.8	−0.1	0.70	....	......	44:	28:	39:	−2.03:

d) SUPPLEMENTARY LIST

857....	B8	5.09	3 39	− 1.5	156	−40	1.7	123	104	146	−0.6	+1.6	0.46	....	......	73	66	71	−0.65
926....	B5	5.25	3 56	− 1.8	160	−37	2.4	100	106	150	+0.2	+2.4	0.36	....	......	93	99	95	+0.14
893....	B8	5.49	3 48	− 5.7	163	−41	1.2	228	98	150	−0.9	−0.8	0.38	....	......	43:	24:	37:	−1.67:
1079....	B2	4.12	4 31	− 3.6	167	−31	0.2	180	105	160	−0.2	+0.1	0.26	+ 9.7 est.	+18.8	59:	40	53:	−2.26:
1123....	B5	4.18	4 41	− 3.4	168	−28	2.2	117	110	162	−0.3	+2.2	0.22	....	......	102	123	109	−0.63
1010....	B8	5.72	4 16	− 7.8	169	−36	2.4	86	95	158	+0.4	+2.4	0.67	....	......	108	128	115	+1.02
1052....	B3	5.50	4 24	−13.3	176	−36	0.7	98	81	158	−0.2	+0.7	0.92	....	......	64	51	60	−0.61
956....	B3	5.45	4 5	−16.6	178	−42	1.3	48	76	153	+0.6	+1.1	1.00	+ 9.0W	+17.8	71	62	68	−0.39

* Mean of two components of bright $H\gamma$ excluded.

agreement with an error-curve extends, with some rough approximation, over the whole curve, though there are signs indicating some excess of extreme luminosities in the case of the B8–B9, and A stars.

NOTATION

The notation used is the same as in *Mount Wilson Contribution*, No. 82, except that t has been substituted for v (Section 11). Only what is necessary for the understanding of Table XXXIX is repeated here.

ν, τ	components of the angular proper motion μ along the great circle toward the antivertex, and at right angles thereto, respectively.
$100\,r$	100 times probable error in the proper motion of any co-ordinate (Section 10).
$\rho-4.3$	the observed radial velocity corrected by the constant amount -4.3 km, in accordance with *Mount Wilson Contribution*, No. 82, p. 28.
W	observed at Mount Wilson.
:	observation uncertain.
orb.	spectroscopic binary, radial velocity of system obtained from determination of orbit.
est.	spectroscopic binary, radial velocity of system estimated.
(1)	only one observation.
$\pi_{2.5}$	parallax on the supposition that $r_u=\pm 2.5$ } See (47) and Section 12, Remark 2.
$\pi_{0.0}$	" " " " $r_u=\ \ 0.0$
π adopted	" " " " $r_u=\pm 1.67$
n	when the parallax is marked with n the star has been assumed to belong to the Nebula-group.

GRONINGEN
January 1917

ANNOUNCING

THE SUMMER QUARTER 1918

AT

THE UNIVERSITY OF CHICAGO

THE war has emphasized as never before the need of the nation for highly skilled men and women, and an obvious obligation rests on every citizen speedily to develop the greatest possible capacity for service.

The Summer Quarter of the University of Chicago affords an unusual opportunity to hasten the completion of any general training already begun, and to secure special intensive training in lines immediately related to war needs, e.g., ordnance supply, military science, food conservation, first aid, spoken French, etc.

In 1918 the Summer Quarter will begin June 17 and close August 30. The First Term will begin June 17; the Second Term, July 25. Students may register for either Term or for both. Students entering at the beginning of the Second Term may register for courses for which they have had the prerequisites. The courses during the Summer Quarter are the same in character, method, and credit value as in other quarters of the year.

A large proportion of the regular Faculty of the University, which numbers over three hundred, and also many instructors from other institutions, offer courses in the Summer Quarter, and in this way many varied points of view are given to students in their chosen fields of study.

ARTS, LITERATURE, AND SCIENCE

The University offers during this Quarter, in the Schools of Art, Literature, and Science, both graduate and undergraduate courses in Philosophy, Psychology, and Education; Political Economy, Political

Science, History, Sociology and Anthropology, and Household Administration; Semitics and Biblical Greek; Comparative Religion; History of Art, Sanskrit, Greek, and Latin; Modern Languages; Public Speaking; Mathematics, Astronomy, Physics, and Chemistry; Geology and Geography; Botany, Zoölogy, Physiology, Physiological Chemistry and Pharmacology, Anatomy, Pathology, Hygiene and Bacteriology; and Military Science.

Divinity

The Divinity School is open to students of all denominations, and the instruction is intended for ministers, missionaries, theological students, Christian teachers, and others intending to take up some kind of religious work. The English Theological Seminary, which is intended for those without college degrees, is in session only during the Summer Quarter. The Graduate Divinity School is designed for college graduates. Pastors, theological teachers, students in other seminaries, candidates for the ministry, and other Christian workers, with requisite training, are admitted in the Summer Quarter.

The Chicago Theological Seminary will also be in session during the Summer Quarter, and its courses are open on the same conditions as those that obtain in the Divinity School.

Law

In the work of the Law School the method of instruction employed—the study and discussion of cases—is designed to give an effective knowledge of legal principles, and to develop the power of independent legal reasoning. The three-year course of study offered constitutes a thorough preparation for the practice of law in any English-speaking jurisdiction. By means of the quarter system students may be graduated in two and one-fourth calendar years. Regular courses of instruction counting toward a degree are continued through the Summer Quarter. The courses are so arranged that students may take one, two, or three quarters in succession in the summer only before continuing in the following Autumn Quarter. The summer work offers particular advantages to teachers, to students who wish to do extra work, and to practitioners who desire to study special subjects.

Medicine

Courses in Medicine constituting the first two years of the four-year course in medicine in Rush Medical College are given at the University of Chicago. For the majority of students taking up medical work for the

first time, it is of decided advantage to enter with the Spring or Autumn Quarter. For the student who is lacking in any of the admission courses, or who seeks advanced standing, it is of especial advantage to enter for the Summer Quarter. All the courses offered are open to practitioners of medicine, who may matriculate as unclassified or as graduate students. Practitioners taking this work may attend the clinics at Rush Medical College without charge.

Education

In the Professional Schools the Graduate Department of Education in the School of Education gives advanced courses in Principles and Theory of Education, Educational Psychology, the Psychology of Retarded and Subnormal Children, History of Education, and Social and Administrative Aspects of Education. The College of Education is a regular college of the University, with all University privileges, and in addition provides professional training for kindergarten-primary, elementary- and secondary-school teachers and supervisors, and for special teachers in Home Economics and in Aesthetic and Industrial Education. It offers undergraduate courses in professional subjects and in the methods of arranging and presenting the various subject-matters which are taken up in the elementary and secondary schools.

Commerce and Administration

The School of Commerce and Administration is an undergraduate-graduate professional school, offering courses arranged to meet the needs of those preparing for various business pursuits, for commercial teaching, for secretarial work, and for philanthropic service. The work for the summer of 1918 will be organized, in co-operation with the School of Education, with especial reference to the needs of commercial teachers. In all the curricula emphasis is placed upon (1) broad foundations of work in history, political economy, sociology, psychology, biology, government and law; (2) an individualized curriculum; (3) contact with practical affairs; and (4) a professional spirit.

A series of public lectures in Literature, History, Sociology, Science, Art, Music, etc., scheduled at late afternoon and evening hours throughout the Summer Quarter, affords an opportunity to students and other members of the University community to hear speakers of authority and distinction in many departments of study and activity. This program will include a number of popular readings and recitals, open-air performances, concerts, and excursions to places and institutions of interest in and near Chicago.

Notable public libraries and museums, highly organized industrial plants, many typical foreign colonies, a large number of settlements, and other significant social institutions make Chicago a peculiarly appropriate center for study and investigation.

In the Frank Dickinson Bartlett Gymnasium for men and the Ida Noyes Gymnasium for women full facilities for indoor exercise are given. Social privileges are offered through the Reynolds Club and Ida Noyes Hall.

The climatic conditions of Chicago during the summer months are excellent, the refreshing lake breeze alleviating even the hottest days. The location of the University is especially fortunate, situated on the Midway Plaisance, the connecting link between two of Chicago's most beautiful parks. These parks are within easy walking distance from the University and contain tennis courts, golf links, bathing-beaches, and lagoons for rowing. These are all open to the public without charge.

The complete ANNOUNCEMENT of courses for the Summer Quarter of 1918 will be mailed upon application to

THE UNIVERSITY OF CHICAGO
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ON THE CORRECTION OF OPTICAL SURFACES

By A. A. MICHELSON

In a recent number of the *Philosophical Magazine* (6), **35**, 49, 1918, an interesting method for correcting optical surfaces by means of the interferometer was developed by F. Twyman. While nothing in the paper indicates that the method is limited to relatively small surfaces, it would appear that such an application to mirrors and lenses of the size of modern astronomical telescopes can hardly be contemplated, as it would involve interferometers of at least equal dimensions.

It was hoped that the modification of Twyman's method represented in Fig. 1 might avoid this difficulty. S is the light-source at the focus of a collimating lens A. Thence the light passes to the dividing surface O, part passing through the achromatic lens B and forming an image of S at the focus of the lens L to be tested. The light returns from the plane mirror M and interferes with the light reflected from O to P and back.

It appears, however, that unless the two optical paths OM and OP are equal—which would involve the presence of a second large lens similar to L—the (circular) interference bands are extremely small and difficult to observe. Even though this difficulty were overcome the assistance of an optically perfect lens B is indispensable.

In view of these difficulties the following simple scheme was put into operation and found entirely satisfactory (Fig. 2). The light from a Nernst glower is concentrated on the slit *S* by means of a

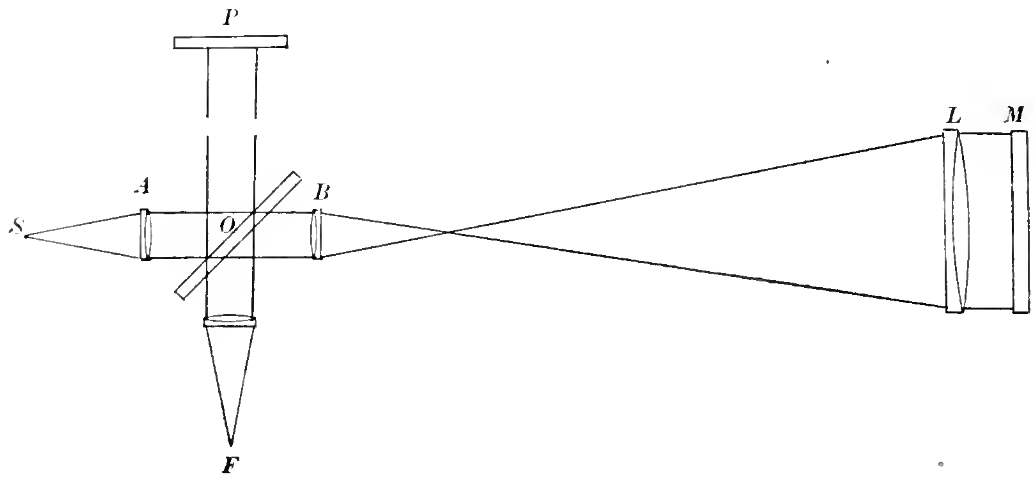

FIG. 1

microscope objective *O*, whence by a total reflection prism it is reflected to a concave mirror 6 inches in diameter and of 36 inches radius.[1] The image of the slit is formed immediately above the

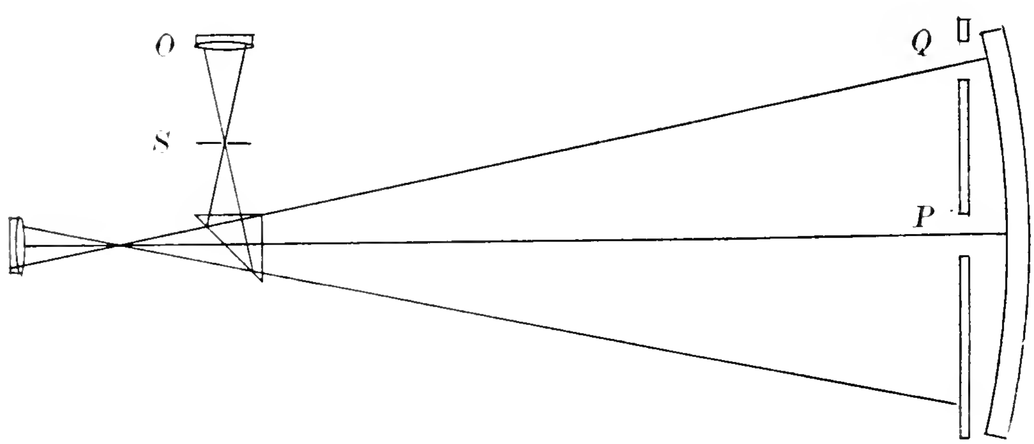

FIG. 2

prism and is viewed by a microscope with a $\frac{1}{12}$-inch objective. A series of screens with two rectangular apertures[2] *Q* and *P* are placed immediately in front of the mirror, one aperture at the center and

[1] This was a mirror in process of polishing and was purposely selected on account of its relatively large errors.

[2] The series of screens could advantageously be replaced by a double-slit mechanism which permits of a continuous variation of the distance *PQ*.

the other at varying distances, producing interference bands, the central band corresponding exactly with the position of the slit image if the mirror is perfect. The distance between the centers of band and the slit image in fractions of the fringe-width gives twice the error of the mirror at the point of the mirror corresponding to the position of the aperture Q in light-waves.

These errors are determined for points Q distant from the center P by 2, 3, 4, 5, 6, and 7 cm. A second set of measurements is taken with Q at the opposite end of the diameter.

Such double sets are repeated at intervals of 45° rotation of the mirror about its axis, through 360°, and the results plotted as represented in Fig. 3. The numbers represent hundredths of a fringe. A second trial gave results differing, on the average, from the first by less than 0.02 fringe or 0.01 of a light-wave.

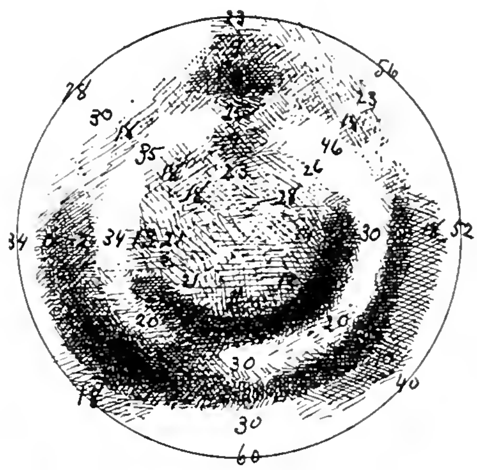

FIG. 3.—6-inch mirror. Radius 35 inches. Mean error 0.19 fringes.

A similar investigation of a 5-inch achromatic lens by O. L. Petitdidier,[1] $f = 12$ ft., backed by a plane mirror[1] used for spectrographic work, showed errors so small that artificial errors were introduced by placing in the path of the pencil a plane-parallel plate 2 inches in diameter, which had been made roughly cylindrical by retouching, at 4 feet from the focus.

The resulting errors are plotted in Fig. 4. The plane-parallel plate was then corrected by local retouching, and the corresponding errors are given in Fig. 5.

Fig. 6 shows a series of photographs of the slit image before correction (under a magnification of about 1000 diameters) at intervals of 45° rotation of the lens and shows clearly the effects of the astigmatism.

[1] On account of the secondary chromatic aberration the light used was approximately homogeneous, corresponding in wave-length with the yellow Hg line.

Fig. 7 gives the corresponding photographs of the slit image after correction. The result is a practically perfect image. It may be added that the entire process of investigation of errors and succeeding corrections took about six hours.

It is clear that such a process of correction may be applied to even the largest astronomical mirrors or lenses, both as an assistance in the original figuring[1] and in the final correction, and that such final corrections may be made without risk of injury to the objective by an auxiliary correcting plate placed anywhere between the objective and focus.[2]

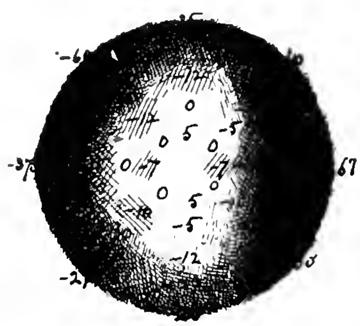

Fig. 4.—Before correcting, mean error 0.29 fringes.

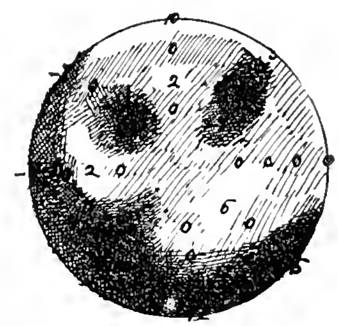

Fig. 5.—After correcting, mean error 0.04 fringes

There is one very important precaution to be observed in these measurements—especially for lenses or mirrors of large aperture: great care must be exercised in the focal adjustment. If, for instance, h is the depth of a concave mirror under observation, r the radius of its disk, and R the radius of curvature, $h=\frac{r^2}{2R}$ and $\delta h=\frac{r^2}{2R^2}\delta R$. Thus if, as in the foregoing investigation, $\frac{r}{R}=\frac{1}{6}$, $\delta R=72\,\delta h$. If then δh is to be correct to within 0.1λ, δR must not exceed 7.2λ or 0.0036 mm.

A similar remark applies (though in less degree) to the selection and maintenance of the wave-length of the light employed if the objective be an achromatic lens.

Paraboloidal or ellipsoidal surfaces as well as spherical.

[2] If, however, the errors are confined to relatively small areas, the correcting plate must be near the objective.

With evident modifications the same method applies to the correction of prisms and gratings. Here of course the order of accuracy attainable depends on the degree of homogeneity of the light-source. A cadmium lamp is amply sufficient for anything below a required resolving power of, say, 500,000, but this may not furnish sufficient light to observe the interference fringes under the high magnification required, especially as the cadmium lines broaden considerably as the luminosity increases.

A quartz mercury lamp has been found to be fairly effective.

Evidently the interferometer method, as shown in Fig. 8, or even more simply a single plane-parallel plate, Fig. 9, may be

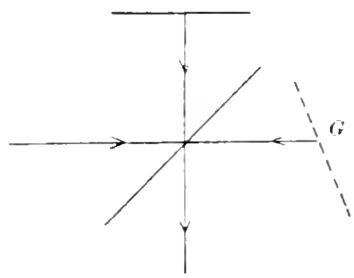

FIG. 8

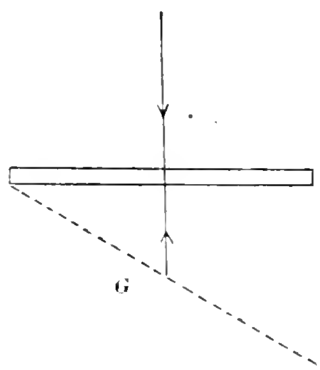

FIG. 9

applied. In either case, with proper adjustment, the interference fringes are concentric circles, and if the grating G (and of course the auxiliary surfaces) be perfect, the diameter of these circular fringes remains constant when the eye or the observing telescope is moved about in any direction. The errors may thus be determined and the auxiliary plate may be corrected at the corresponding points.[1]

In the method of Fig. 9 there is no singularity to be met, such as occurs in the shape of the interference fringes in the method of Fig. 8 in the vicinity of zero difference in path. These singular figures are shown in the photographs[2] (Fig. 10) for various adjustments of the angles of the mirror.

[1] This would be good for only one order of spectrum, though by rotating the correcting plate in the same sense as the grating there would still be partial compensation.

[2] It should be noted that these photographs are taken with a small diaphragm. If the entire aperture is used (focused for parallel rays), the entire interference system reduces to the single circular fringe.

The difference in path of the two interfering pencils is given by

$$\Delta=\frac{2c}{S^2}(y-y_0)(x^2+y^2)+\alpha x+\beta y$$

in which c is the tangent of the angle which the grating makes with the wave-front;

S = distance of the eye from the mirror;

α = "error" of adjustment in altitude;

β = "error" of adjustment in azimuth.

(Grating rulings vertical.)

Fig. 11 gives two cases which have been calculated for $\Delta=1$, 2, 3, etc. 11*a* for $\alpha=0$, $\beta=-2$, $y_0=1$, and 11*b* for $\alpha=0.1$, $\beta=-2$, $y_0=0$.

Ryerson Physical Laboratory
University of Chicago
May 1918

ON THE CAUSE UNDERLYING THE SPECTRAL DIFFERENCES OF THE STARS

By C. D. PERRINE

Some recent investigations have led to an explanation of the strong affinity of the stars of Class B for the Milky Way and of a relation of the spectral differences in the components of double stars to their distance from the sun. Briefly it is that the spectral and perhaps other characteristics of these stars, in common with the novae, are due chiefly to conditions existing in the more distant regions of the galaxy instead of wholly to a process of development through radiation and condensation. This explanation was suggested largely by the fact that Nova Persei No. 2 (the only nova observed before the appearance of bright lines) passed through the B-type stage at or about the time of its greatest brightness.

Such a hypothesis involves the entire question of evolution, as it tends to make location in the system and external causes of equal or greater importance than the elements of time or mass in determining the physical conditions of the stars belonging to our system. It appears to have but little bearing on the ultimate life-history of the system as a whole, its beginning, or its end.

Further consideration and study have brought to light other facts which appear to bear also upon this question and which will be discussed in what follows from the broader standpoint of their relation to the processes of evolution.

Relation of the spectral differences in the components of double stars to distance as indicated by proper motion.—It seemed desirable to investigate what was believed to be contrary evidence afforded by the spectral differences of the components of double stars. It was seen almost at once that there was a relation between these spectral differences other than the well-known conclusion that in contrasted pairs the fainter component is generally of the earlier spectral type.[1]

[1] Clerke, *Problems in Astrophysics*, p. 263.

It was found that this condition appears to be related to the size of the proper motions and therefore to the distance of the stars from the sun. Other investigators have noticed that a few of the fainter components of double stars give spectra of later type, but appear to have ascribed little or no importance to the contradiction, none so far as I know attempting other explanation than a purely accidental one.

The data used for this investigation are taken from the observations of the spectra of 745 double stars made at Harvard and classified by Miss Cannon.[1]

For obvious reasons this preliminary investigation was limited to stars known either to be binary or to have common proper motion whose differences of brightness are approximately half a magnitude or greater. There were found 78 of such stars in that list.

It was first observed that the stars in which the fainter component was of the earlier type were in general closer together than those in which the companion was of a later type. Before being able to draw any conclusions it was desirable to get some idea of the effect of distance from the observer on the actual separations. This was investigated through the medium of the proper motions. The result has been almost as startling as it was unexpected in indicating that *the relative spectral types of the components depend upon their distances from the sun, those pairs in which the fainter component is of earlier type being distant, whereas those in which the fainter component is of later type are much nearer.* I have encountered few facts more significant than this appears to be. It is, in my opinion, difficult to exaggerate its importance in connection with the physical condition of the various bodies composing our stellar system. For this reason the data will be examined in considerable detail.

The stars selected as suitable from the Harvard results are given in the accompanying tables, together with the total proper motions of the systems as computed from data given in Boss's *Preliminary General Catalogue.*

Of these stars, 26 have the fainter components of later type, 28 have them of earlier type, and 24 are of the same type.

[1] *Harvard Annals,* **56**, Pt. 7.

The stars were grouped according to whether the fainter component is of later spectral type than the primary (Table I), or whether it is of earlier type (Table II). Although undoubtedly of much interest and importance in detailed studies of the spectral

TABLE I

FAINTER COMPONENT OF LATER TYPE

Star	1900.0 α	1900.0 δ	Mag. Brighter Component	Δ Mag.	Spectral Class Brighter	Spectral Class Fainter	μ	s
ζ Piscium......	$1^h\ 8^m.5$	$+\ 7°\ 3'$	5.6	0.9	A5	G5	0″.143	23″.7
37 Ceti.......	1 9.4	− 8 28	5.2	2.6	F5	G	.295	49.4
γ Arietis......	1 48.0	+18 49	4.8	4.8	Ap	K	.138	222.8
59 Andromedae	2 4.8	+38 34	6.0	0.7	A0	A2	.025	16.6
η Tauri.......	3 41.5	+23 48	3.0	3.3	B5	A0	.052	117.3
f Eridani......	3 44.9	−37 55	4.9	0.6	B8	A0	.082	7.8
θ Tauri.......	4 22.9	+15 39	3.6	0.5	A5	K0	.107	337.1
α Leonis......	10 3.0	+12 27	1.3	6.3	B8	G	.247	176.7
H.R. 4191.....	10 37.7	+46 44	5.3	1.8	F0	G0	.289	288.1
17 Comae Ber .	12 23.9	+26 28	5.4	1.4	A0p	A3	.029	145.3
α Centauri.....	14 32.8	−60 25	0.3	1.4	G0	K5	3.67	21.9
α Librae......	14 45.3	−15 38	2.9	2.2	A2	F5	0.131	231″.0
κ Lupi........	15 5.0	−48 21	4.2	1.7	B9	A	.121	26.9
μ Boötis......	15 20.7	+37 44	4.5	2.2	F0	K0	.169	108.3
ν Scorpii......	16 6.2	−19 12	4.3	2.2	B3	A	.034	41.3
ε Normae......	16 19.8	−47 20	4.8	2.7	B5	A	.010	22.8
b Draconis....	18 22.5	+58 45	4.9	2.7	A2	F	.065	88.8
κ Coronae Aust.	18 26.5	−38 48	6.0	0.6	B8	A	.038	21.7
β Lyrae.......	18 46.4	+33 15	3.4*	4.4	B2	B3	.008	46.0
57 Aquilae.....	19 49.3	− 8 29	5.8	0.8	B3	A	.022	35.7
15 Cephei.....	22 0.6	+59 19	6.7	0.9	B0	B9	.013	183.4
15 Cephei.....	22 0.6	+59 19	6.8	0.8	B5	B9	.013	136.1
15 Cephei.....	22 0.6	+59 19	6.7	0.1	B0	B5	.013	236.3
15 Cephei.....	22 0.6	+59 19	6.8	1.0	B5	B9	.013	192.4
ξ Cephei......	22 0.9	+64 8	4.6	1.9	A3	G?	.230	6.9
8 Lacertae.....	22 31.4	+39 7	5.8	0.7	B3	B5	.018	22.4

* Var.

characteristics of double stars, including the present problem, those having both components alike in this respect will not be considered especially, although several peculiarities have been noticed.

An examination of the individual stars in Table I shows that all of those which have proper motions under 0″.050, without exception, belong to Classes B and A. The differences of spectral class and of magnitude are also consistently smaller than for the stars having larger proper motions. This appears to be too consistent to be mere coincidence even in so small a number of stars, especially

in connection with the peculiarities of those stars whose components are of earlier types.

The consistency with which the 28 systems having their fainter components of earlier types show small proper motions is remarkable, especially when it is considered that all but four of these stars

TABLE II

FAINTER COMPONENT OF EARLIER TYPE

Star	1900.0		Mag. Brighter Component	Δ Mag.	Spectral Class		μ	s
	α	δ			Brighter	Fainter		
1 Arietis	$1^h44^m.6$	$+21°47'$	6.2	1.2	G0	A	0″.018	2″.9
α Piscium	1 56.9	+ 2 17	4.3	0.9	A3	A	.042	2.6
γ Andromedae	1 57.8	+41 51	2.3	2.8	K0	A	.070	10.7
w Eridani	3 49.2	− 3 14	5.0	1.4	G5	B?	.034	6.9
H.R. 1771	5 17.7	−24 52	5.5	1.2	G	A3	.029	3.0
H.R. 2174	6 3.8	+ 2 31	5.9	1.1	A0	B9	.027	28.9
14 Lyncis	6 44.2	+59 34	5.8	1.2	G	A	.047	0.4
μ Can. Maj.	6 51.5	−13 55	5.4	3.1	K	A	.007	2.3
γ Volantis	7 9.6	−70 20	3.9	1.9	K0	G	.097	13.3
H.R. 3428	8 34.6	+20 1	6.4	0.5	G	A2	.041	63.4
ι Cancri	8 40.6	+29 8	4.2	2.4	G5	A2	.054	30.6
24 Comae Ber.	12 30.1	+18 56	5.2	1.5	K0	A3	.017	20.6
H.R. 4893	12 48.4	+83 57	5.3	0.5	A2	A	.033	21.6
a Lupi	14 30.8	−45 42	5.6	3.3	K0	A	.034	19.8
ϵ Boötis	14 40.6	+27 30	2.7	2.4	K0	A	.049	2.8
δ Boötis	15 11.5	+33 41	3.5	4.5	K0	G0	.155	104.8
α Scorpii	16 23.3	−26 13	1.2	5.8	Ma	A	.034	3.2
H.R. 6575	17 34.1	+ 2 5	6.4	1.1	K	F5	.048	111.2
H.R. 6803	18 5.7	+16 27	6.5	1.1	F	A	Small	1.2
H.R. 7140	18 51.2	+33 51	6.1	1.7	G	A	.011	45.5
H.R. 7300	19 10.8	+14 54	5.7	2.1	G0	A0	.020	89.9
β Cygni	19 26.7	+27 45	3.2	2.1	K	B9	.009	34.7
χ Aquilae	19 37.9	+11 35	5.6	1.2	F5	A	.013	0.6
H.R. 7548	19 44.7	−55 13	6.1	0.6	G5	A2	.025	23.0
o^1 Cygni	20 10.5	+46 26	3.9	3.2	K	B8	.002	107.1
β Capricorni	20 15.4	−15 6	3.3	2.8	G0	A0	.028	205.2
— Cephei	22 18.8	+66 12	7.1	1.0	G	A	.042	4.1
H.R. 9094	23 57.5	+65 33	6.0	1.5	F5	A2	.018	15.0

(brighter components) belong to *the Classes F, G, and K, which have shown the largest average proper motions of any.*

The average of the entire group is but 0″.037, and if we omit the largest two (0″.097 and 0″.155) the average is reduced to 0″.030. *This criterion of proper motion indicates that these stars are at the same general distance as the stars of Class B.*

The stars of Table II show a decided preference for the galaxy, half of them being within 15°, and three-fourths of them within 40°, of the galactic plane. This is significant when it is considered that the principal stars of these pairs belong almost entirely to the middle and later types of spectra.

Further evidence in this matter is found in 19 stars observed by Harvard College Observatory to have composite spectra and in 62 stars in Campbell's *Catalogue of Spectroscopic Binaries*,[1] in which the spectra of both components have been observed or strongly suspected.

For both spectra to appear on a photograph the difference in brightness of the components of close binary systems will usually be small. On account of the tendency for stars of nearly the same brightness to have similar spectra, the evidence from these two sources can scarcely be expected to be of as great weight as the preceding. It proves to be quite definite, however.

Of the 19 Harvard composite stars, 2 appear to be somewhat uncertain as to the differences of spectral class, both being of early type. Of the remainder the one having the largest proper motion (0."15) has the fainter component of the later type. The remaining 16 all have very small proper motions. Of these, 5 have the fainter component of later type and 11 of earlier type. All of the most strongly marked spectral contrasts have the fainter components of earlier type, and all but one of the 11 are strongly marked with respect to spectral contrasts.

This evidence is confirmatory, therefore, of the condition noted among the double stars whose components had been separately observed.

Of the 62 stars of Campbell's *Catalogue*, 30 belong to types O and B and 23 to type A. It is reasonably certain, therefore, that all of the O and B stars are distant, and the probability is strong that most of the 23 A stars are distant also.

The following statement[2] I understand to apply to the 62 stars in question:

From the published descriptions of the double spectra it is fairly well established that when the two spectra are substantially equal in brightness

[1] *Lick Observatory Bulletins*, **6**, 46, 1910.

[2] Campbell, *Lick Observatory Bulletins*, **6**, 47, 1910.

they are identical in type: and when one spectrum is considerably fainter than the other, the spectrum of the secondary is apparently of a slightly earlier type than the spectrum of the primary. There appear to be no exceptions to this rule, though the difficulty in the way of giving accurate descriptions of the fainter spectrum must be recognized.

As there is also good reason to believe that the great majority of these stars are distant, we are led directly to the conclusion that these stars, as far as they are competent, also confirm the previous evidence.

In addition to the stars of the Harvard list, for which the spectra of both components have been observed, I have examined several other well-known stars whose colors have been observed, viz., α Herculis, η Geminorum, γ Delphini, η Cassiopaeiae, ξ Boötis, β Cephei, and γ Leonis. These stars also show in general the same relation to distance as the Harvard stars. There seems room for doubt as to the spectral class which may correspond to some of the colors observed. For this reason it seems best to omit all such from the classifications until the spectra have been observed.

Careful consideration of the data seems to show beyond doubt that the relation observed is to distance alone (or coupled with low galactic latitudes). It cannot well be with the actual separations of the stars, for the range in that direction seems to be about the same in all of the groups. It can scarcely depend upon the differences of mass of the systems as indicated by the differences of brightness, for a similar reason. There is a difference in absolute magnitude between the two groups, the nearer ones being on the whole the fainter. But there is no direct evidence of a relation.

There appears to be a consistently smaller difference in brightness in the stars whose components are of the same spectral type and also of a smaller average absolute separation than for the stars whose components differ in spectral class only. The difference of brightness seems quite definite and consistent, and seems to be corroborated in the cases of the stars with large differences of brightness, which show in general a large difference of spectral class also. The differences of separations are fairly marked for the groups, but are not very consistent. Both conclusions require confirmation from more extensive data.

The conclusion that the relations of spectral type in double stars of unequal magnitude depend in general upon their distance or, what is more probable, upon their location in our stellar system, seems to be shown so definitely as to leave no doubt of its reality. The data, although not extensive, seem sufficient, together with their consistency, to insure against mere coincidence.

We do not know the earlier condition of these double stars, whether both components were of the same spectral type or not. Investigators have generally assumed that the components were originally of the same spectral type. This appears to be the most natural assumption. It is not possible to say, therefore, just what the course of change has been. (That changes have taken place no one will doubt.) However, we can justly conclude, I think, that the conditions are such in these regions as to produce *opposite* spectral effects in the components. There can be little doubt that the fainter components are in general also of smaller mass. We may therefore state our conclusion in the following form: that the conditions appear to be such that if two stars of unequal mass were introduced into the near region the smaller body would progress more rapidly toward the later stage than the larger one, whereas in the relatively distant galactic regions the tendency would be for the smaller body to become of earlier type more rapidly than the larger one.

If the smallness of the absolute separations of the components of the same spectral type, which have been alluded to, should prove to be general, it would be evidence more or less strong that the spectral types of both components were the same at the time of their origin as binaries, and further that the middle types were more favorable to their formation.

The bearing of these phenomena on spectral classes in general will be considered in a later portion of this paper.

An attempt to trace the cause of spectral differences in the stars and nebulae.—In the history of the novae (imperfect as it is), particularly of Nova Persei No. 2, we have, it seems to me, a clue of inestimable value as to the course pursued by stellar bodies under conditions operating to produce the different types of spectra. The order in which their changes occur is at least *one* order in

which the greatest of nature's forces operate, and in fact the only order of such changes of which we have indisputable evidence. The novae give us the best (and perhaps the only) clue to changes under conditions which so far are impossible to produce in the laboratory. The belief gains force with me that the phenomena of the novae are not those of an isolated and very peculiar class of bodies only, but that their processes are closely related to the life-histories of all stars, the greatest difference being that they pass through their cycle in such an incredibly short time. Is not the fact that the various stages of the novae are paralleled almost perfectly by similar stages in the spectra of ordinary stars the strongest kind of evidence of the closeness of relationship in their essential processes?

It has been known for some time in a general way that stars of different spectral class preferred different regions, the solar and later types being most numerous in the nearer regions, whereas the early-type stars, particularly those of type B, show a strong affinity for more distant regions in the direction of the galaxy.

The peculiar dependence upon distance from the sun of the spectral relation of the components of double stars differing in brightness, which has just been discovered, adds considerable weight to the theory that spectral class depends very largely upon location in the system.

Dependence upon location in the system requires an external cause and does away at once with the sufficiency of universal natural laws alone, such as gravitation, radiation, contraction, electro-magnetic, and other forces. These may, and undoubtedly do, play important parts. But they are not the controlling factor. It is equally doubtful, in my opinion, that *variations* in such universal natural laws can be invoked, such as variable gravitation, peculiarities in radiation, contraction, etc., in different parts of the system. They seem unlikely. If there were no more reasonable explanation they might be. We have, however, I think, a more reasonable and probable cause, viz., that of cosmical matter. It is not difficult to understand how stellar bodies encountering sufficient of such matter may be caused to change their surface conditions (if no more) radically. We do not understand all of the

possibilities of such a condition and its results, but we have strong evidence not only that such matter exists, but also that it causes some such changes of spectral type. By a process of elimination as well as by direct evidence some such process has been tacitly accepted to account for the novae.

Distribution.—An examination of the galactic distribution observed by Pickering[1] of 6106 stars brighter than magnitude $6\frac{1}{4}$ shows practically the same (and nearly uniform) distribution for the stars of spectral classes F, G, K, and M, but a decided preference for the galactic regions is shown by the A stars, a preference which is strongly increased in the B stars. These results indicate not only a preference of B and A stars for the galaxy, but almost an equal *avoidance* by these classes of the non-galactic regions. These characteristics are well shown in Table III and Fig. 1, where the quantities have been reduced to percentages. The intermediate position of Class A is well marked.

TABLE III

DISTRIBUTION OF STARS BRIGHTER THAN MAGNITUDE $6\frac{1}{4}$ WITH RESPECT TO GALACTIC LATITUDE

	Spectral Class						Total
	B	A	F	G	K	M	
No. of Stars........	716	1885	720	609	1719	457	6106
$\pm 62^\circ.3$.........	5	16	22	21	22	22	
± 39.8.........	12	18	21	21	22	24	
± 21.6.........	32	28	28	28	27	27	
± 8.1.........	51	38	29	30	29	27	

It is a well-known fact that nearly all of the novae, the stars of Class O, the planetary nebulae, the gaseous nebulae, and the short-period variables are found in the region of the Milky Way.

Distance.—Direct determinations of parallax are not available for the more distant stars and hence recourse is had to proper motions. It should be pointed out that Kapteyn and Adams' conclusion[2] that there is a direct relation between proper motion

[1] *Harvard Annals*, 64, 143-44, 1909.

[2] *Proceedings National Academy of Sciences*, 1, 14, 1915

and radial velocity tends to weaken the assumption as a measure of distance, but not to destroy it.

The proper motions taken from Boss's paper[1] show a marked decrease from the stars of Classes F and late A to those of Class B, indicating a general increase of distance also, as we approach the early B stars. This peculiarity is made plain in Table IV. The proper motions of the novae, the stars of Class O, the planetary

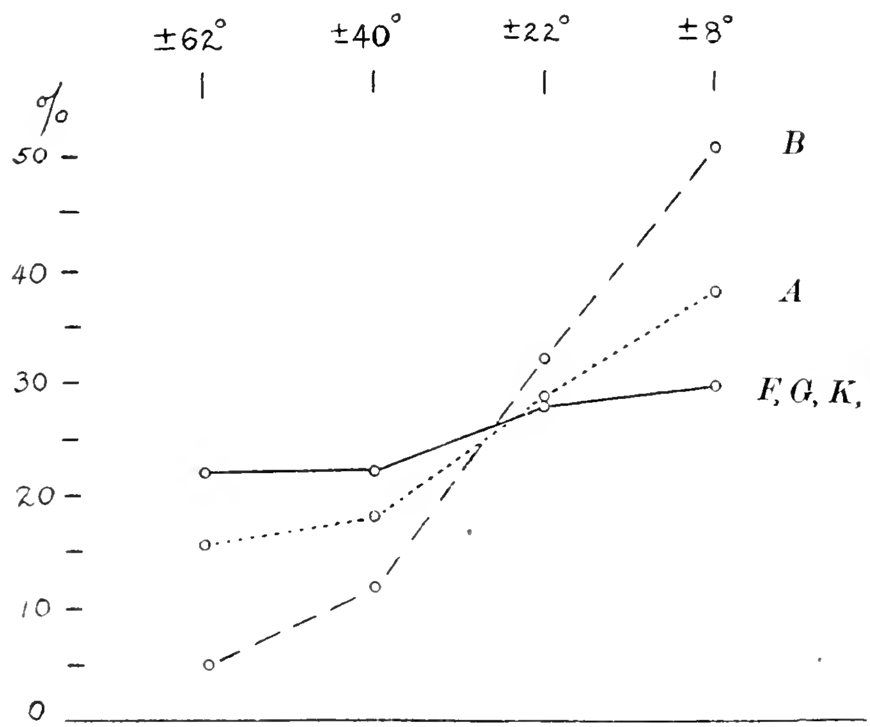

FIG. 1.—Galactic distribution by spectral classes

nebulae, and the short-period variables are very small. Hence their distances must be great.

The distributions and distances seem to confirm the belief that the more distant region in the direction of the galactic plane is the seat of conditions which tend to produce activity and a change of spectrum toward the earlier types.

Brightness.—Under this head will be considered: *a*) the absolute brightness of the stars of different spectral classes; *b*) the changes in brightness of the novae.

a) No exact figures are available to me of the average magnitudes and proper motions of the different spectral classes from which to

[1] *Astronomical Journal*, **26**, 188, 1911.

derive relative absolute magnitudes. The general condition is, however, well known from the researches of E. C. Pickering,[1] who states with regard to stars of Class B: ". . . . indicates that of the bright stars one out of four belongs to this class, while of the stars of the sixth magnitude there is only one out of twenty, and that few, if any, would be found fainter than the seventh or eighth magnitude." This appears to have a deep significance. Why are the B stars so few and all so bright?

TABLE IV

Mean Proper Motion of Stars by Spectral Class

	Number of Stars	Mean Centennial* Proper Motion
Oe5 to B5	490	2.″40
B8 and B9	217	3.82
A	1157	4.62
A2 to A4	273	5.53
A5 to A8	164	7.07
F	287	7.91
F2 to F8	205	7.93
G entire	444	5.24
K entire	1227	5.74
M entire	222	4.99
Total	4686	....

* Excluding proper motions greater than 20″ per century.

With regard to the A stars, a consideration of their average brightness and their proper motions seems to indicate an average intrinsic brightness for them greater than for the stars of later types and intermediate between them and the B stars. A consideration of their spectra also indicates a gradual diminution in brightness after the B class.

With the exception of γ Argus and ζ Puppis, all of the stars of Class O are relatively much fainter than those of Class B or other spectral types. There is good reason, therefore, to conclude that stars in the stage of Class O are, on the whole, very much fainter than stars in the stage of Class B.

There is also reason to believe that unless the planetary nebulae are enormously remote their absolute brightness is much less than

[1] *Harvard Annals*, 56, 37, 1905.

that of the O stars—that there is a sudden decline in brightness from B to O stars and a further decline to the planetary nebulae.

b) In Nova Persei No. 2 maximum brightness occurred at about the time it reached the B stage, from which it declined sharply as the bright-line stage appeared. Between the bright-line stage and the nebular stage there was a further and greater decline in brightness. In no other novae were spectroscopic observations obtained before the bright-line stage, but in all which were sufficiently observed the loss of light was very marked between the bright-line "nova" stage and the nebular stage. Considering, therefore, changes in brightness, the order appears to be the same in the novae as in the early spectral classes in the direction B, O, planetary nebulae.

Spectral changes in the novae.—With the exception of Nova Persei No. 2, the novae have not been observed spectroscopically until after the appearance of the bright bands. These emissions embrace hydrogen, helium, calcium, sodium, and a substance giving lines at λλ 463 and 468 which form one of the chief characteristics of the Wolf-Rayet or Class O stars. These emissions are often wide and distorted, probably more or less in proportion to the magnitude of the outburst, if we may judge by the brighter nova in Perseus, and are usually in pairs, an absorption line on the more refrangible side of an emission line. The strong, more or less continuous spectrum which is present at these stages gradually disappears and the lines characteristic of the nebulae make their appearance. The lines which are highest in the spectrum, particularly those at λλ 339 and 346, in some of the novae at least, were the last to appear and the first to disappear. This, together with the peculiar behavior of the bright lines of hydrogen and helium in early-type stars and the well-known shifting of the maximum of intensity in the ordinary incandescent spectrum with temperature, gives rise to the belief that the activity in these stars is progressive with wave-length and that the chief factor in these spectral changes is temperature.

With the fading of the continuous spectrum and general loss of light comes the nebular stage, when the condition of the nova is essentially a stellar planetary nebula. With continued loss of

light the pronounced nebular characteristics gradually disappear and the Wolf-Rayet conditions again become dominant. Finally some one of the ordinary stellar stages is reached as the star becomes very faint.

The more or less empirical explanation of the preference of B stars for the Milky Way and the relation to evolution in general rests very largely upon the behavior of Nova Persei No. 2. It is necessary, therefore, to review the early stages of this star briefly.

We do not know its early history further than that it must have been extremely faint previous to its very sudden outburst. Of its spectrum we know nothing until its observation on February 22, 1901, when it was of Class B, with typical hydrogen and helium absorption. This general condition it maintained through February 23. Between February 23 and 24 the great change occurred which placed its spectrum in the same class as the other novae previously observed.[1]

With its later well-known bright-line history we are not especially concerned, as it was essentially that followed by these stars as a class. It is most unfortunate that we have no earlier observations of the spectrum of Nova Persei which would enable us to say definitely what stages it had passed through on its way from obscurity to magnificence. Such a course may or may not have been in the inverse order to the generally accepted order of evolution, i.e., B, A, F, G, K, M. It may be doubted, however, if any stage "later" than A appears in the early stages of novae on account of the rapidity of the increase of light.

However this may be, we have the definite fact that at about the time of its maximum brightness the nova was of spectral class B, from which it passed over very rapidly to the banded spectrum of the typical nova, with some characteristics in common with the bright-line stars, and finally became essentially a nebula.

Observations by Hartmann[2] in 1907 and by Adams and Pease[3] in 1913 show that it had lost the strong nebular characteristics and again become a Wolf-Rayet star of magnitude 12½.

[1] *Harvard Annals*, **56**, Pt. 3.

[2] *Astronomische Nachrichten*, **177**, 113, 1908.

[3] *Astrophysical Journal*, **40**, 294, 1914.

Probable cause of the outburst in the novae.—The various theories to account for the phenomena of the novae have, with one exception, been found wanting and have been abandoned. The one exception is that proposed first, I believe, by Monck[1] (and perhaps others), and elaborated by various investigators, which, although not conceded in all the details worked out from it, has been generally accepted, I think, as the most probable basis upon which to build a detailed theory. As time passes and data accumulate, the underlying assumption of this theory appears to gather strength, and I have little doubt that ultimately it will be found to satisfy observed conditions.

An attempt will be made at a future time to trace in some detail the action which takes place in the outburst and subsequent subsidence of the novae. For our present purpose it is only necessary to outline the theory in as simple a form as possible. It will only be assumed, therefore, that a body of stellar proportions has penetrated a mass of finely divided matter, and that the outburst has been caused by the conversion of kinetic into radiant energy. Nothing will be assumed as to the previous condition or movements of the stellar mass or of the actual condition of the cosmic matter. It is conceivable that such encounters may take place between stellar masses in all conditions with either finely divided solid matter, gaseous nebulae, or with a mixture of both, and that according to the conditions and masses of the bodies concerned we may have variations in the characteristics and duration of the resulting outburst.

In the case of Nova Persei the evidence favors the view that the finely divided matter was solid rather than gaseous before its encounter with the stellar body. The spectrum of some of the nebulosity, at least about the nova, was not gaseous, but resembled that of the star at about the time of its greatest brightness.[2] This is understood to mean that the matter was finely divided and solid and that it emitted no light of its own.

The fact that this type of spectrum occurred at about the time of maximum brightness of the nova is important and significant

[1] Clerke, *Problems in Astrophysics*, p. 379.

[2] *Lick Observatory Bulletins*, **2**, 32, 1903.

in view of the apparent culmination of brightness generally in the stars of Class B.

In attempting to trace the action and relations in the novae it seems undesirable to lay much stress on small details and differences at the present time. In my opinion the larger and more important factors only should be considered in the first attempts to get a perspective of the true theory. Too much attention to small details at first (too near a view, in other words) is likely to prevent an adequate view of the whole. If we consider the tremendous forces which must of necessity be in action in such an outburst, what is more reasonable than to expect almost any kind of variation in the structure of such bands, the more violent the outburst the more complex the structure? Will there not be most terrific pressures, velocities, and abnormal distributions, particularly of the lighter gases? When I contemplate the magnitude of such forces I marvel that we should be able to recognize even hydrogen under such conditions.

The widening of spectral lines as an indication of activity.—Miss Maury[1] in her discussion of spectral types in the Draper Memorial has classified the 681 stars of that paper according to the width of spectral lines. She draws this conclusion: "It is therefore probable that spectra distinctly belonging to Division *b* (wide lines) are confined to stars of the Orion type and of Secchi's first type," i.e., to spectral classes O, B, A, and early F. A number of the brighter southern stars of these early types have been observed here recently, and one of the most suggestive things about their spectra is this peculiarity of narrow and wide lines. The wide lines appear to be especially significant, for in many cases they show a tendency to have bright borders. Practically all of them have the appearance of being the result of unusual activity, similar to the novae, only on a much reduced scale. It was so striking as to attract my attention to essentially the same conclusion arrived at by Miss Maury before I was aware of hers.

The greatest significance appears to be that this characteristic of broad disturbance lines, if we may call them such, is only found

[1] *Harvard Annals*, 28, 10 (Table I).

in the spectral classes which show the greatest preferences for the Milky Way, B and A.

There is another peculiarity which appears to be related to this widening of the lines in the early type spectra. It is the frequent occurrence of bright edges to the lines, both wide and narrow. These brightenings are more often of both edges of the lines, but sometimes are of only one edge, or one edge is much brighter than the other. They have been noticed in a large number of cases, and in many others very weak brightenings have been observed without any absorption. These phenomena give rise to the belief that they indicate unusual activity in the atmospheres or photospheres of these stars.

The characteristics and distributions of these stars are not due to pure chance. Interpreted directly they tend to confirm the theory that there are two courses being pursued in these early types and also that the underlying cause is largely due to some special attribute of the Milky Way.

Are the two sets of spectral lines due to an upward and a downward course as regards activity? Or are they due to two classes of stars differing widely in mass in some such way as the component stars of the globular clusters?

An examination was made of the proper motions of the stars whose lines were known to be wide, to see if they would show evidence of being more distant than the average stars of their respective classes. So far as the evidence from the few stars available goes, these stars appear to be at essentially the same distances as the other stars of the same spectral classes.

The distribution of these wide-line stars with respect to the galaxy was also examined. There appears to be no peculiarity in this respect beyond their spectral classes—the distribution of these wide-line stars is essentially that of the respective spectral classes to which they belong.

The maximum intensity in spectra and its relation to spectral order.—Another consideration which seems of importance in this connection is that of the intensity of different parts of the spectrum as an index of stage in evolution. An examination of the behavior of hydrogen and helium indicates that increasing activity is accom-

panied by progressive increase of intensity toward the violet, decreasing activity acting in a contrary way. Some of the elements present in the nebulae have been found to behave in a similar way. These, coupled with the well-known fact that in an incandescent source a rise or fall of temperature is accompanied by a shifting of the maximum of intensity in the spectrum toward the violet or toward the red, respectively, lead to the conclusion that the continuous spectrum alone furnishes an independent method of arranging the stars in order of activity. Now it is well known that in the order B, A, F, G, K, M the intensity of the upper portion of the spectrum continually diminishes. I have examined a number of spectra of Class O and find that the continuous spectrum in these stars, although weak, appears to be relatively stronger than in the B stars and much stronger than in the A and later types. This tends also to confirm Pickering's conclusion[1] that the fifth-type stars, Class O, form a connecting link between the B stars and the nebulae. It seems to me that this conclusion is still further strengthened by the intensity and activity in the higher parts of the spectrum of the nebulae.

The cause of Cepheid variation.—The fact that the Cepheid variables are distant and show a marked preference for the Milky Way raises the question whether the cause of their variation can be related to the conditions under discussion. The most important condition to be accounted for in this type of variation is that the maximum brightness occurs at approximately the time when the star is approaching the observer with its maximum velocity, and the minimum of brightness at the time when the star is receding with its maximum velocity. The spectrograph shows these stars to be binaries whose masses are widely different.

A number of hypotheses have been proposed to account for this type of variation.[2] The finding of evidence of action in distant galactic regions due to external matter has suggested an alternative

[1] *Astronomische Nachrichten*, **127**, 1, 1891.

[2] The author proposed a hypothesis also on the ground of increased activity of the secondary after periastron passage (*Astrophysical Journal*, **41**, 307, 1915). It was afterward found that a similar explanation had been proposed by Roberts twenty years before (*ibid.*, **2**, 283, 1895).

cause of this type of variation. It is that the advancing faces of both stars will encounter a larger amount of such matter and become brighter in general than the following faces. If the two components were of equal size and mass, little or no variation in their total light would result. But as there is such a marked difference, at the time of approach of the primary such an increase of light due to its advancing face being turned toward the observer will be greater than the corresponding diminution due to the receding and darker face of the smaller star being turned in the same direction. The contrary would occur at the time the two stars were in the opposite parts of their orbits, i.e., the primary receding when the minimum of brightness occurs. If this is true, then the amplitude of the variation in brightness should depend upon the relative sizes and masses of the two components. To test this point the stars of this class whose orbits have been computed were examined. The result was indeterminate, owing perhaps to the small changes in brightness of these stars and to other factors which may be conceived to modify the results, such as density of matter, for example. A cursory examination of available data seems to indicate that the great majority, if not all, of the stars of great contrast in mass are of types B, A, F, and G, the short-period variables being confined to Classes F and G.

The matter seems well worth a close investigation as a possibly decisive test of the cause of this type of variability as well as for other bearings which may exist.

The foregoing explanation of Cepheid variation implies (what has been observed) that only binary stars of widely different masses can in the early spectral conditions show a periodic variation in brightness.

In this connection arises the very important question as to whether there is any physical relation between the Cepheid variables and the long-period variables of the late types which show bright lines at maximum. The marked differences in behavior of these two kinds of variables, coupled with their great difference in spectral type and with the fact that one group shows a strong preference for the Milky Way, whereas the other shows no such preference, seem to justify the conclusion that their activities are due to fundamentally different causes.

Some consequences of unusual interest would follow from the presence of considerable quantities of finely divided matter in the regions of double stars. For example, such matter would act as a resisting medium, and with the lapse of time the periods of these stars would become shorter. It appears to be significant that there is a progressive preference of the stars of early type for short periods.[1] Should such retarding action continue long enough we should expect the secondary body finally to be driven down upon the primary. What would be the result? It could hardly be a nova, whose disturbance must be very superficial and thin. Could it be a catastrophe of the magnitude to form the large gaseous nebulae?

As the hypothesis of cosmic matter in the Milky Way to account for the novae and the B stars appears to have a wide bearing, an attempt will be made to see if it will satisfy the observed spectral differences among the stars generally.

No attempt is made to account for the complete evolution of the system in the broadest sense, but only for the present cycle of changes and conditions. This may be only a limited portion of the total history, but nevertheless it appears to be more or less self-perpetuating.

The chief facts may be recapitulated as follows:

1. The strong preference of the B stars for the Milky Way, and their considerable distance.

2. The great absolute brightness of the B stars, and their small number.

3. The intermediate position in both respects of the A stars.

4. The strong preference of the novae, the O stars, the planetary and gaseous nebulae, for the Milky Way, and their distance.

5. The appearance of the B spectrum in Nova Persei No. 2 at the time of its greatest brightness.

6. The strong preference of the phenomena causing the outbursts in novae for the Milky Way.

7. The almost identical courses, both as to spectrum and relative brightness, followed by the novae and brighter stars, if arranged in the order B, O, planetary and irregular nebulae.

[1] Campbell, *Lick Observatory Bulletins*, **6**, 38, 1910.

8. The broadening of spectral lines which is confined to the classes A, B, and O. This can scarcely be interpreted in any other way than as a sign of great activity.

9. The chief characteristics of the Cepheid variables—preference for galaxy, great distance, movement of maximum of intensity in spectrum toward violet at time of greatest brightness.

10. The dependence of the relation of spectral type of the components of double stars upon distance.

The foregoing facts, in connection with the course followed by Nova Persei, lead to the theory *that the stage of Class B in stellar spectra results from the same general cause as the similar stage in Nova Persei, and that the principal difference in the two cases is one of intensity affecting the element of time.* In other words, that the B stars are confined in general to the Milky Way, because there the conditions are favorable to the production or maintenance of that spectral stage in a manner allied to the outbursts of the novae.

This explanation appears to satisfy all the best-established facts, requires no violent assumptions, and on the whole appears to have a good degree of probability.

It is true that the preference of the B stars for the Milky Way may be explained also by evolution from the gaseous nebulae which are limited to the galaxy, and that the absence of the B stars in the nearer regions of sky may be naturally explained by assuming that all the stars in this region are older. The great brightness and small number of the B stars, the peculiar behavior of the double stars in the near and distant regions, together with the many signs of activity among the early-type stars in the galaxy, seem to indicate decisively, however, that the course of evolution in the brighter, early-type stars is toward the nebulae rather than away from them. The notorious fact of the great brightness of the B stars points clearly to the upward stage as the one to which they belong.

One of the strongest arguments for such a partial theory of the evolutionary processes as that indicated here is the number of previously disconnected facts which it appears to harmonize. For example, it not only explains the preference of the B stars for the Milky Way, but also explains the intermediate position of A stars, the greater distance and brightness of the B stars, the preference

of solar types for the region of the sun, a logical connection of the early spectral types including the planetary and irregular gaseous nebulae, the relation of the novae to ordinary stellar processes, and provides the basis for a simple and logical cycle of changes for the greater part of our stellar system.

Such an evolutionary process suggests a possible explanation also for the general preference of the bright stars of later types for the galaxy. It is simply that they are in general of larger mass, having swept up the matter which has always been most plentiful in the near as well as the more distant galactic regions, thus retarding development toward the later and fainter types.

The sudden decrease in brightness between the average B and the average O stars is very suggestive when compared with the great loss of light which occurred in Nova Persei about the time that, and shortly after, the bright lines and bands made their appearance.

It is true that the theory depends largely upon a stage of spectral activity which has been observed in but a single star and is incomplete. This one case is so definite, however, that taken in connection with other facts there seems good reason to think that a somewhat similar stage may have been passed through in the other novae before they became known. It seems all the more probable that such a stage may have escaped detection in other novae because of its early appearance and short duration.

However this may be, the evidence as far as it goes is very definite on the appearance of the B stage at or just before the maximum brightness is reached, and there is nothing to contradict the possibility that such a stage may be general in the novae.

Further consideration shows that it does not seem necessary to assume that all stars so affected would reach the bright-line stage of the novae or even the B stage; that if the changes depend upon the density of the cosmical matter (or other conceivable conditions) we should expect that the region of more sparsely distributed cosmical matter would be much more extensive than the regions of great density. It is perhaps significant in this connection that the number of A stars is so much greater than of B stars, which are in general undoubtedly of greater brightness. It seems not

impossible that those stars which only reach the A or B stage after the maximum pursue a decline through more or less the same stages as on the rise. Such a condition would account also for a large part of the observed excess of A stars over B stars.

Analogy in the case of the novae points to the very faint stars rather than the bright stars as being the returning stages to obscurity.

This hypothesis rests upon three assumptions as a result of the foregoing discussion:

1. That the spectral class of the stars is determined *in general* by location in the system, being a function of galactic latitude and distance.

2. That the more distant regions in the direction of the galaxy contain a larger quantity of finely divided solid matter than the near regions.

3. That the cause underlying the spectral changes among the stars is essentially the same as that producing the phenomena of the novae.

Hypothesis.—The general hypothesis to account for the present spectral conditions of the stars may be stated as follows:

The cause is dual, depending upon the amount of cosmic matter and upon radiation and condensation phenomena. The stars of Classes A, B, and O, the planetary and irregular nebulae, the novae, and perhaps the Cepheid variables, are confined to the galaxy because there the matter is sufficiently plentiful to cause an increase of energy, the energy from the matter swept up being in excess of the energy lost by radiation. The direction of spectral change under such conditions is *toward the nebulae.*

In the regions (distant or near) where there is little or no cosmic matter radiation will overpower the energy received from external sources and the direction of change will be *toward the late types.*

In a considerable portion of the system the changes of spectral class may be due to retardation.

The close relationship between the Wolf-Rayet stars and the planetary nebulae has been shown by the work of E. C. Pickering, Keeler, Wright, and others. The close connection of Class B and Wolf-Rayet stars has also been shown, particularly by the

work of the Harvard astronomers. There can be little doubt that the Wolf-Rayet stars belong between the nebulae and the B stars. The question is, In which direction has the change occurred? The course followed in the novae indicates very strongly that the nebular condition is the climax of evolutionary activity. If this be so, we have in the novae the general course of evolution pursued in regions where cosmic matter is plentiful.

Briefly stated, it is that essentially stellar bodies encounter cosmic matter in the form of clouds;[1] that these encounters produce disturbances in the star's atmosphere, or, what seems equally possible, the formation of an atmosphere on a solid body; that the stage of spectral type reached will depend upon the masses of matter concerned and the violence of the encounter, the most violent encounters reaching the nebular stage, on the way to which the body passes the stage of Class B, then of Class O; that the planetary nebulae are the relatively moderate outbursts in which gaseous nebulosity has been generated, whereas the large irregular nebulae have resulted from the most violent outbursts of all; that the declining stages are pursued in a more or less inverse spectral order to that of the rise; that the increasing stages of activity and the culmination are, in general, marked by a broadening of the lines and bands, both of absorption and emission, possibly in the larger masses only, and that the declining stages are marked in general by narrow lines.

Hypothesis to account for the planetary nebulae.—If the order of evolution proposed above is correct, a not improbable explanation of the formation of the planetary nebulae presents itself. It is that the outburst, if of sufficient violence, may pass beyond the ordinary Wolf-Rayet stage and evolve a large amount of gaseous nebulosity which is thrown off as a shell. Such a shell will appear as a more or less circular ring with a web of faint nebulosity inside the ring, and a central star. The detail within the rings, which is remarkably varied, can be accounted for as streamers and irregular masses thrown out from exceptionally disturbed areas on the star,

[1] It may be possible that an encounter with purely gaseous matter may produce similar phenomena. On the whole it seems much more probable, in my opinion, that the encounters have been with finely divided matter in a solid state.

much as we account for the streamers and masses of the solar corona.

This explanation seems to imply that the forces acting on the nebulosity ejected reach their limit at the outer edges of the ring. The outward forces I conceive to be eruption and light-pressure, opposed by gravity. Combinations of these appear to be sufficient to account for the observed phenomena.

Electromagnetic forces may play a part also, but of their presence or action in such cases we know little at present.

If this be the process of evolution of the planetary nebulae, then a central star *must have been* a necessary factor. It does not follow that it is visible now. Indeed such an origin would lead us to expect some of the brightest central stars to be found in the smaller or fainter nebulae, where the activity may have been less than in the large or bright objects in which the activity might be expected to have been greater and the material of the originating star more completely dissipated. The complete absence even of such a central star in some of these bodies can be accounted for in this way.

Hypothesis to account for the irregular nebulae.—If the initial outburst is great enough, the gaseous nebulosity will be driven off, both by the force of the outburst and by light-pressure, at a speed which will overcome the attraction of the central mass and carry to great distances. It would seem logical in such cases to expect less regularity in form of the resulting nebulosity than in those where the activity has not been so great.

This leads to the suspicion that the dark, finely divided matter which is believed to exist in the distant galactic regions may be none other than such condensed nebulosity—that, in place of the early Orion stars, for example, being wholly in the process of condensing from their inclosing nebulous envelope, this nebulosity is in fact largely the *result* of a great catastrophe, the nebulosity having been thrown off in the process.

Is this also true of the nebulosity in the Pleiades, with the difference that in the Pleiades the stars are slightly "older" in type and the nebulosity not self-luminous? Has this nebulosity frozen from a gaseous state? It is very suggestive that many

bright stars of early type have masses of nebulosity near, which appear to be connected with them.

Upon this hypothesis the effect on double stars would be somewhat as follows: The smaller bodies, because of their larger relative surface areas, will sweep up more of this matter than the large ones in proportion to their masses. If we assume that the increase of energy is greater by such accretions from external matter than the energy lost by radiation, the activity (temperature probably) of the smaller body will increase more rapidly than that of the large body. If, therefore, the two bodies start in the same condition, the smaller should move more rapidly toward the early type of spectrum. This, of course, implies that the effect in these stars is not merely a surface one.

On the other hand, in the near regions such a hypothesis accounts for the change of spectra of the secondaries toward later types by assuming that radiation is greater than the energy absorbed from the external matter.

The stars whose components are of the same spectral class show much smaller differences of magnitude than do the other groups and an almost complete limitation of such as have small proper motions to Classes B and A. These peculiarities, as well as that of the pairs of small μ in which the fainter star is of the later type, do not appear to be contradictory to the hypothesis given. Two stars of the same size acted upon by the same conditions should leave them of the same class still, even if they were in the region rich in cosmic matter. On the other hand, in distant regions where matter was not so plentiful the smaller and fainter star of a pair would be of later type, just as appears to be the case in the nearer regions.

The evidence of these double stars seems to show that no considerable portion of the supposedly high surface temperatures of the early-type stars can be due to the working of Lane's law. No such law can reasonably be supposed to be selective in its action.

Many questions suggest themselves in this connection. Does the bright-line stage in the early-type stars mark the culmination of their activity? Or is it merely an accidental phase?

What part does the matter composing the zodiacal light play in the spectral condition of our sun? If it were not for this matter, would our sun be of K type or later?

If such is in reality the chief cause of the differences in spectral type, may it not be that the changes in many of the early-type stars occur in relatively short intervals?

What is the relation of the globular clusters to the other members of our stellar system? How have they been formed? Why are the component stars so uniform in brightness and type?

The recent researches of Nicholson seem to me to have a bearing on the conditions underlying spectral changes in general, but particularly of the Wolf-Rayet stars and the gaseous nebulae. May not his conclusion that the spectrum of coronium is that of a substance known on the earth which has been raised to incandescence by some special method of excitation be extended to substances raised to super-temperatures such as appear to exist in the gaseous nebulae?

Some three or four years ago the idea came to me that some such conditions best satisfied all of the principal known facts and that the few observable radiations in the spectra of the nebulae might be due to the high temperature of these bodies (combined perhaps with electromagnetic forces), where the greater part of the energy had been pushed out, as it were, at the upper end of the spectrum, where the atmosphere and the weakness of our observing methods make it nearly or quite impossible for us to detect its presence. This idea has not weakened as time passes. The facts surrounding the appearance of the nebular spectrum in the novae seem to me a strong argument for some such condition.

The question arises whether it is possible, under the conditions believed to exist, for development of stellar bodies to take place from masses of *gaseous* nebulosity in the way which is usually assumed, i.e., by condensation and accumulation of the nebulous matter itself. For were a nucleus to be formed in the mass of gas, would not the accumulation of some of the surrounding nebulosity and its condensation upon this nucleus, according to Lane's law, cause a rise of temperature which would again disrupt the mass until an equilibrium of temperature was reached, then to go through

a similar cycle. but without the possibility for a body of any considerable size to be formed *from the inside?* Formation through condensation and radiation from the outside also seems to present some difficulty.

If temperature is the dominating factor in stellar activities and conditions, as there is strong reason to believe, and the nebular condition is the culminating form of matter as temperature rises, as appears to be the case judging from the novae, some rather curious consequences appear to result from the application of the theory of evolution from the gaseous nebulae. According to Lane's law the temperature will rise in the gaseous mass as soon as condensation sets in. If this happens, what form will the nebulosity take with the increase? Would it not have to take some "super" form, of which we appear to have no knowledge? If this reasoning is correct, how can a mass of gaseous nebulosity ever condense as such? This seems to lead to an *impasse*—either Lane's law is not true in such cases, or the gaseous nebulosity must change its form or condition in some manner before again playing a part in condensation phenomena. Perhaps the nebular stage is not strictly a gaseous condition.

General condensation does not seem to harmonize with the observed conditions, which are that the great nebulous masses have one or more large and often bright stellar bodies within or nearly centrally located in the nebulous mass. It is possible to see how the nebulous matter has been largely or wholly ejected from the stars, but not how these stellar bodies have been formed wholly from the nebulous masses.

A most suggestive conclusion has recently been reached by Seares.[1] From photographs taken with and without a color-filter he found that in several spiral nebulae the outer regions in general and the more dense condensations in the spirals had a larger proportion of light of shorter wave-length than the central regions of these nebulae.

The significance of this observation in the present case lies in the fact that a considerable number of very bright B stars or a general tendency to the early type of spectrum in the bodies of the outer

[1] *Proceedings National Academy of Sciences,* **2**, 553, 1916.

galactic regions of our system would tend to produce some such effect as that observed by Seares. The researches of Pickering[1] and Kapteyn[2] show exactly such a condition for stars down to the tenth magnitude. Although the stars fainter than this have not been investigated, it seems probable that this effect may continue to fainter magnitudes also.

Conclusions.—The results of this investigation may be summarized as follows:

1. The relation of the spectral classes of the two components of a binary system is a function of distance from the sun, those at distances generally beyond $\mu = 0''.05$ (excepting some stars with both components of Classes B and A) having the fainter components of *earlier* spectral type, while those nearer have the fainter components of *later* type.

2. The preference of the B stars for the galaxy can be explained upon the assumption that they have passed through stages similar to that of Nova Persei No. 2 at or just before its maximum brightness.

3. The foregoing phenomena can be explained as due to the same causes which produce the outbursts in the novae, viz., encounters with finely divided solid matter or gaseous nebulosity or both.

4. Such a hypothesis, if confirmed, has a bearing on the order and cause of the evolutionary processes in our stellar system, and tends to substitute location and external causes for time as of the first importance.

5. The observed facts appear not to be inconsistent with the hypothesis that a large part of the characteristics of spectral class among the stars generally may be due to some external condition which is largely a function of both galactic latitude and distance, combined with phenomena of radiation and condensation.

Observatorio Nacional Argentino, Córdoba
October 31, 1917

[1] *Harvard Annals*, **26**, 152.

[2] *Annals of the Royal Observatory, Cape of Good Hope*, **3**, 22 (Introduction).

ADDENDUM

The list of luminosities and parallaxes of 500 stars, by Adams and Joy,[1] which has just come, contains 24 double stars and wide pairs with common proper motions, whose spectra for both components are given. These have an important bearing upon the part of the foregoing paper which deals with the variation of spectral class of the components of double stars with distance. The principal data regarding these 24 stars are collected in Table V.

These stars are practically all of large proper motion and the parallaxes show that most of them are near, as was to be expected. According to the criterion established in the preceding paper, essentially all of these stars should have the fainter component of later type. Of these stars twenty-one show such a condition; one (γ Virginis) has both components equal in brightness and of the same spectral type, and 2 show the fainter component to be of earlier type. Of the latter two, one (Boss 4892–3) has a difference of only 0.2 magnitude, a spectral difference of 3/10 of a unit, and a moderate sized parallax. The other exception (ϵ Hydrae) has a large difference of magnitude (4.0), a small difference of spectral type for so large a difference in brightness, the proper motion is only medium sized, and the trigonometrical parallax small. The spectrographic parallax of the fainter component comes out rather small also (0."016). The other stars call for no special comment further than that the nearest and those of largest difference in brightness have also in general the greatest difference in spectral type.

These stars, therefore, confirm the previous conclusion that in the nearer stars the fainter component is of the later spectral type. Not a single one of these stars is really contradictory if we make a reasonable allowance for limits in which the transmission may occur. The small change of type in the case of ϵ Hydrae with so large a difference in brightness is not necessarily inconsistent with the theory proposed, as this star is in the region where the transition appears to occur.

This material addition to the data and the very valuable parallaxes make it possible to obtain some information regarding the distance at which the transition in relative type of spectrum of the

[1] *Mt. Wilson Contr.*, No. 142; *Astrophysical Journal*, **46**, 313, 1917.

TABLE V*

Name	Vis. Mag.		1900.0		Spectrum		Abs. Vis. Mag.		Lum.	μ	π	
	Br.	Fr.	α	δ	Br.	Fr.	Br.	Fr.			Spec.	Trig.
Σ 3062	6.9	8.0	$0^h\ 1^m.0$	$+57°\ 53'$	G4	G0	5.2	5.3	0.832	0″.266	0″.046	+0″.036
η Cassiopeiae	3.6	7.6	0 43.0	+57 17	F9	K4	5.0	8.7	1.00	1.242	.190	+ .191
Boss 592–3	6.6	7.4	2 31.2	+24 13	F3	F5	3.9	3.7	2.75	0.140	.028	+ .030
γ Leporis	3.8	6.4	5 40.3	−22 29	F7	K5	4.7	7.5	1.32	0.468	.151	+ .108
G Cancri	5.2	6.0	8 6.5	+17 57	F8	G0	3.4	4.1	4.37	0.155	.044	+ .031
ϵ Hydrae	3.5	7.5	8 41.5	+ 6 47	F9	F4	2.5	3.6	10.0	0.196	.063	+ .004
Fed. 1384	9.2	9.2	8 46.0	+71 11	K7	K8	9.2	9.7	0.021	1.40	.126	+ .086
Lal. 18115	7.0	7.9	9 7.6	+53 7	K8	K8	8.6	9.2	0.036	1.69	.138	+ .152
ξ Ursae Majoris	4.0	4.9	11 12.9	+32 6	F9	G1	5.0	5.6	1.0	0.732	.158	+ .158
83 Leonis	6.5	7.6	11 21.7	+ 3 33	G9	K4	4.9	6.3	1.10	0.743	.048	+ .023
γ Virginis	3.7	3.7	12 36.6	− 0 54	F0	F0	3.1	3.2	5.75	0.564	.076	+ .068
ξ Boötis	4.7	6.6	14 46.8	+19 31	G6	K3	5.5	7.6	0.631	0.168	.145	+ .230
Pi. 14^h 212	5.8	8.7	14 51.6	−20 58	K5	Ma	6.8	10.1	0.191	2.039	.158	+ .174
A.Oe. 14318–20	9.2	9.6	15 4.7	−15 54	G8	K0	6.0	6.2	0.398	3.75	.023	+ .054
Lal. 27742–3	6.8	7.6	15 8.2	+19 39	G6	G9	3.9	4.9	2.75	0.68	.026	+ .018
σ Coronae	5.8	6.8	16 10.9	+34 7	F8p	F9	4.4	5.1	1.74	0.302	.046	+ .031
ν Cor. Bor	5.3	5.4	16 18.7	+33 56	K6	Ma	1.6	0.9	22.0	0.055	.018	......
μ Herculis	3.5	9.7	17 42.5	+27 47	G5	Mb	3.2	9.6	5.25	0.817	.087	+ .095
70 Ophiuchi	4.1	6.0	18 0.4	+ 2 31	K0	K5	5.6	7.6	0.575	1.131	.200	+ .187
Gr. 10 Area II, 66.75	7.7	8.1	18 21.4	+ 8 44	G5	G8	6.3	4.3	1.91	0.498	.052	+ .037
Boss 4892–3	6.6	6.8	19 9.5	+40 40	G6	G4	5.5	5.5	0.631	0.654	.060	+ .052
Boss 5037–8	6.3	6.4	19 39.2	+50 18	G1	G4	5.3	4.4	0.759	0.217	.063	+ .048
61 Cygni	5.6	6.3	21 2.4	+38 15	K7	K8	7.9	8.7	0.069	5.271	.288	+ .313
Lal. 45455–6	8.6	9.0	23 8.9	− 9 28	F4	G0	5.1	5.8	0.912	0.56	0.020	+0.019

* The α, δ, Lum., μ, and π refer to the brighter components of the pairs.

components occurs, and of the rate of change of spectral type with difference of brightness. The condensed results are given in Table VI. It does not appear to be necessary, at this time, to give all of the individual results and the names of the stars.

TABLE VI

	Harvard Stars			Mt. Wilson Stars	
	Companion Earlier Type	Companion Later Type		Companion Later Type	
		Large μ	Small μ	Large π	Small π
Difference of visual magnitudes	1.9	2.6	1.8	2.1	0.8
Difference of spectra	−1.02	+0.63	+0.25	+0.34	+0.26
Mean μ	0.″037	0.″19	0.″033	1.″08	0.″39
Mean π	....	....	....	0.″164	0.″044
No. stars	28	11	12	10	14

The pairs of both lists, in which the fainter components are of later type, were divided roughly into two classes, larger and smaller proper motions. There appears to be a relation between the differences of magnitudes and spectral type depending upon the size of the proper motions, the larger μ (nearer) stars showing a larger mean difference of magnitude and a larger variation of spectral type than the smaller μ (more distant) stars. There are reasons for thinking that this condition may be real, but the evidence as given in the table cannot be considered to have much weight because the mean μ of the Mt. Wilson group of *smaller* μ is double the size of the μ of the *larger* μ group of the Harvard stars. An examination of the presumably critical stars seems to confirm the reality of such a peculiarity. The results are shown graphically in Figs. 2 and 3.[1] The small range in spectral type for the smaller μ stars is very marked in Fig. 3, where all of them (26 in number) without exception have a range of spectrum of less than one unit,[2] and two stars have a small reversal of spectral difference.

[1] The black disks in Figs. 2, 3, and 4 represent Harvard stars; the open circles, Mt. Wilson stars.

[2] The unit is assumed to be equal to the spectral interval A0 to F0, F0 to G0, etc., or G, 1, 2, 3, etc., to K, 1, 2, 3, etc., and is divided into ten parts to conform to the subscripts in the spectral classes.

On the other hand, the variation of spectral type with difference of visual magnitude is much greater in the stars whose fainter components are of *earlier* type, as is seen from Fig. 4. The reality

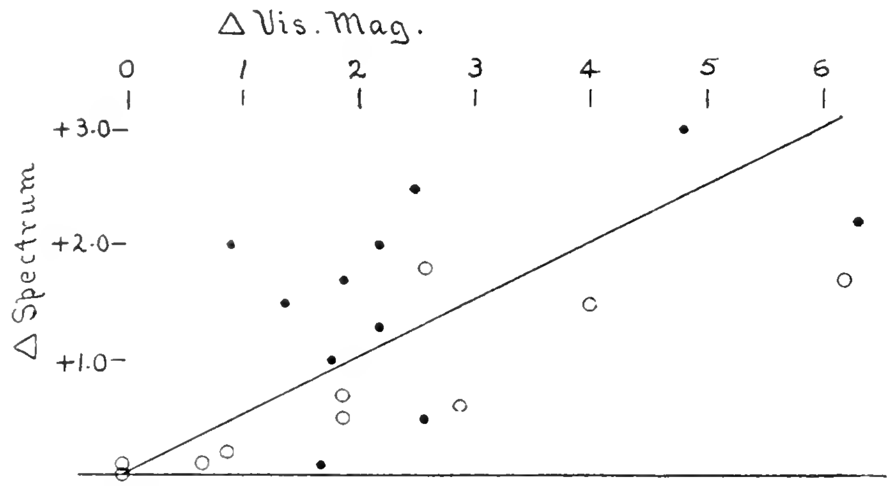

FIG. 2.—Curve of spectral changes, larger μ and π

of the large variation is confirmed by the stars showing large differences of brightness, which also show large differences of spectral class (where the effect of accidental error is much reduced). Only one star of large difference of brightness (δ Boötis) shows a small

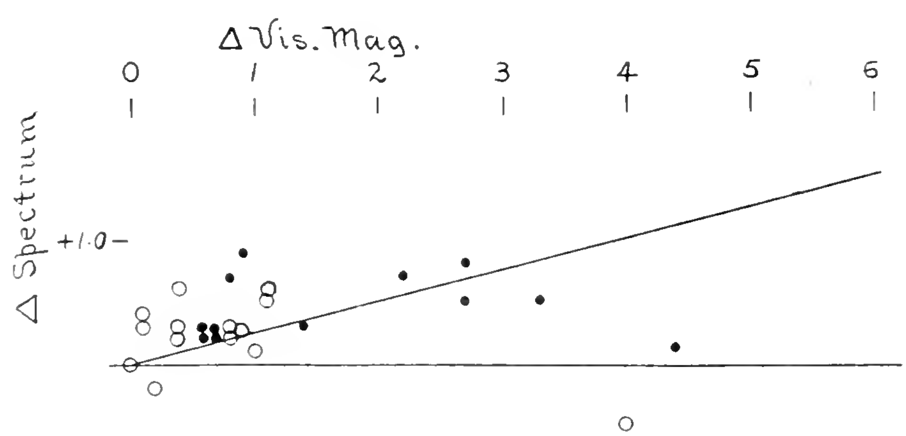

FIG. 3.—Curve of spectral change, smaller μ and π

difference of spectral type, and its comparatively large μ (the largest of the group) seems to place it in the region of transition. It should be said that the classifications were arbitrary without any suspicion that such a peculiarity might exist.

Fig. 5 shows the curves of the variation of spectral type with difference of brightness where the companions are of earlier and also of later type. For this purpose all of the latter stars were

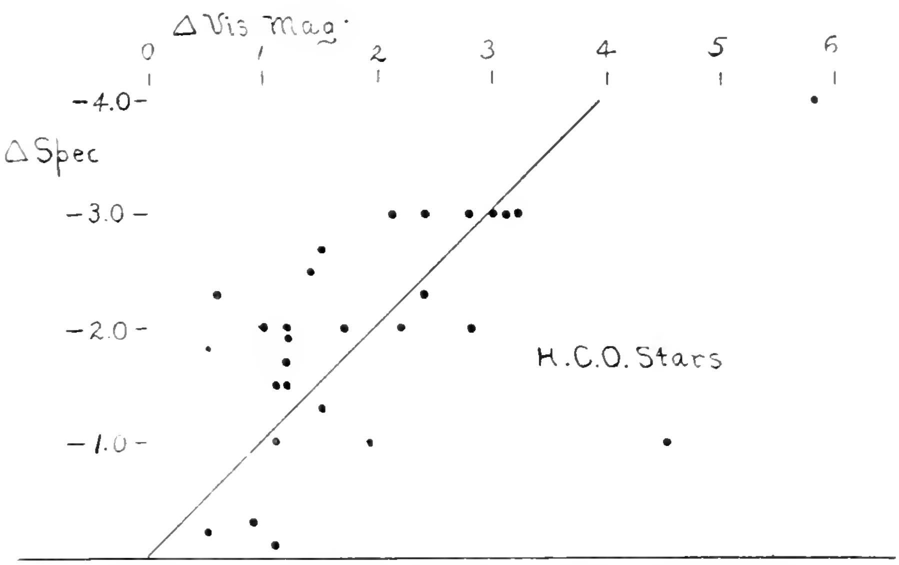

FIG. 4.—Curve of spectral change. companions earlier type

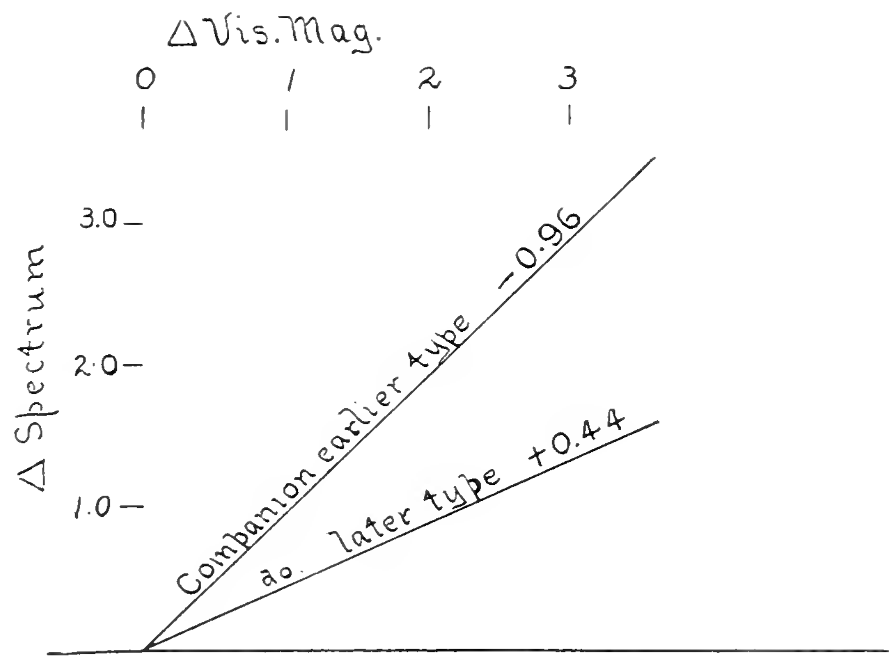

FIG. 5.—Curves of spectral change

combined. I see no reason to suspect that systematic differences in the determination of spectral class can have caused this rather consistent difference in rate of spectral change. It seems not

difficult to account for such a difference on the general hypothesis already formulated. Notwithstanding these presumptions in favor of its reality it seems best to await further confirmation before discussing it.

It is not probable that the distance is the same in all directions, but the data do not seem to justify an attempt to determine the law of distribution and distance. Such a knowledge would doubtless be very interesting and important, and as soon as sufficient data or a better knowledge of the matter is obtained it will be attempted.

After the completion of the earlier paper, an article by H. E. Lau,[1] "Über die blauen Doppelsternbegleiter," was received, in which he calls attention to the peculiarity of spectral relation of the components of double stars, and ascribes it to phenomena connected with "giant" and "dwarf" stars.

In any groupings of very large and very small proper motion, among the naked-eye stars, there will almost always be a large difference in absolute magnitude. This is the case with the stars in Lau's lists, the difference being 6 magnitudes or more. The differences are not so great in my own lists. On the basis of proper motions the difference in the case of the Harvard stars is not over two magnitudes. That this is not a phenomenon limited to giant and dwarf stars is clearly shown, in my opinion, by two things:

a) The peculiarity is found among all absolute magnitudes and among all spectral classes, as an inspection of Tables I and II of my paper shows.

b) Four stars of mean apparent magnitude 6.4 and $\mu = 0''044$ were found among the stars having the fainter components of *earlier* spectral type and four stars of mean apparent magnitude 2.7 and $\mu = 0''134$ among the stars having the fainter components of *later* type.

When reduced to a common distance on the basis of their proper motions, the stars of the group with the companions of *earlier* type are found to be a magnitude fainter than those of the group with companions of *later* type, thus reversing the giant-dwarf relationship assumed by Lau.

[1] *Astronomische Nachrichten*, **205**, 29, 1917.

CONCLUSIONS

I. The 24 pairs of stars of Adams and Joy's Mount Wilson list of the luminosities and parallaxes of 500 stars fully corroborate the conclusion reached in the foregoing paper that the fainter components of such stars, when near, are of a later spectral type than the primaries.

II. The transitional stage where a change in this phenomenon takes place and the fainter components become of *earlier* type appears to be at a general distance represented by a parallax of $0''.01$ or $0''.02$, corresponding roughly to 200 or 300 light-years.

III. At the general distance assumed to be that of the transition of the fainter components from later to earlier spectral type there appears to be a smaller change of spectral type generally for a given difference of magnitude than is the case with either the nearer stars or those in which the fainter components are of earlier type.

IV. The change of spectral type for a given difference of magnitude appears to be less on the average in the stars having the fainter components of later type than in the stars having the fainter components of the earlier type.

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
February 13, 1918

KNIFE-EDGE SHADOWS

PHOTOGRAPHY AS AN AID IN TESTING MIRRORS

By RUSSELL W. PORTER

The characteristic and somewhat startling shadow (so called) that covers a paraboloidal mirror when a knife-edge intercepts the cone of light rays from its surface entering the eye affords one of the principal optical tests in determining the precise figure of the speculum in the reflecting telescope. It is a rather complex shadow to describe to one who has never seen it, and the attempts to depict it in treatises on speculum-making are deplorably weak in presenting the illusion of reality and give but little help to one anxious to figure his own mirror.

Such considerations as these led the writer recently to interpose a sensitive plate in place of the eye behind the knife-edge at the center of curvature of a concave mirror. It seemed reasonable to suppose that the shadow would in some degree be impressed on the plate; and after many failures, due for the most part to faulty illumination, he succeeded in securing the photographs accompanying this article. To his great surprise they revealed imperfections of the glass, unsuspected and unseen by the naked eye; and it was at once apparent that such photographs would be a great aid in the final figuring and provide a perfect guaranty of the optical excellence of the mirror.

Photographic records of optical surfaces of revolution have been obtained in the case of lenses by J. Hartmann, who investigated the 15-inch objective at Potsdam and the 40-inch of the Yerkes Observatory from photographs made at Williams Bay.[1] It would seem that the writer has been trying to do with the reflector what Hartmann had already accomplished for the refractor. There is, however, this difference: in the Hartmann "focographs" the plate is focused on the objective itself by the introduction of a camera lens, while the concluding photographs accompanying this account

[1] *Zeitschrift für Instrumentenkunde*, **29**, 217, 1909; *Astrophysical Journal*, **27**, 237, 257, 1908.

are secured directly without focusing, thus freeing the results of any errors due to the addition of another lens in the optical train.

The stand carrying the knife-edge and artificial star are shown in Fig. 1. At the left is a brass tube inclosing an acetylene flame, opposite which, attached to the tube, is a small slide carrying several

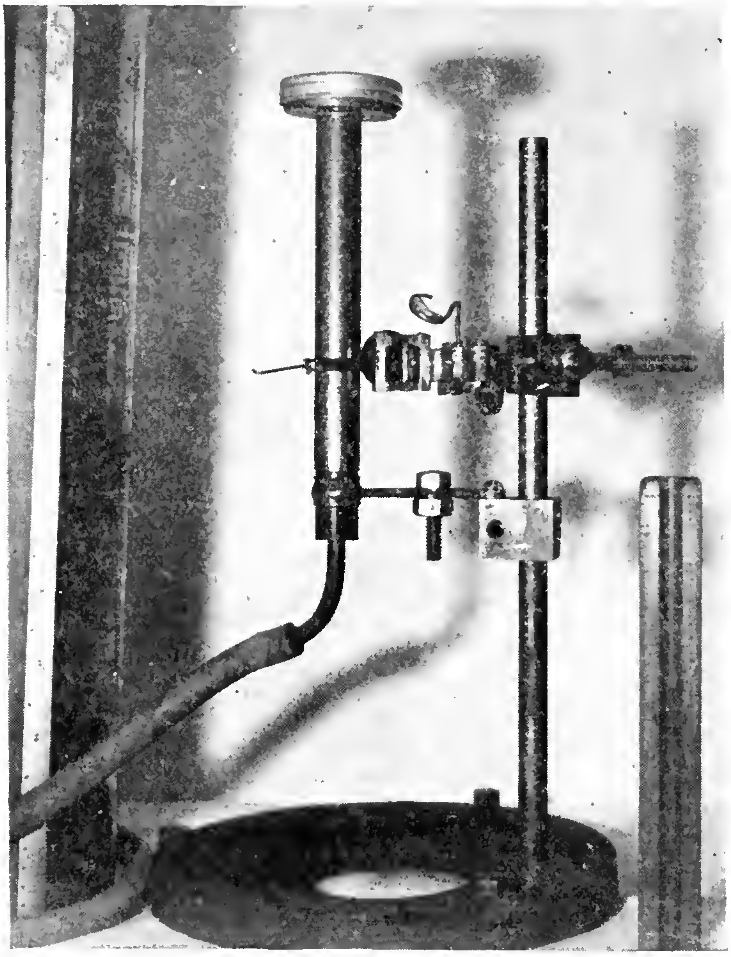

FIG. 1

perforations of diameters varying from 1/0.0005 to 1/0.05 of an inch. Between the tube and stand-support is seen the knife-edge inclosed in a square frame traveling on a track parallel to the optical axis of the mirror. Behind the square frame is a removable eyepiece for examining images in the plane of the knife-edge. Transverse motion of the knife-edge is obtained with the leveling screws on the stand base. Just above the knife-edge is a small magnifying glass used in keeping the knife-edge in its proper

position on the cone of reflected light during exposures. The camera is not shown in the photograph.

Following the method of Hartmann by focusing the plate on the objective by means of a telephoto lens, the series of focographs near the center of curvature were obtained (Plate VII). The mirror was 12 inches in diameter and 4 feet in focal length. The figures show in an interesting way how the characteristic parabolic shadow develops and changes from a position 0.15 inch inside the mean center to a like position outside.

In the experiment, doing away entirely with focusing and simply exposing the plate to the reflected cone, the photographs in Plate VIII were secured. The speculum used was one of the writer's first mirrors, of 10 inches' diameter. He had subsequently converted it to the Cassegrain form and bored out its center, as with the previous glass, without noticing any distortion of surface. He knew that there was a pronounced turned-down edge, but when this zone was diaphragmed out the performance of the glass was quite satisfactory and produced a creditable image.

The three upper focographs of Plate VIII are taken with the knife-edge at the center of curvature of the mirror, as in the lower left-hand diagram. The parabolic shadow is formed on the ground glass immediately when the knife-edge begins to cut into the cone of reflected light. By looking at the knife-edge with the magnifying glass, where the cone impinges upon it, it is easy to determine when it has reached the center of the cone section thus cut. The mean center of curvature can be found almost as readily on the ground glass as with the naked eye.

Focograph No. I was taken inside the center of curvature, No. III outside, 0.12 inch apart along the optical axis. No. II was intended to be at the center itself but is in reality, as the focograph shows, a little inside.

The character of these extra-focal negatives is beautifully brought out. Their respective apparent sections are drawn above them, assuming the light to graze the surface from the direction opposite to the knife. This is the remarkable delusion of Foucault's shadows—that the mirror seems to be illuminated by light coming in at right angles to the axis and not along it, as it really does.

Superposed over these parabolic shadows there is a faint trace of the characteristic shadow of a warped surface. The glass was not hung in a strap but rested on its edge. Distortion might be expected, and it is there and evident to the optician familiar with it.

Now as to the markings on the glass surface itself. Most noticeable is the turned-down edge. There was probably an attempt to rectify this by local polishing, which produced the small, round, intensely white spots seen strung along the rim. Next are the marks of the small rose tool, small epicycloidal strokes of the polisher hitherto unsuspected, and then the shadow coming in from the upper edge, also unsuspected and perhaps due to internal strains in the glass. Even the accidental scratches on the mirror are very noticeable. Finally one recognizes a slight turning up of the edge around the hole bored out of the center, which, as before remarked, could not be seen when tested with the eye after the disk had been perforated.

The lower and larger focograph was taken with the knife-edge and artificial star at the focus, midway between the center of curvature and the mirror. It might be obtained at the telescope by removing the eyepiece and keeping the image of a bright circumpolar star on an interposed knife-edge. But atmospheric disturbances and uneven driving of the clock would certainly render it difficult. In the laboratory, however, the best conditions are realized, and parallel rays are secured by the use of a flat, as shown in the right-hand diagram.

The optical train is as follows: The concave and flat mirrors face each other at a distance apart of about one-half the focal length of the former. Artificial star, knife-edge, and plate are behind the parabolic mirror. The beam of light from the pinhole strikes the flat, is reflected to the concave surface, and is thrown back to the flat in parallel rays. It returns to the knife-edge in the same way but in reverse order. There are in all five reflections, so that inequalities of the parabolic surface reach the plate exaggerated by double their true value. Much light is lost by absorption; hence for photographic purposes the mirrors should both be freshly silvered and a brilliant light source used.

Theoretically, with a perfectly parabolized concave and a perfect flat, light from a point-source will return by reflection to a point; and the interposition of a knife-edge at this point will

produce instant extinction. The point-source having the appreciable diameter of the pinhole (one-hundredth inch), obliteration of the disk does not occur at once, and the occultation takes place progressively through a uniform fading away of the illuminated disk. This is the condition striven for by the optician. Had the mirror under examination been perfectly parabolized, its focograph would have presented an absolutely flat disk. But the shadow indicates clearly that the mirror was overcorrected and that its defining qualities would be improved by refiguring.

The phenomenon of diffraction is well shown in these focographs. If a magnifying glass is used, fringes will be seen accompanying all the stronger markings.

It is also interesting to consider the minuteness of the quantities we are dealing with. This mirror was a long-focus one and the amount taken off its surface near the center, in passing from the sphere to the paraboloid, was something like one hundred-thousandth of an inch (0.00025 mm). In other words, the large shadow over the right side of focograph No. II is due to this depressing of the surface by this amount. The shadows of the depressions produced by the rose tool, for instance, are of an order very much less than that of the large shadow, say one-tenth, and indicate a departure from the true parabolic surface of only a few millionths of an inch.

Is it surprising, then, if such minute inequalities are so palpably sufficient to impair the defining qualities of a mirror, that larger departures from a true surface should arise through deformation by flexure due to faulty support? It is therefore important that these grosser distortions due to flexure and temperature be watched for and guarded against. If each night before getting down to work the observer will apply the knife-edge test with a bright star by removing the eyepiece, he can tell at once if his mirror is as it should be; if not, the character of the shadow will point the way to the source of the trouble. He will find it an excellent index to the state of the atmosphere, which, if unsteady, will make the mirror appear to boil like molten silver.

LAND'S END OBSERVATORY
PORT CLYDE, MAINE
December 1917

THE ORBIT OF THE SPECTROSCOPIC BINARY BOSS 46[1]

By WALTER S. ADAMS and GUSTAF STRÖMBERG

The star Boss 46 = B.D. 50°46 ($\alpha = 0^h 12^m.4$; $\delta = +50° 53'$, 1900.0) belongs to the class of spectroscopic binaries in which the calcium lines give values of the radial velocity differing widely from those furnished by the other lines in the spectrum. The variable velocity of the star was first found from spectrograms obtained in 1914 at Mount Wilson, and the announcement of the variation was made in the *Annual Report of the Observatory* for the year 1916. The visual magnitude of the star is 6.0 on the Harvard scale; its total proper motion is 0".011 annually; and its spectral type as determined from our photographs is B3p. The lines in its spectrum, with the exception of those due to calcium, are faint and diffuse, and a spectrograph of low dispersion has been employed for nearly all of the observations. The precision obtained for the calcium lines is considerably higher than for the remainder of the lines in the spectrum.

The list of photographs is given in Table I, together with the Greenwich Mean Time of the middle of each exposure and the phase based upon a period of 3.5225 days and referred to the epoch 1917, January 1.0. In view of the long interval covered by the observations the period is known with a considerable degree of accuracy. The results for the velocity, as derived from the hydrogen and helium lines on the one hand and the calcium lines on the other, are combined into mean ualues for plates taken near the same phase. These normal values, together with the mean phases, are given in Table I, the number of plates being indicated by the figures in parentheses. It was not possible to measure the calcium lines upon all of the negatives, since they occur near the end of the spectral region photographed, and a dense negative is required for the purpose. For this reason the mean phase and the number of plates differ in several cases for the hydrogen and the calcium results.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 149.

TABLE I

Plate No.	Date	G.M.T.	Phase	Mean Phase H and He	Mean Phase Ca	Mean Velocity H and He	Mean Velocity Ca	O−C H and He	O−C Ca	Remarks
						km	km	km	km	
4840	1916, May 20	23h 42m	0d.424	0d.408 (2)	0d.392 (1)	−241	−34	+ 0.8	−1.6	
5514	1917, Feb. 5	14 48	0.392							
5384	1917, Jan. 1	15 51	0.660	0.678 (4)	0.672 (3)	−251	−32	−10.6	+2.8	
6278	1917, Oct. 6	21 42	0.627							
6279	1917, Oct. 6	22 23	0.656							
6329	1917, Oct. 31	15 05	0.70							
5287	1916, Dec. 4	17 15	0.899	1.010 (4)	1.010 (4)	−173	−37	+ 9.8	−3.0	
6376	1917, Nov. 25	14 51	1.03							
6377	1917, Nov. 25	15 19	1.05							
6378	1917, Nov. 25	15 48	1.06							
4854	1916, May 21	23 27	1.417	1.434 (5)	1.426 (3)	− 25	−24	+ 0.1	+5.0	
5472	1917, Jan. 30	15 04	1.448							
6229	1917, Sept. 30	16 41	1.463							
6230	1917, Sept. 30	17 00	1.476							
6281	1917, Oct. 7	15 28	1.367							
4305	1915, Aug. 18	0 00	1.718	1.699 (3)	1.699 (3)	+ 81	−24	− 7.7	−3.1	
6237	1917, Sept. 30	21 51	1.678							
6238	1917, Sept. 30	22 23	1.700							
6291	1917, Oct. 25	18 22	1.88	1.885 (2)	1.885 (2)	+140	−26	−10.9	−8.1	
6292	1917, Oct. 25	18 42	1.89							
4916	1916, July 10	23 13	2.092	.....	.....	.....	.....	.....	.....	Process Plate
5461	1917, Jan. 13	14 47	2.050	2.057 (5)	2.072 (3)	+194	−18	+10.8	−1.7	
5462	1917, Jan. 13	15 52	2.094							
6296	1917, Oct. 25	21 45	2.02							
6297	1917, Oct. 25	22 02	2.103							
5016	1916, Aug. 18	20 51	2.247	2.246 (2)	.....	+189	.....	+ 3.1	.....	40″ Camera
5029	1916, Sept. 9	0 03	2.245							
3950	1914, Dec. 25	16 10	2.399	2.382 (2)	2.399 (1)	+153	−13	−15.3	+2.5	
5018	1916, Aug. 18	23 40	2.364	.....	.....	.....	.....	.....	.....	40″ Camera
4922	1916, July 11	23 16	3.094	.....	.....	.....	.....	.....	.....	Seed 23 Plate
				3.164 (2)	3.094 (1)	− 80	−14	− 2.4	+2.0	
5034	1916, Sept. 9	23 48	3.233	.....	.....	.....	.....	.....	.....	40″ Camera
5338	1916, Dec. 10	18 03	3.410	3.486 (3)	3.486 (3)	−164	−27	+10.1	−3.2	
5452	1917, Jan. 11	15 09	3.586							
5493	1917, Feb. 1	15 25	3.462							

Except as noted in the table a dispersion of one prism and a camera of 18 inches (45.7 cm) were used in the spectrograph. The plates with but two exceptions were Seed Gilt Edge 27. With this combination the exposure times averaged from 15 to 25 minutes. This relatively short exposure is advantageous in view of the very rapid change of velocity in some portions of the velocity-curve.

The lines measured upon the photographs and the wave-lengths employed are as follows:

Ca	3933.825	He	4388.100
Ca	3968.625	He	4471.646
He	4026.342	Mg	4481.400
H	4101.900	Fe	4549.642
He	4143.919	Hβ	4861.527
Hγ	4340.634	He	4922.096

The spectrum of the iron arc was used for comparison purposes on all of the spectrograms.

THE ORBIT AS DERIVED FROM THE HYDROGEN AND HELIUM LINES

The well-known graphical method of Lehmann-Filhés was used for the derivation of the approximate elements of the system. These elements were then corrected by means of differential formulae, a least-squares solution being used and the eleven normal values of Table I being assigned weights according to the number of plates involved. A convenient summary of the formulae to be employed is given by Plummer in his article on the determination of the orbits of spectroscopic binaries.[1] The resulting definitive elements are as follows, the letters having their usual significance. The errors given are probable errors.

$$U = 3.5225 \text{ days (assumed)}$$
$$K = 217.4 \pm 3.4 \text{ km}$$
$$\omega = 323^{\circ}.0 \pm 10^{\circ}.2$$
$$e = 0.094 \pm 0.020$$
$$T = 1917, \text{ January } 2.865 \pm 0.098 \text{ G.M.T.}$$
$$\gamma = -44.9 \pm 5.5 \text{ km}$$
$$a \sin i = 10{,}480{,}000 \text{ km}$$
$$\frac{m_1^3}{(m+m_1)^2} \sin^3 i = 3.71 \text{ sun's mass}$$

[1] *Astrophysical Journal*, **28**, 212, 1908.

The differences between the observed values of the radial velocity and those computed from these elements are given under O−C in Table I. The average deviation is 7.4 km. A graphical representation of the velocity-curve and of the observed velocities is given in Fig. 1.

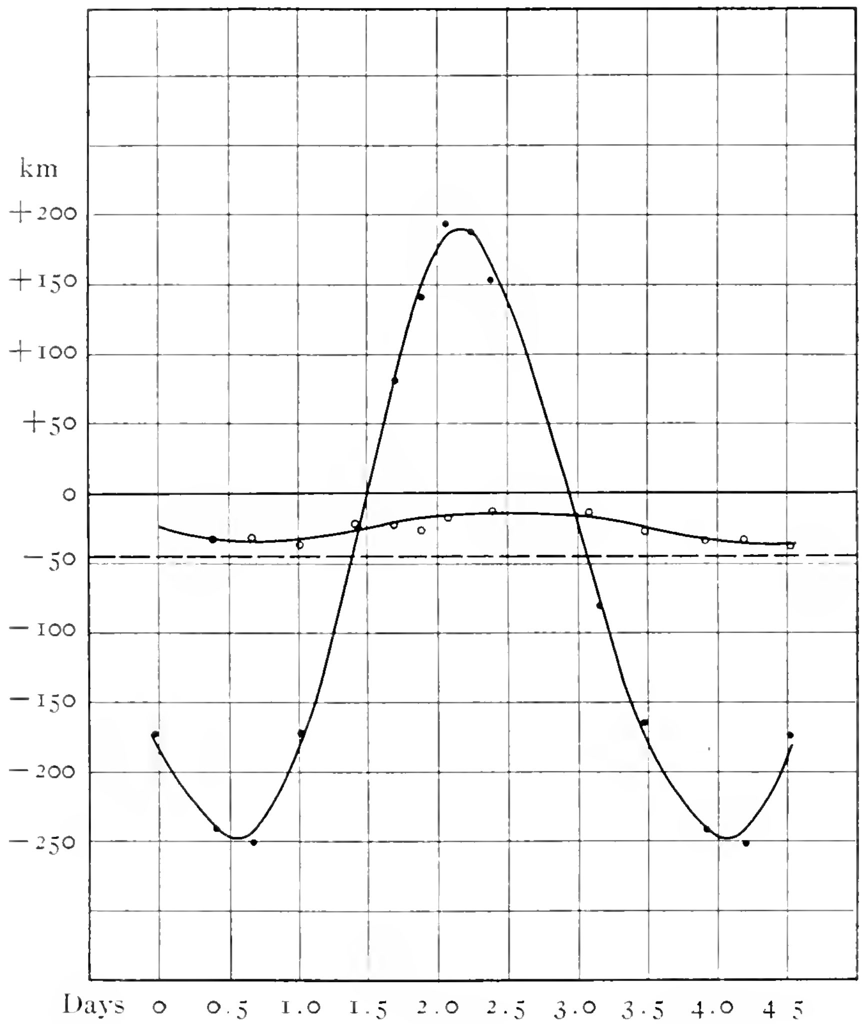

FIG. 1

The most convenient form for the calculation of the radial velocity at any phase of the period, and hence for the comparison of future observations with these results, is by means of a Fourier's series connecting velocity with phase. If we put $\tau = \frac{2\pi}{U}(t-t_0)$ we may represent the velocity V by the equation

$$V = a_0 + a_1 \cos \tau + a_2 \cos 2\tau + \ldots .$$
$$+ b_1 \sin \tau + b_2 \sin 2\tau + \ldots .$$

The epoch t_0 has been taken as 1917, January 1.0. The coefficients in this series have been determined in the usual way with the following result:

$$V = -49.3 - 145.3 \cos\tau + 18.2 \cos 2\tau + 10.0 \cos 3\tau - 5.2 \cos 4\tau$$
$$- 147.0 \sin\tau + 18.2 \sin 2\tau - 7.3 \sin 3\tau + 6.1 \sin 4\tau$$

The agreement of the radial velocities computed from this formula with those observed is considerably better than that of the velocities calculated from the elements, the average deviation being 3.0 km instead of 7.4. This result is due to the use of nine constants as compared with the five constants of the orbit.

THE CALCIUM LINES

An examination of the radial velocities obtained from the calcium lines seems to give distinct evidence of a small variation with the same period as that for the hydrogen lines within the limits of error of the observations. If we apply to these results the method of representation by a Fourier series, we obtain the following equation:

$$V = -23.5 - 1.88 \cos\tau + 1.29 \cos 2\tau - 0.58 \cos 3\tau$$
$$- 10.41 \sin\tau - 0.94 \sin 2\tau - 0.42 \sin 3\tau$$

The velocities calculated from this formula when compared with the observed values give the residuals shown in Table I. The average deviation is 3.3 km. The velocity-curve for the calcium lines is shown in Fig. 1, the observations being represented by small circles. The motion of the system is -23.5 km.

THE SPECTRUM OF THE SECOND COMPONENT

Several of the photographs show evidences of the presence of the second component of the system, and on a very few plates we have been able to secure measurements of velocity. They are, however, too few in number to be used in the calculation of the orbit. The secondary spectrum appears to be of nearly the same type as the primary, and the lines are seen most clearly when the principal star shows its maximum positive velocity.

DISCUSSION OF THE RESULTS

The principal features of interest in connection with these results are as follows:

1. The variation of velocity shown by the hydrogen and helium lines is remarkably large, amounting to nearly 450 km.

2. The calcium lines indicate a variation in velocity of about 20 km with the same period as that shown by the hydrogen and helium lines.

3. The motion of the system as derived from the two sets of lines differs by about 20 km.

It is evident that the variation of velocity indicated by the calcium lines favors the view that the calcium gas producing these lines is connected with the binary system and is not a detached cloud in space. This is in accordance with the observations of Jordan,[1] Lee,[2] and others on spectroscopic binaries of similar character. On the other hand, the difference in the motion of the system as derived from the hydrogen and the calcium lines would seem to indicate an independent source of origin for the latter in accordance with the original suggestion of Hartmann.[3] It seems to us probable that the calcium gas is actually connected with the stellar system, and that the difference in the apparent motion of the system is not to be interpreted wholly on the basis of velocity. A systematic displacement of the calcium lines relative to the other lines in the spectrum might readily come about from a marked difference in the distribution of the gases around the stars. Whether it could amount to as much as 20 km is, however, doubtful. A very probable source of difference may be the presence of the spectrum of the secondary component. As we have already stated, this is seen most clearly at the time of maximum positive velocity of the principal component. At other times it would blend with the stronger spectrum and thus introduce a possible source of systematic error into the measured displacements.

An important result which may have a direct bearing upon this question was obtained by Beal at the Allegheny Observatory in

[1] *Publications of the Allegheny Observatory*, **2**, 63, 1910.

[2] *Astrophysical Journal*, **37**, 1, 1913. [3] *Astrophysical Journal*, **19**, 268, 1904.

1915.[1] A series of observations upon the spectroscopic binary η Orionis, which belongs to the class with abnormal calcium lines, showed that the motion of the system had undergone a change of 25 km in the interval between 1902 and 1915. Whether this result is interpreted as the effect of the presence of a third body, or as a change due to variations in the physical and mechanical conditions in the system, it is clear that a factor is introduced which may go far toward explaining the results found in the case of Boss 46.

MOUNT WILSON SOLAR OBSERVATORY
March 1918

[1] *Report of the Eighteenth Meeting of the American Astronomical Society.*

REVIEWS

Table of Log_{10} sec^2 θ for Determining Photographic Densities by Means of Nicol Prisms. Computed by PAUL S. HELMICK. University of Iowa Monographs, No. 4. Iowa City: The University, 1917. Pp. 14.

This table is intended for the use of those desiring to measure photographic densities with apparatus employing nicol prisms in its construction. The table gives the photographic density of the plate, i.e.,

$$\log_{10} \frac{\text{Incident light}}{\text{Transmitted light}}$$

directly in terms of the angle of rotation of the nicols. The value of the function is given for every $0\overset{\circ}{.}5$ from 0° to 98° and for every $0\overset{\circ}{.}01$ from 89° to 90°, together with tabular differences.

The table may be obtained free by addressing the Librarian of the State University, Iowa City, Iowa.

This table will doubtless be found useful for workers in this field, although a reviewer can easily find points of criticism. The table could be issued in a more compact form, and the densities rounded off to 0.001, as the third decimal is quite uncertain in actual measures of photographs. Certain improvements in the typographical appearance could be suggested, but will not be necessary for persons making regular use of the table.

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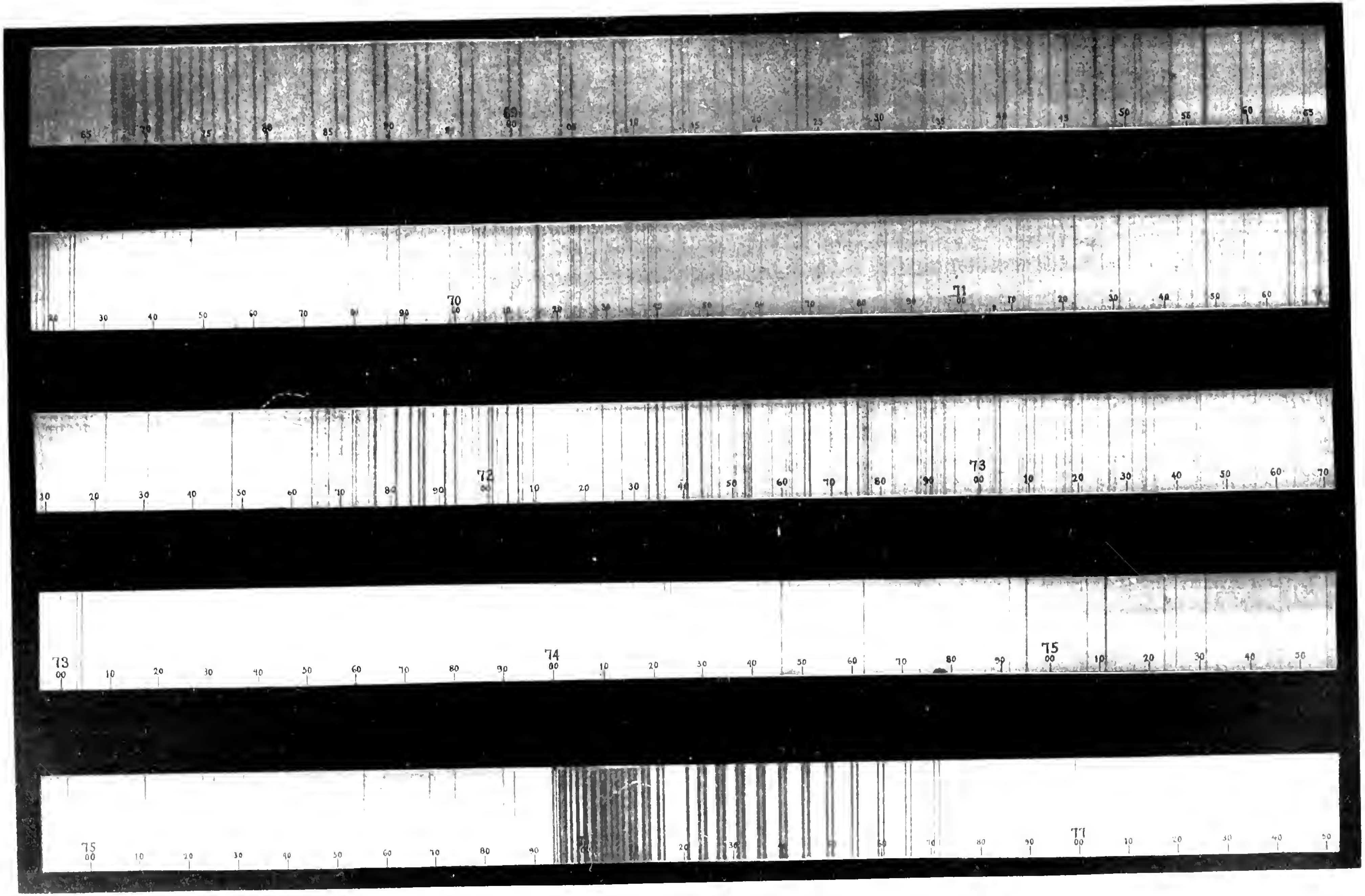
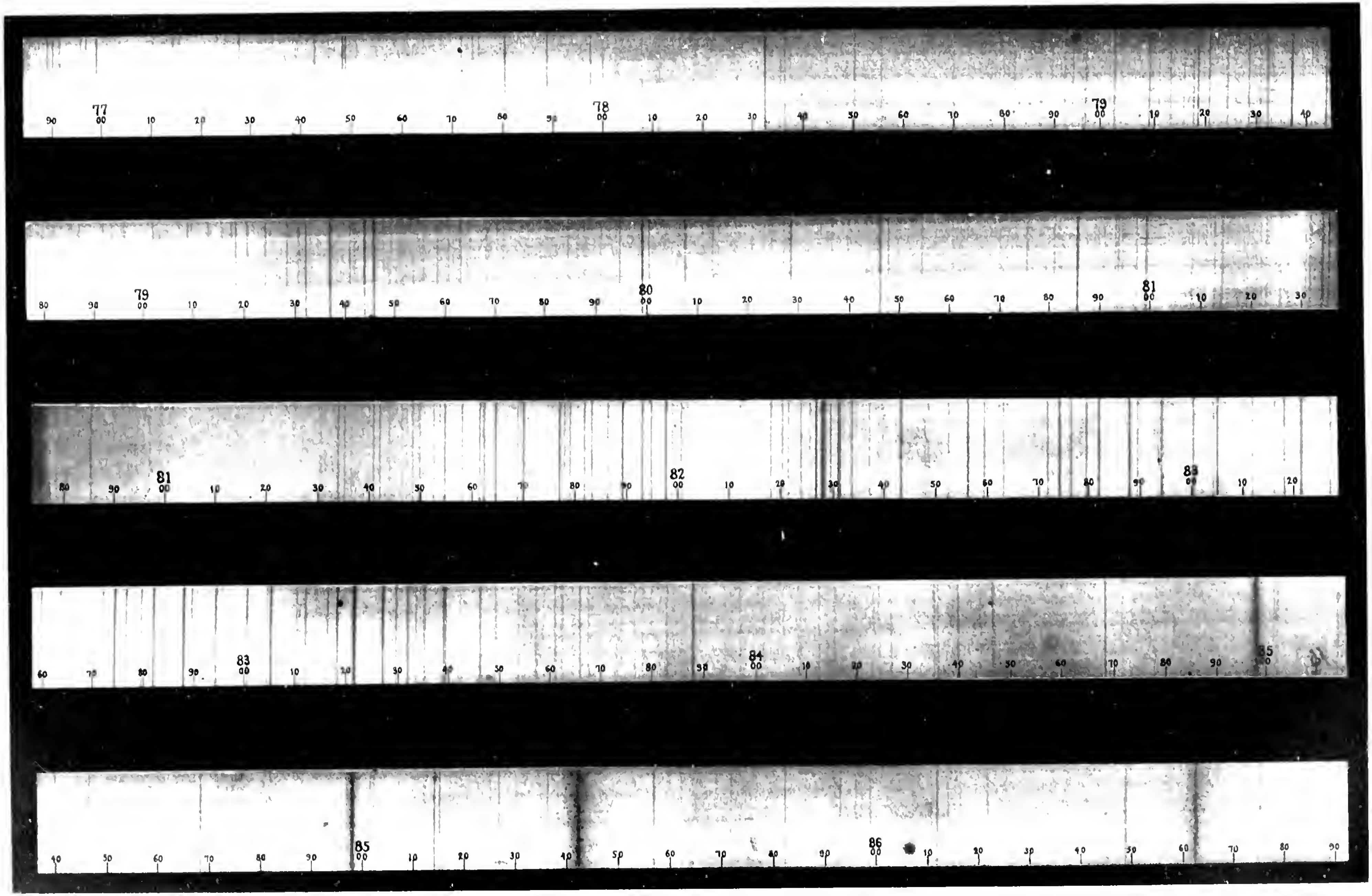


THE INFRA RED SOLAR SPECTRUM, λ6860–7750 A



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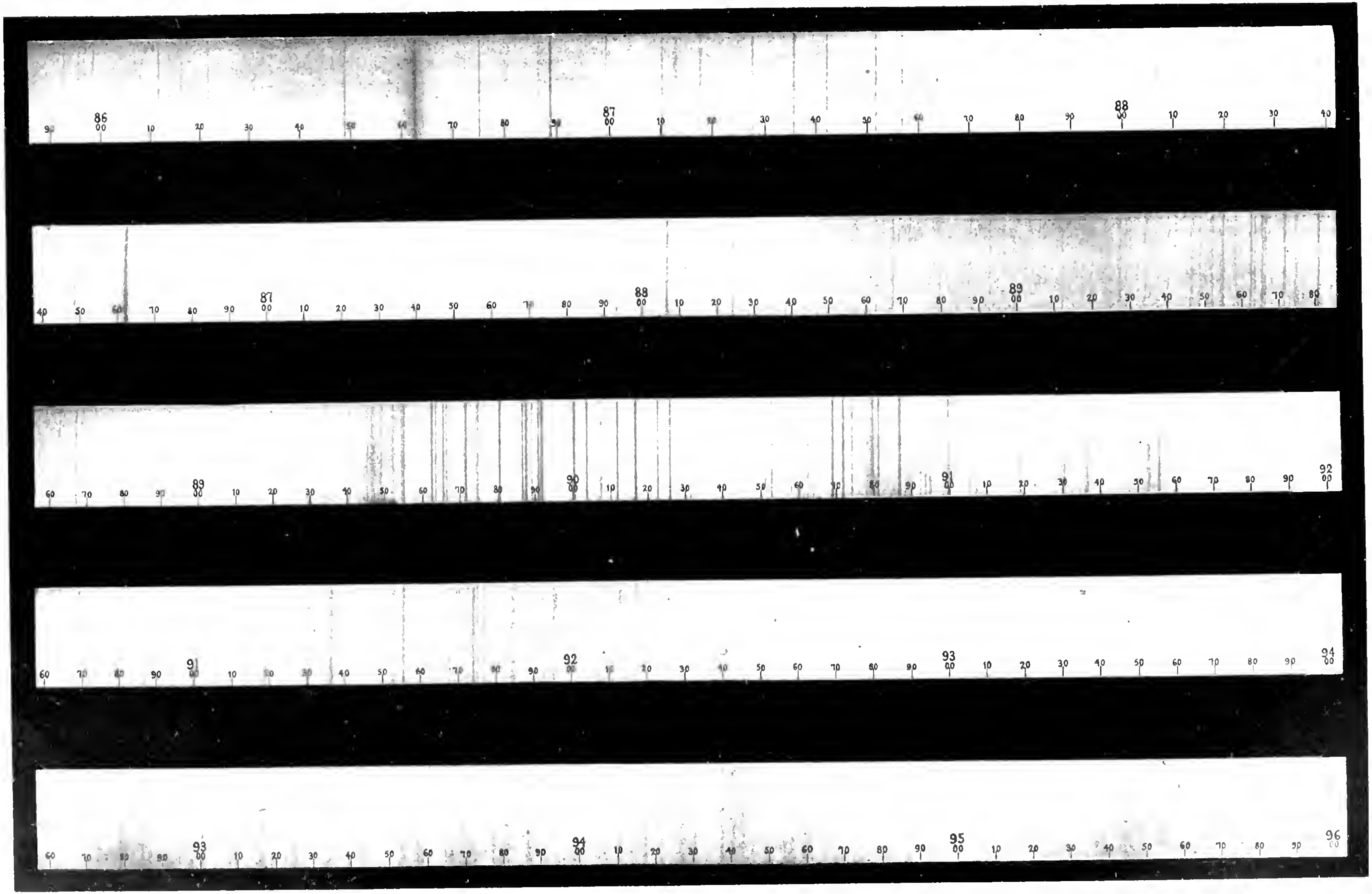

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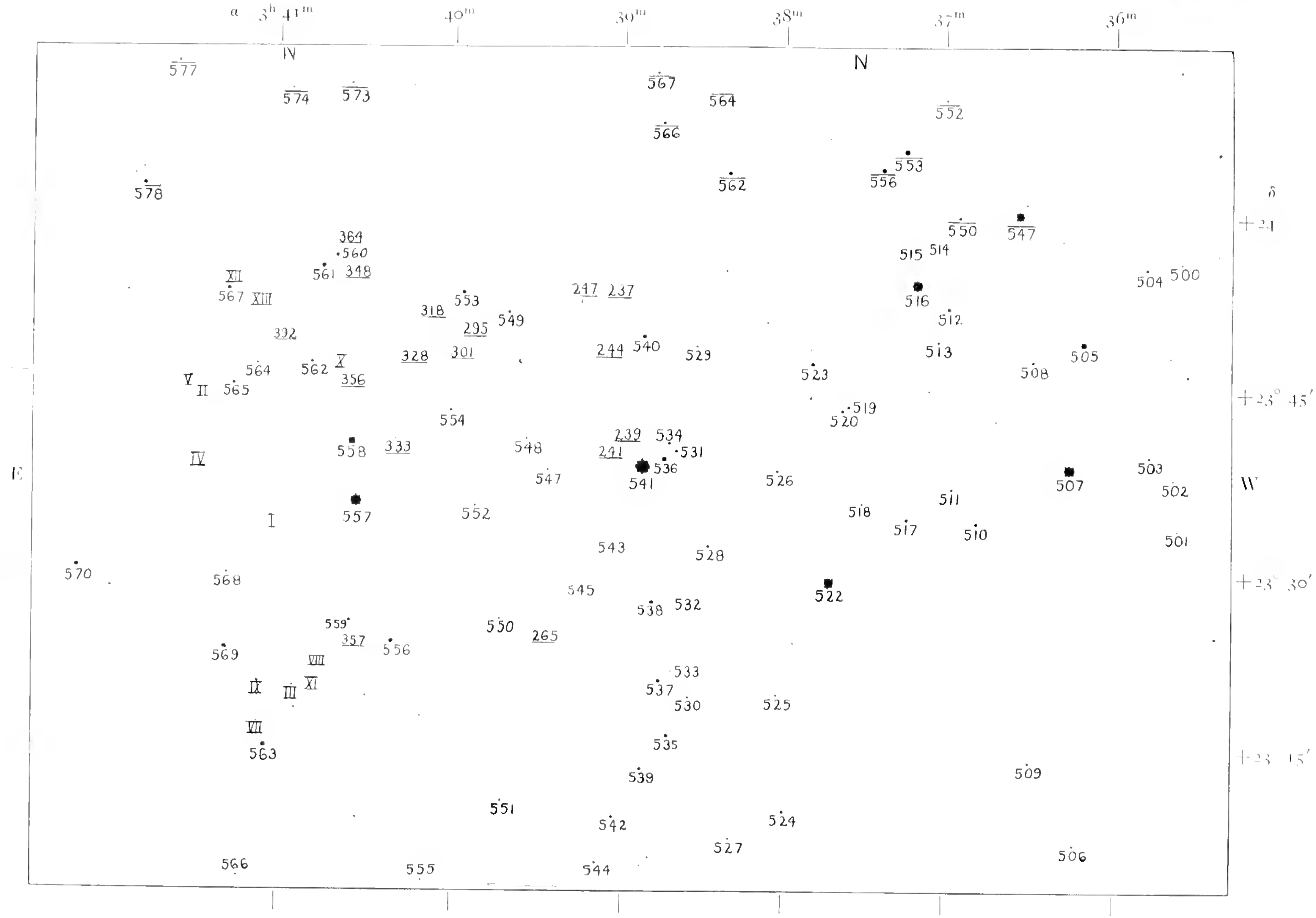


Photographic Chart of the Pleiades, Taken with 2-Foot Reflector

PLATE V

Fig. 1.—Structure of Bi $\lambda 4722$

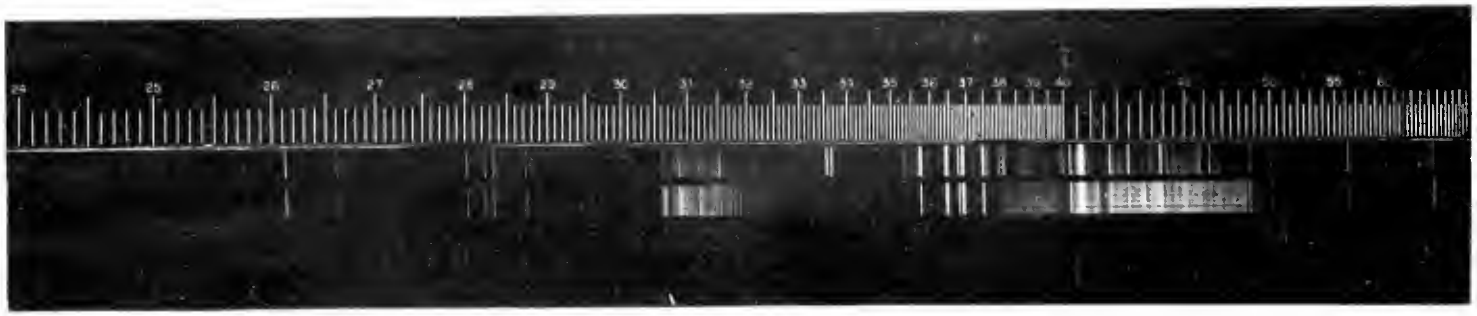


Fig. 2.—Spectrum of Lead (*a*) and Radio-Lead (*b*)

PLATE VI

Fig. 6

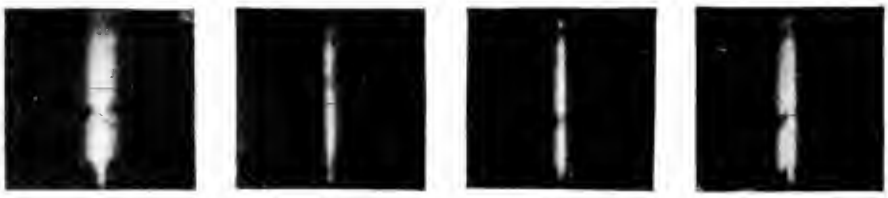
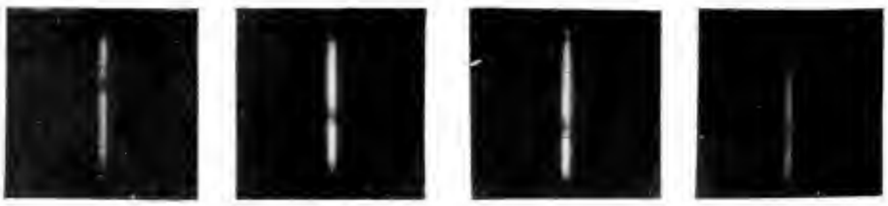
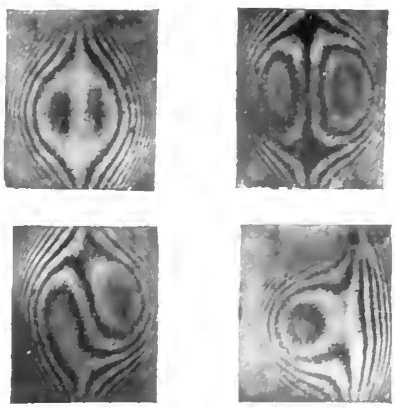


Fig. 7



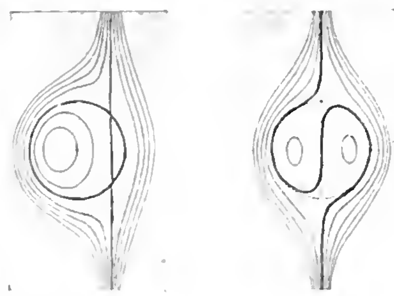
c, *d*

e, *f*



e *f*

Fig. 10



a *b*

Fig. 11

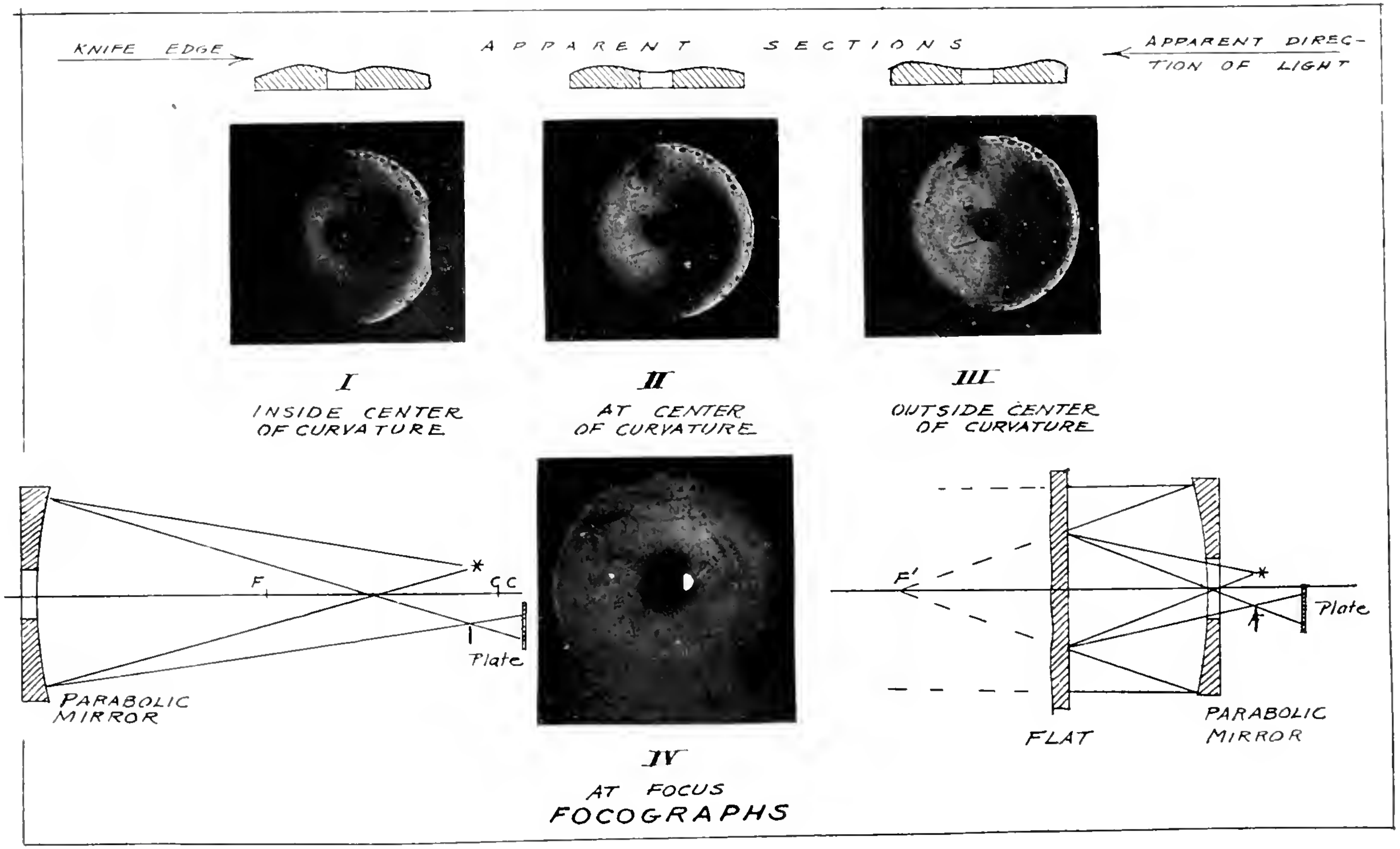
The Correction of Optical Surfaces
(*f* corresponds with *a*; *e* with *b*)

PLATE VII

Focographs Near Center of Curvature

Taken from positions from 0.15 inch within to 0.15 inch outside the mean center

PLATE VIII



FOCOGRAPHS